An In-Depth Analysis of Demographic Shifts and their Effects on HIV/AIDS Prevalence in Eastern African Countries

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ABSTRACT

Eastern African countries rank in the top ten most highly infected nations globally. To shed light on this, therefore this study investigated the impact of population dynamics on Human Immunodeficiency Virus (HIV) prevalence using a two-stage residual inclusion (2SRI) regression model to address potential endogeneity. The Hausman Test confirmed endogeneity among the variables, justifying the 2SRI model's application. Key findings indicate that population growth rate, fertility rate, and the population aged 15-64 significantly and positively affect Human Immunodeficiency Virus (HIV) prevalence. The instrumental variable, Children ages 0 to 14 young individual infected by HIV, 15 to 24 newly affected by Human Immunodeficiency Virus (HIV), and Children 0 to 14 living with Human Immunodeficiency Virus (HIV), was validated for strength, demonstrating robustness against weak instrument issues. Results revealed that a 1% increase in population growth rate leads to a 53% increase in Human Immunodeficiency Virus (HIV) prevalence. Additionally, the Dumitrescu & Hurlin Granger non-causality test established that population growth rate Granger causes Human Immunodeficiency Virus (HIV) prevalence. These findings underscore the critical role of demographic factors in shaping Human Immunodeficiency Virus (HIV) prevalence and highlight the necessity for targeted public health interventions to manage and reduce Human Immunodeficiency Virus (HIV) prevalence effectively.

Keywords: Control Function Approach, Demographic Shift, Eastern African Countries, HIV Prevalence, Two-Stage Residual Inclusion (2SRI)

I. INTRODUCTION

HIV and AIDS continue to be a significance major global health concern despite tremendous efforts made in both international and local efforts to combat the pandemic. At a UN Special General Assembly in 2001, it was voted to recognize HIV/AIDS as a global public health emergency, increase international action against the pandemic, and mobilize resources (Govender et al., 2021). An international effort was launched by the Millennium Development Goal 6 of the Millennium Declaration of 2000 to address the HIV/AIDS epidemic. Two decades and US$ 109.8 billion in donor support later, it is time for the international community to reassess regional success.

The frequency of AIDS-related deaths has dropped because of antiretroviral medication (ART); nonetheless, not everyone has access to treatment, and there is still uncertainty around the availability of effective vaccines and curative treatments. Programs for awareness and prevention have been suggested to be a more practical strategy (Zhao et al., 2018)(Zhao et al., 2018). Sadly, these promises were not kept because of the high prevalence of HIV and the limited accessibility of antiretroviral drugs among key communities. The multifactorial spread of HIV is significantly influenced by social factors. HIV testing had to adhere to ethical standards protecting patient confidentiality; this, coupled with stigma and discrimination, may have concealed the illness in many communities and may continue to do so. Developing countries were ill-equipped to handle the additional burden that HIV/AIDS imposed on their already fragile healthcare systems, particularly those in Africa (Bogart et al., 2021).

There were other factors at play when the outbreak turned into a pandemic. Perhaps because of unresolved systemic concerns, preventive interventions like the use of condoms, stopping mother-to-child transmission, voluntary male medical circumcision, and community awareness campaigns have not been as effective as expected (Elias & Tchuenche, 2012). Since more People Living with HIV (PLHIV) were receiving antiretroviral medication, a decline in

incidence and death was expected. It was anticipated that preexposure prophylaxis, and viral suppression which gave rise to the theory that "undetectable equals untransmutable" and prenatal regimens would further inhibit viral transmission. HIV infections differ geographically and even among nations. The attempts to contain the HIV pandemic are significantly impacted by these disparities in HIV incidence. (PEPFAR, 2023)

Between 2017 and 2050, it is predicted that the East African region will rank among the ten with the greatest population growth. This will affect regional mortality, fertility, and life expectancy (Kitole et al., 2023: Tile et al., 2023). The EAC is the region with the highest number of HIV/AIDS victims, according to the UN, accounting for 45% of all HIV infections and 53% of all HIV-positive individuals worldwide, despite the region's rapid population transition (Govender et al., 2021). Of these victims, 35 million are adults and 1.7 million are children under the age of fifteen.

The study objective was to examine how population dynamics affect HIV/AIDS prevalence in eastern African nations is the goal of this research.

II. LITERATURE REVIEW

2.1 Empirical Reviews

The daily residency status, HIV status, and HIV care status of every one were determined by (Larmarange et al., 2018) using data from clinic visits, ART prescriptions, viral load and CD4 count, migration, and deaths. The study focused on the influence of population dynamics on the population's HIV care cascade. The study was carried out in KwaZulu-Natal, a rural region of South Africa. Position was determined using scores ranging from 0 (undiagnosed) to 4 (at the top of the HIV treatment continuum). (Suppressed by viruses). Every adult resident living with HIV who entered or left the community had their cascade score compared to the average score of their cluster at the time of entry or departure. Then, determine the annualized overall contribution, broken down by component of change, of each entry and exit to the average cascade score. The research discovered that population dynamics limited the average cascade score's growth over time in all clusters.

Furthermore, based on national HIV estimate data, (Khalifa et al., 2019) study on the Demographic Change and HIV pandemic Projections to 2050 for Adolescents and Young People Aged 15-24, 148 projects that the HIV pandemic would peak in 2050. By sex and five-year age group, the numbers of HIV-positive individuals and new HIV infections were predicted. Projections were made based on three main hypotheses: future patterns in HIV incidence, antiretroviral therapy coverage, and antiretroviral coverage for preventing mother-to-child transmission. The results cover nine different geographical areas. According to the study, between 2010 and 2050, there will be 10% more adolescents and young people, but there will be 61% fewer individuals with HIV. It is anticipated that between 2010 and 2050, new HIV infections among adolescents and young people will drop by 84% in Eastern and Southern Africa, which has the worst HIV epidemic in the entire globe. In West and Central Africa, which is where the second-largest HIV pandemic is located, fewer new cases are anticipated to occur by 35%.

According to (Nsuami & Witbooi, 2018) research, pre-exposure prophylaxis and ARV therapy are included in a model of HIV/AIDS population dynamics. The management of HIV infection has lately been accomplished with the use of oral pre-exposure prophylaxis (PrEP) and antiretroviral therapy (ART). Utilizing an antiretroviral drug is what pre-exposure prevention entails to shield those who are HIV-negative from contracting the disease. A new HIV/AIDS transmission model, incorporating ART and PrEP, is proposed in the research. Through simulations, we show that our model can be used to evaluate the effects of ART and PrEP uptake in each population. Future HIV prevalence estimates can also be calculated using the model. The research establishes the global stability of the disease-free equilibrium as well as the global stability of the endemic equilibrium in the model's most general instance, which permits PrEP users to default. We also provide thought-provoking simulations based on newly released South African data.

Bloom & Luca, (2021) researched the dynamics and evidence of the HIV epidemic's effects on population and household structure. She used earlier examples of this impact to predict future changes using demographic theory and reviewed more recent evidence to look for signs of these changes. According to the research, there are only a few age groups where slight increases in the male-to-female ratio are starting to become noticeable (about 15% among 25–34-year-olds). In some age pyramids, similar-sized increases in the percentage of people aged 15 to 29 compared to people aged 30 to 54 are seen. These "youth bulges" are anticipated to disappear, while an ageing effect gradually sets in because the epidemic influences fertility. Over time, the size of all age groups will decrease, but middle-aged adults will experience this change comparatively less. In the most affected nations, the percentages of widows and orphans have grown. For widows, fewer remarriage odds were seen.

Mkwashapi et al., (2023)conducted a study on how the HIV epidemic has affected fertility in sub-Saharan Africa. The information for sub-Saharan Africa was gathered from Demographic and Health Surveys, published

studies, and individual correspondence. The effect of the HIV/AIDS pandemic on the number of births in Uganda was shown using a mathematical model. utilizing a research methodology that involves data review and analysis from the numerous individual studies that have looked at the relationships among HIV/AIDS and fertility. Research revealed that fertility was lower among HIV-infected women than HIV-uninfected women, with the exception of those between the ages of 15 and 19, when the selective pressure of sexual debut on pregnancy and HIV infection led to greater fertility rates among the HIV-infected. Because of this fertility disparity, there was a 0.37% (95% confidence interval 0.30%, 0.44%) decline in total fertility that could be attributed to the community for every 1% increase in HIV prevalence.

2.2 Theoretical Foundation

2.2.1 Epidemiological Theory

A comprehension of the relationship between infectious disease epidemiology and demography, it is important to consider the kind and calibre of the evidence at our disposal as well as the steps we take to go from anecdote to general guidelines and from hypothesis to theory (Paz & Dujardin, 2005). We can make educated guesses about how changes in host population structure may have impacted the epidemiology of certain illnesses based on a thorough understanding of the natural history and transmissibility of certain infections as shown by observational studies (Silk et al., 2019). Examples of coincident changes are provided by historical and archaeological records of population size and organization, as well as patterns of disease and mortality. These include the introduction of new pathogens during the formation of civilizations and the decline in disease linked to better living conditions and hygiene (Manego et al., 2017)

Examples are used to illustrate general concepts in the study of the relationship between infections and demographics, as well as the opposite. However, since infections like HIV, malaria, TB, influenza, and bubonic plague dominate the relationship between diseases and demography, these cases are seldom typical (Manego et al., 2017). Therefore, by looking at patterns and factors of health and illness within communities, epidemiological theory seeks to understand how diseases spread and why some groups are more vulnerable than others.

2.2.2 Theoretical Model

You can adapt the traditional SIR model to incorporate birth and death processes when thinking about a scenario in which the population is not constant. As a result, the SIRS model is created, which has terms for the birth rate (μ) and death rate (δ). The SIRS model equations are as follows.

This equation shows the rate of change of the number of vulnerable individuals (v) over time (t). It is a function of the transmission rate (β), the number of people who have been disposed of (v), and the number of people who have spread the disease to others (d), divided by the total population (N). In this case, the rate at which people are born into the exposed class is proportionate to the size of the population at that time.

$$\frac{\partial V}{\partial t} = \mu N - \beta \frac{V}{n} - aV \quad \text{..... (1)}$$

The rate of change of the number of diseased persons (HIV) with time (t) is represented by this equation. Individuals who transfer from the exposed class to the diseased class are represented by the first part of the equation, while those who recover from the illness are represented by the second part of the equation. The recovery rate is expressed as y.

$$\frac{\partial HIV}{\partial t} = \beta \frac{HIV}{n} - yHIV - aHIV \quad \text{..... (2)}$$

III. METHODOLOGY

The study employed a non-experimental research approach by Examine the recorded data sets from the World Data Bank for each nation between 1990 and 2021. The United Nations Population Division, worldwide population forecasts, census reports, statistical materials from the National Statistics Office, demographic statistics, and population data were some of the sources from which these global numbers were compiled.

This method is especially helpful in situations where conducting experiments would be immoral or impossible, like when examining the dynamics of HIV commonness in the overall population. The non-experimental research design of this study contributes to a realistic portrayal of how population dynamics affect HIV prevalence. Utilizing World Bank data was a cost-effective, reliable, and appropriate choice for the study, especially when the dataset was gathered by reputable organizations (Johnston, 2014; Tile et al., 2024). The Eastern African countries

Tanzania, Uganda, Kenya, Burundi, and Rwanda were chosen for the unity of analysis, and the availability of a large and representative sample of countries also enhanced the learning's peripheral legitimacy.

This implies that the study's findings might apply to comparable circumstances in other Sub-Saharan African nations. nevertheless, a few disadvantages of experimental study methods include the inability to modify relevant variables and the lack of control over irrelevant aspects, are lessened by the non-experimental research design.

3.1 Model Estimation

The study uses a two-stage residual inclusion model to address the endogeneity between HIV prevalence and random error, this is because population dynamics have an important effect on HIV prevalence, on the other hand, HIV prevalence has important effects on the population dynamics (Kitole et al., 2022). Therefore, there are feedback effects between population dynamics and HIV prevalence. Consider the model estimation below

$$HIVP_t=\beta_0+\beta_1 PD_t+ \beta_1 HIVP_{t-1}+ \omega_t \quad (3)$$

But population dynamic can be affected by the other factors together with the HIV prevalence and the equation now becomes.

$$PD_t= \beta_0+ \beta_1 HIVP_t+ \beta_2 FR_t+ \beta_3 DR_t+ \beta_4 LER_t+ \upsilon_t \quad (4)$$

Therefore, by substituting equation 33 into equation 32, the equation becomes.

$$HIVP_t=\beta_0 + \beta_0+\beta_1 HIVP_t+\beta_1 HIVP_{t-1}+\beta_2 FR_t+\beta_3 DR_t+\beta_4 LER_t+(\upsilon_t+ \omega_t) \quad (5)$$

Now Let

$K_0 = (\beta_0 + \beta_0)$

$H_1 = (\beta_1 HIVP_t+ \beta_1 HIVP_{t-1})$

$PD_t=(\beta_2 FR_t+ \beta_3 DR_t+ \beta_4 LER_t)$

$\hat{u} = (\upsilon_t+ \omega_t)$

Now the equation becomes.

$$HIVP_t=K_0+ H_1+ PD_t+ \hat{u} \quad (6)$$

Nonetheless, the instrumental variable (IV) and the two-stage residual inclusion (2SRI) have been used to ease these difficulties. According to (Kitole et al., 2022), 2SRI is the most effective method for managing endogeneity in both linear and non-linear models. Two steps are involved in 2SRI. Firstly, residuals from the estimated functions are calculated to a reduced form, and the second-stage regression then includes these residuals as extra explanatory variables.

The control function technique, which helps to manage both endogeneity and heterogeneity, is generally an extension of the two-stage residuals with null hypothesis testing of the exogeneity. It is more commonly referred to as the new stage reduction equation (Kitole et al., 2022) The HIV prevalence, for which Equations (3) and (4) were found to be precisely characterized, was the tool employed to suppress this endogeneity that emerged throughout the estimation process. Nevertheless, considering HIV prevalence and population trends.

The identification of possible endogeneity problems that might have an impact on the estimate of Equations (3) and (4) motivated the use of an instrumental variable model in this investigation. Endogeneity arises when there is a correlation between the variables of interest and the error term in a statistical model (Gammadigbe, 2021). If this endogeneity is not addressed, it may induce bias and inconsistent parameter estimations, which could compromise the validity of the study's conclusions.

The study utilized a robust econometric technique, the two-stage residual inclusion and control function approach regression method, to address the issues posed by endogeneity. This approach was selected because of its exceptional capacity to estimate Equation (3) and (4) simultaneously, thereby resolving the endogeneity and heterogeneity issue (Kitole et al., 2023: Tile et al., 2023)

The key component of this strategy is its ability to instrument for the potentially endogenous variable (HIV prevalence) by using an instrumental variable, in this case, children under 14 newly infected with HIV, young adults 15 to 24 newly infected with HIV, and children 0 to 14 living with HIV. A variable that has a correlation with the endogenous variable of interest but is unaffected directly by the error term is called an instrumental variable. The study aimed to decipher the intricate association between HIV prevalence and prevalence by using this instrumental variable (Kitole et al., 2023: Tile et al., 2024)

HIV prevalence, which is the incidence of HIV/AIDS, was regarded as a latent dependent variable in Equation (7). It is impacted by a collection of explanatory variables X_1 that are thought to have an impact on the prevalence of HIV/AIDS in addition to the latent population dynamics (PD). This equation's disturbance term, (ω), is meant to have a normal distribution and takes into account any unexplained fluctuation. The dependent variable in equation (4) is

population dynamics. It is thought to depend on latent HIV/AIDS prevalence in addition to other external factors represented by the symbols$\beta_2 FR_t$, $\beta_3 DR_t$, $\beta_4 LER_t$.. Population dynamics are influenced by the latent HIV/AIDS variable and these exogenous influences together.

To sum up, Equations (3) and (4) were simultaneously estimated in this work using the instrumental variable model, namely the two-stage residual inclusion regression approach. By incorporating an instrumental variable to separate the relationship between HIV prevalence and population dynamics in the context of East African countries, our strategy addressed the endogeneity dilemma.

Table 1

Variable Measurement

Variables	Measurements	Expected sign
Dependent variables		
HIV prevalence	Total number of people living with HIV	
Independent variables		
Fertility rate	Number of births per thousand population per year	+
Net migration Death rate	Number of deaths per thousand population per year	-
Life expectancy rate	The mean number of years a cohort of people might expect to live	+
Population growth Number of children living with HIV	Total number of children living with HIV	+
Number of adults living with HIV	Total number of adults living with HIV	+
Number of males living with HIV	Total number of males living with HIV	+
Number of females living with HIV	Total number of females living with HIV	+

IV. FINDINGS & DISCUSSIONS

4.1 Response Rates

Stipulated results in Table 2 show that on average East African fertility rate is 5 per cent per year and the minimum fertility recorded in all selected East African countries from 1980 to 2020, moreover, the lowest fertility rate recorded was 0.785 while the maximum was 7.405 per cent per. Also, the results that on average the mortality rate in East African countries was 105.8987 people per thousand between 1980 to 2020 and the minimum amount of the mortality rate recorded was 0.973 people per thousand moreover the maximum record was 341.2 people per thousand this was the highest mortality rate recorded so far.

Table 2 Descriptive Statistics

Variable	Observation	Mean	Minimum	Maximum
Fertility rate	160	5.469575	0.785	7.405
Mortality rate	160	105.8987	0.973	341.2
Net migration	160	-16713.3	-137427	1244966
Life expectancy	160	52.74191	32	69.329
Prevalence of HIV Total	160	5.058125	4	10.1
Children Infected with HIV	160	92836.88	50056	250000
Adults infected with HIV	160	46573.75	25000	180000
Prevalence rate Female	160	3.320625	2	10.4
Prevalence rate Male	160	1.24625	0.5	3.2
Antiretroviral therapy coverage	160	36.34375	20	100

Additionally, net migration on average in all selected East African countries recorded 16713 people per year where there is more outmigration to other countries than in-migration and the minimum amount of net migration record between the mentioned year is 137427 outmigrants peoples per year, and the maximum amount of the net migration shows that there is 1244966 people migrated to East Africa from outside the selected counties. Moreover, Table 2 shows that the life expectancy in East African countries on average is 52 years while the minimum life expectancy rate is 32 years, and the maximum is 69 years. Additionally, HIV/AIDS prevalence is the total number of people living with HIV in a given population on average shows that the total number of people living with HIV/AIDS is 508997% per year, while the minimum is 4% per year, and the maximum is 10% in a year.

Number of children infected with HIV in each population on average shows that the total number of children infected with HIV/AIDS is 92836.88 children, while the minimum is 50056 children, and the maximum number of

children infected with HIV is 250000. Additionally, the number of adults infected with HIV on average results shows that the total number of adults infected with HIV/AIDS is 46573.75 adults, while the minimum is 25000 adults, and the maximum number of adults infected with HIV is 180000. Moreover, the rate of HIV infection to males in a given population on average shows that the rate of males infected with HIV/AIDS in East African countries is 1.2 per cent per year, while the minimum rate of males infected with HIV/AIDS in East African countries is 0.5 per cent per year, and the maximum rate of male infected with HIV/AIDS in East African countries is 3percent per year. Additionally, the rate of HIV infection to females in a given population on average shows that the rate of females infected with HIV/AIDS in East African countries is 3 per cent per year, while the minimum rate of females infected with HIV/AIDS in East African countries is 2 per cent per year, and the maximum rate of female infected with HIV/AIDS in East African countries is 10 per cent per year and the antiviral therapy coverage to peoples living with HIV on average it shows that 36 people per year attend the service while the minimum attendant of people living with HIV is 20 peoples per year and the maximum number of people attending the antiviral therapy was 100 peoples per year.

The findings show the population growth rate in East African countries from the period of 1990 to 2021. The population is still growing in East Africa where the trends in its countries show a horizontal trend with a declining sign where the population is growing at a declining rate as shown that the highest record population growth rate was recorded by Rwanda in 1999s where it recorded the population growth rate of 7 per cent. But also, Rwanda has the highest record population growth rate but it also has the lowest population growth rate where in 1993 it recorded a population growth rate of negative 4 per cent and later its population continued to grow at a higher speed than the other countries.

population growth rate is still in the positive region where shows that the population is still growing at a positive rate, therefore apart from efforts of the global to reduce the population growth rate by introducing several programs in developing countries such as family planning programs but still East Africa records a positive population growth rate, and this will continue for several years to come. This may be attributed to several factors such as an increase in life expectancy in most of the East African countries, Migration from the neighbouring countries, and a decline in the death rate of children which increases the number of live persons per year in the countries Tile et al., (2023)

Since the 1990s, results show that most of the countries started at a low rate during the 1990s and the later number of cases in HIV positive increased until the maximum point in the 2000s which was a mark of turning point to most of the East African countries. Now about all countries in East Africa show a declining trend of HIV prevalence means the total number of people living with HIV has been declining in recent years compared to the previous years of 1990. But the number is increasing at a decreasing rate which means there are still a few people living with HIV it does not mean that the number is declining negatively. This may be attributed to several programs that are introduced to fight against HIV/AIDS in most developing countries such as the provision of free condoms to all areas with a high risk of transmitting HIV such as in markets, bars, schools, and universities. (Yoshikawa et al., 2020)

4.2 Estimation Results

A two-stage residual inclusion regression model was used in this study to examine how population dynamics affect the prevalence of HIV/AIDS in each of the countries under investigation. Concerns about possible endogeneity between the independent variables (factors influencing population dynamics) and the dependent variable (HIV prevalence) are the reason for using this modelling approach. The study used children ages 0 to 14 who were newly infected with HIV and young people ages 15 to 24 who were newly infected as an instrumental variable to address this issue.

Nonetheless, the Hausman Test was used in the study to confirm the existence of endogeneity. The results of this diagnostic evaluation, which are shown in Table 3, showed that at a significance level of 1%, the null hypothesis was categorically rejected. This rejection indicates strong evidence of endogeneity between the variables being studied. As a result, it was decided that the use of a two-stage residual inclusion model would be suitable for determining how these variables affected the dependent variable. This modelling strategy was thought to be beneficial since it provides more accurate estimations of the interrelationships between the variables by successfully addressing the endogeneity issue.

Table 3

Durbin Score and Wu-Hausman Test

Test	Test score	P-values
Durbin (score) chi2 (1)	36.8897	(0.0000)
Wu-Hausman F (1,151)	45.6173	(0.0000)

Creating a suitable tool to explain the structural equation is one of the main obstacles to endogeneity control with the IV approach. As a result, the strongest and most legitimate instruments can be identified by evaluating them. According to academics (Kitole et al., 2022), excellent instruments are exogenous in the model specification and have a high degree of correlation to the endogenous variable without having any causal direct influence on outcome measures. Determining whether an instrument possesses these three characteristics is made easier by choosing to check for validity.

The instruments used in this instance were Children 0 to 14 living with HIV, Young People ages 15 to 24, and Children 0 to 14 newly infected. Its validity and strength needed to be confirmed. A weak instrument test was carried out to determine the instrument's strength in this inquiry; the findings are shown in Table 4. Even at a 5% significance level, the eigenvalue statistic outperformed all other statistics in Table 4, according to the results of the weak instrument test. This result suggests that the selected instrument is robust and has the strength required to be used in the investigation. Therefore, it is clear that the problem of a weak instrument is not present in this investigation. There is no direct impact of the instrument used in our model on the outcome variable. Instead, it only has an indirect effect that is mediated by the treatment variable. This confirms that our instrumental variable approach is valid for assessing our independent factors' causal effects on the dependent variable.

Table 4

Instrument Strength by Eigenvalue Statistic

Minimum eigenvalue statistic = 91.8609							
Critical Values			Number of endogenous regressors:1				
Ho: Instruments are weak			Number of excluded instruments: 1				
					5%	10% 20%	30%
2SLS relative bias					13.91	9.08 6.46	5.39
					10%	15% 20%	25%
2SLS	Size of nominal	5%	Wald	test	22.30	12.83 9.54	7.80
LIML	Size of nominal	5%	Wald	test	6.46	4.36 3.69	3.32

The 2SRI has been utilized to control the endogeneity issue in the models, improving the results and making them more suitable in column 1. Because heterogeneity has been suppressed, the interaction term insignificance explains why heterogeneity is not an issue in this model. While residuals themselves are significant in column 1 of 2SRI and inconsequential in column 2 of the control function approach, the coefficient of multiplicative interaction effects of residual and HIV/AIDS prevalence is insignificant in column 2. As a result, 2SRI will be used for both the estimation and interpretation of the results in this case.

Therefore, the findings in Table 5 show that the population growth rate is significantly and positively affecting the HIV prevalence of the region, such that a one per cent increase in population growth rate in the region per year, increases the total number of people living with HIV by 53 per cent, other factors remain constant. Population growth rate comes from several factors such as net migration, death rate, and fertility rate once the population grows at a certain percentage where initially the population had some number of people living with HIV hence it increases the number of people living with HIV at a current period. This result is not in line with that of Khalifa et al., (2019) who researched projections of the adolescent and young adult population's demographic changes and the HIV epidemic through the year 2050 for those aged 15 to 24 showed that while the proportion of adolescents and Between 2010 and 2050, there will be a 10% increase in young adults overall, whereas the percentage of HIV-positive young people will decrease by 61%. The world's largest HIV epidemic is currently located in Eastern and Southern Africa; projections indicate that between 2010 and 2050, the number of new HIV infections among adolescents and young people in this region will decrease by 84%.

The fertility rate is significant and positively affects the total number of people living with HIV in the region, that is as the number of births women increase per year increases the HIV prevalence by 22 per cent in the region. This may be attributed to the number of new babies born with HIV infections that is mother to child infections also fertility increases the population in a certain area and once the population increases means urbanization which accelerates the rate of new infection in the region hence the total number of people living with HIV increases. This result is in line with that of Khalifa et al., (2019) who conducted a study on Demographic change and HIV epidemic projections for adolescents and young adults aged 15 to 24 using national HIV estimates files. The study projects that between 2010 and 2050, the number of adolescents and young people will increase by 10%, while the number of HIV-positive individuals will decrease by 61%. The largest HIV epidemic in the world is found in Eastern and Southern Africa, where it is predicted that between 2010 and 2050, the number of new HIV infections among adolescents and young adults will drop by 84%.

Population ages 15 to 64 is positive and significantly affects the HIV prevalence in the region, that is as the number of people aged 15 to 64 increases it also increases the number of people living with HIV/AIDS, other factors remain constant. This may be attributed to the fact that the population aged 15 to 64 are in danger of practising HIV/AIDS risk behaviours such as sexual intercourse, drinking alcohol behaviour and other prostitution behaviours. This result is consistent with that of (Swai et al., 2017), who investigated the prevalence of HIV and risk factors for the virus among individuals 50 years of age and older in the Rombo district of Northern Tanzania. The study found that the virus was more common in women (2.1%) than in men (1.3%), with a prevalence of 1.7% (n=10). Only 40% (n=216) of respondents knew anything about HIV prevention, compared to 60% (n=350) who knew about HIV transmission. Additionally, the study supports that of (Mahy et al., 2014) who found that adults 50 years of age and older have HIV after researching Increasing trends in HIV prevalence among people aged 50 and older: evidence from estimations and survey data. Since around 1995, all age groups under 5 have seen an increase in the worldwide HIV prevalence among older people.

Table 5

Effects of Population Dynamics on HIV/AIDS Prevalence

Dependent variable HIV/AIDS prevalence	Estimation Technique 2SRI	Estimation Technique Control function approach
Population growth rate	0.535***	0.536***
	(0.0514)	(0.0532)
Fertility rate	0.221***	0.221***
	(0.0141)	(0.0141)
Population_ages15-64	0.425***	0.425***
	(0.0283)	(0.0283)
Life expectancy	-0.000383	-0.000383
	(.55E-05)	(2.55E-05)
Net migration	9.75E-08	9.70E-08
	(9.23E-08)	(9.25E-08)
Residual	0.136***	0.136***
	(0.0119)	(0.0123)
HIV prevalence * Residual		-5.82E-05
		(0.00057)
Constant	-6.848***	-6.857***
	(0.442)	(0.447)
Observations	**159**	**159**
R-squared	**0.846**	**0.846**

Standard errors in parentheses ***p<0.01, **p<0.05, *p<0.1

4.3 Causality Test

Panel data causation tests are used to investigate the causal connections among the variables in a panel dataset. Hence, examining causality in panel data can offer a significant understanding of the dynamics and correlations across variables in this organized dataset. The purpose of this study was to evaluate if population changes are a cause or effect of HIV prevalence by applying a causality test to find the causal relationships between population growth and HIV/AIDS prevalence (Lopez & Weber, 2017).

Dumitrescu & Hurlin Granger's non-causality test introduced a single lag by default. The test's result in this instance refutes the null hypothesis that Population growth rate does not Granger-cause Prevalence of HIV and accepts the null hypothesis that Population growth rate does Granger cause Prevalence of HIV for at least one-panel var (country ID)(Lopez & Weber, 2017). The values acquired for W (W-bar), Z (Z bar), and (Z-bar tilde) are reported in the output. The P-values for the last two statistics are given using the conventional normal distribution as a basis.

Table 6

Dumitrescu & Hurlin Granger Non-Causality Test

Lag order: 1		
W-bar	28.7197	
Z-bar	43.8287	(p-value = 0.0000)
Z-bar tilde	38.264	(p-value = 0.0000)

H_0: Population growth rate does not Granger-cause Prevalence of HIV.

H_1: Population growth rate does Granger cause Prevalence of HIV for at least one-panel var (country ID).

V. CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

The examination of the relationship between population dynamics and HIV prevalence has ultimately revealed a robust relationship between population growth, fertility rates, and the age distribution of the HIV-positive population, which is defined as those who are 15 to 64 years old. The important policy implications of these findings should be noted by governments and organizations fighting the HIV/AIDS epidemic. The findings showed a positive correlation between the rise in population and the prevalence of HIV. When there is a sharp rise in the population and a strain on healthcare resources, it may be challenging to provide comprehensive HIV prevention and treatment services.

Moreover, an important determinant of HIV prevalence is the age distribution of the population, especially that portion in the 15–64 age group. The high prevalence in this age group may imply a significant risk of transmission. These findings emphasize the need for an all-encompassing approach to HIV prevention and control. In addition to healthcare and treatment, good policies should address broader socioeconomic and demographic concerns. By approaching these issues comprehensively, governments and organizations can work to reduce the prevalence of HIV and ultimately improve public health outcomes.

5.2 Policy implications

Governments must therefore pass legislation promoting family planning, access to contraception, and sexual education to effectively control population increase and reduce the burden on healthcare systems. Furthermore, as indicated by the positive relationship between HIV prevalence and reproduction rates, thorough sexual and reproductive health education is crucial. Legislators should provide the public with the knowledge and resources needed to make informed decisions about family planning, including HIV prevention and safe sex practices. In this regard, it is essential to have access to affordable, reliable contraception as well as healthcare services.

Public health policies and programs should be developed with this age group in mind, offering easy access to diagnosis and treatment in addition to information and awareness-raising. Aside from drug abuse, gender inequality, and other factors that contribute to HIV transmission in this community, interventions should also focus on other socioeconomic determinants of health.

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Analysis of the Impact of Crude Oil Price Changes on Economic Growth in Tanzania: ARDL Econometric Model

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ABSTRACT

Crude oil is a valuable resource for boosting global economic growth. In this regard, this study examined the effect of crude oil price fluctuations on Tanzania's economic growth from 1989 to 2022. Empirically this study utilized time-series data extracted from World Bank in particular, GDP per capital from World Development Indicators (WDI) as proxy of economic growth and Statistical Review of World Energy for Crude Oil prices. In this study, Renaissance growth theory, Autoregressive distributed lag model (ARDL), Dickey and Fuller tests, The Johansen test for cointegration, Breusch- Godfrey test for Serial correlation LM, Breusch-Pagan heteroscedasticity test, Jarque-Bera normality test, Error Correction Model (ECM), Granger Causality test, CUSUM and CUSUMSQ curves tests were used to analyse the data. This study revealed that crude oil price changes have positive and significant impact on Tanzania's economic growth. A dollar increase in crude oil prices brings about 0.232791unit increase Tanzania's economic upturn in the short run. The ARDL results also shows that error correction model (ECM) of -0.1000 (p= 0.0025) is significant with its value, suggesting a moderate speed of convergence to equilibrium after shock. Thus, this study recommends to policy maker and the government to ensure that they use fiscal policies that will reduce the adverse impact of the world crude oil prices hikes, finding other sources of energy and promoting research and development to explore and harness the oil fossils to produce oil and reduce oil importations and increase oil exportations to further increase economic growth.

Keywords: ARDL, Crude Oil Price, Economic Growth, Tanzania

I. INTRODUCTION

Crude oil is an essential commodity for the growth of an economy because of its substantial use as a source of energy in numerous production and operation processes (Muthalib et al., 2019). This influences the world demand for crude oil (Adam et al., 2021). For instance, world's oil consumption has increased from 84.56 million barrels every day 2016 to 100.02 million barrels every day in 2019 (Rumbia & Azis, 2020). According to Rumbia & Azis (2020), demand for crude oil is expected to increase from 103.78 million barrels every day by 2024 to 109.67 million barrels every day by 2035. In this regard, demand for crude oil is increasing while the supply remains short. This causes a rise in price of crude oil approximately $86.04 per barrel in 2024 succeeding Covid-19 pandemic and Russia-Ukraine conflict, resulting from the embargoes imposed against Russia by European Union (EU).This unpredictable tendency of changes in crude oil prices is anticipated to rise further over the next years (African Energy Chamber, 2023).

In Africa, the main crude oil-producing countries includes Nigeria, Libya, Algeria, and Angola. However, they produce a small percentage of the world's crude oil, which cannot accommodate Africa's oil demand (African Energy Chamber, 2023). Following the Covid-19 outbreaks and trade embargoes imposed by the European Union (EU) against Russia after the Ukraine-Russia conflict, the world crude oil supply was reduced since Russia produces approximately one-third of the total oil production. The increase in crude oil prices has also impacted African countries, particularly those in Eastern Africa that import oil from the rest of the world, leading to inflationary pressures and increased transportation costs. Consequently, investors and producers raise the prices of commodities to cover increased production costs, leading to decreased investments and lower net exports, which subsequently affects the economic growth of these countries (African Energy Chamber, 2023).

In Tanzania on average approximately 35,000 barrels of crude oil are demanded every day from which all are imported (Gasper & Mbwambo, 2023). This cultivates an increase in crude oil prices as publicized by Energy and Water Utilities Regulatory Authority (EWURA) of Tanzania from time to time. This is anticipated by increased tariff rates and transportation costs which subsequently leads to high inflationary pressures, high rate of unemployment, lack of confidence on saving and investments leading to low production and hence reducing overall economic growth (Gasper & Mbwambo, 2023).

High oil prices have significant benefits to OPEC as it increases their trade values which are made with large prices, hence boosting their profit margins which increase economic prosperity as determined by GDP(Gross Domestic Product) (Dinh, 2022). Additionally, OPEC earns foreign reserves and revenues which increases domestic productivity necessary for participating in international trade and spurs their GDPs (Nwanna & Eyedayi, 2016).

However, the effects of increasing crude oil prices has significant cost for oil importing countries all over the world, since it manipulates the prices of commodities through increase of cost inputs, transportations, increasing the consumption spending, reducing level of investments ultimately affects the economic growth of the countries adversely (Muthalib et al., 2019).

Changes in the price of fossil fuels have a significant effect on both supply and demand sides of all sectors of the economy (Ali et al., 2017). As per International Energy Agency (IEA) (2023), increased price of crude oil affects the price of gasoline, manufacturing, electricity power production and domestic cooking oil. In Tanzania particular, high oil prices negatively affected consumption of individual households, rural and urban transportation streams (Gasper & Mbwambo, 2023). Moreover, high crude oil price has accelerated unemployment patterns especially from the private sector which abstain from high crude oil prices by reducing their employment patterns and consequently affecting economic growth (Gasper & Mbwambo, 2023). To date, researchers are interested to examine a linkage between the price of unrefined oil and expansion of economies (Rumbia & Azis, 2020). Number of empirical findings shows the presence of relationship between price of unprocessed oil and growth of the economy (Adam et al., 2021).

This study aims to investigate the impact of crude oil prices on economic growth of Tanzania as empirical evidence on existing linkages between these variables. Since many studies have been conducted in Africa especially Crude oil producing countries like Nigeria, Libya, South Sudan, Ghana and Congo by Awunyo-vitor et al. (2018), Anthony (2020), Nwanna & Ayenajeh Eyedayi (2016), Msafiri & Yurievich, (2018), Ftiti et al.(2016) and (Ogboru et al., 2017), only few studies have been done in oil-importing nations like Tanzania. Thus, the fundamental goal of this research is to make analysis on the impact of fluctuation in crude oil prices towards economic expansion in Tanzania. Specifically, the investigation aims to: (i) determine whether fluctuations in price of unrefined oil affects economic upswing of Tanzania and (ii) determine whether there exists a long run relationship between economic growth and price of crude oil.

1.1 Research Hypotheses

H0: Changes on crude oil prices has no effect on economic growth
H1: Changes on crude oil prices has effect on economic growth

H0: No long run association between prices of crude oil and economic expansion.
H1: There's long run association between prices of crude oil and economic expansion

II. LITERATURE REVIEW

2.1 Theoretical Review

2.1.1 Renaissance Growth Theory

Renaissance growth theory was developed in the early 1980s. According this theory, shocks to the price of crude oil have unfavorable impact on economic growth. The theory further suggests that oil prices impacts both advanced economies and growing economies in different ways varies from one country to another due to the available stabilization policies toward oil price shocks (Ogboru et al., 2017). According to Oriakhi & Osaze (2013), changes in price of fossil oil and volatileness on price of fossil oil both have an adverse impacts on economic prosperity, while fossil oil price fluctuation present a gradual effect on economic growth, volatileness has instantaneous negative and remarkably impact on it. This study therefore is built on the Renaissance model of growth. The selection of this model is of paramount importance due to clear relationship between changes in crude oil price and economic growth which is the key concern of this study. This theory was developed from the symmetry and asymmetry effects on economic growth which pause an ambiguity on how asymmetry changes on government policy and controlled variables that affects the economy (Smets

& Wouters, 2007) and (Stiglitz, 1989). Since renaissance put clear relationship between changes in fossil oil prices and economic hike of the country, this approves its superiority over the two theories (Ogboru et al., 2017).

2.2 Empirical Review

Awunyo-vitor et al. (2018) investigated the nexus between changes in the price of crude oil and economic expansion. Secondary data from International Financial Statistics and World Development Indicators were used in this investigation. Furthermore, descriptive statistics, Granger causality test, Johansen cointegration test and unit root test were employed for data analysis. The study findings showed that changes in oil prices and Ghana's economic growth are negatively correlated. Likewise, Benli et al. (2016) explored the impact of variations in crude oil prices against Turkey's real economic productivity. Nonlinear Autoregressive Distributed Lags (NARDL) model was used for this study, allowing us to evaluate the corresponding response of economic growth to changes in its regressors and to examine the long-run and short-run asymmetries simultaneously. The divergent impact of fluctuations in oil prices on economic growth is supported by the empirical findings. In particular, rising oil costs have a sustained detrimental impact on economic expansion.

Conversely, Ogboru et al. (2017) determined how Nigeria's economic growth was affected by fluctuations in crude oil prices between 1986 and 2015. Data from secondary source were employed. Johansen's co-integration Test, Zivot-Andrews Tests, the Vector Error Correction Model and Granger Causality Test were employed to analyse the data. The study revealed that Crude oil prices affect positively the economic growth of Nigeria. Correspondingly, Berument et al.(2010) explored how shocks in oil prices affects the growth of country aggregate output among selected MENA nations. Data from World Bank was used. Vector Autoregressive, Unit root test using Dicky Fuller and Granger causality were used as the methods of data analysis. The analysis showed that price of unrefined oil has a substantial impact on economic growth in selected countries (United Arab Emirates, Iraq, Oman, Libya, Iran and Algeria). In line with, Mhamad & Saeed (2016) investigated the adverse impact of price of fossil oils on economic upturn. It employed different channels like spending level, inflation rate and unemployment which consequently affects the economic expansion of Iraq as the oil producing country. The study used Ordinary Least Square (OLS) and secondary data running from 2000-2015. Multiple linear regression together with its assumption was used to analyze the data. Study showed that oil price and oil exports positively determine the economic growth of Iraq.

Antagonistically, Awunyo-vitor et al.(2018) investigated the relationship between variations in oil prices and Ghana's economic expansion. Secondary data from International Financial Statistics (IFS) and World Development Indicators (WDI) were used in the study. Further, data was analyzed using descriptive statistics, unit root test, Granger causality test, and Johansen cointegration test. The investigation showed that fluctuations in oil prices had no effect on Ghanaian economic growth. Furthermore, Dinh (2022) investigated the linkage between the price of crude oil and the GDP (Gross Domestic Product) of various countries. Time series running from 1991 to 2020 was used to determine this relationship. The Autoregressive Distributed Lag model (ARDL), Unit root test, Pearson's correlation (two tailed) was employed to analyse the data. The study found that Crude oil prices affect Vietnam, China, South Korea highly negatively while it has a minimal negative effect on Thailand, Singapore, Malysia Japan and America. The investigation showed that in Indonesia, prices of fossil oil influence positively economic upswing. Additionally, Ftiti et al.(2016) examined how shock in crude oil prices has affected the economic growth of Nigeria. The research employed second hand data from different source from 1980 to 2014. Multiple regression was used to analyze the data. According to the study, there is positive and substantial correlation between rising oil prices and economic expansion.

Separately, Ali et al.(2017) investigated how Pakistan's household spending, political stability, and remittances affected the country's economic growth. The study examined how 2008 oil price shock, both before and after, and its destructive effects on general output. The study compared the influence of the oil price shock and economic upswing before and after shocks. The findings showed that there is a negative association between economic expansion and prices of crude oil. Additionally, structural break dummy confirms undetectable shift in economic growth before and after shocks to oil prices. Conclusively, Brucal et al.(2018) explored the power of price volatility of fossil oil toward state-level towards economic prosperity. The study employed 48 contiguous states for the period 1973-2013 and Structural decomposition of supply and demand factors that drives the real crude oil prices. Heterogeneity of neighboring state in production and consumption of crude oil and natural gases were used to analyze this relationship. The study showed that oil-exporting states are more vulnerable to unanticipated changes in oil prices, and the direct effect of oil price shocks can also effect on neighboring states through production and consumption sensitivity to crude oil prices through supply shocks.

III. METHODOLOGY

3.1 Data

This study utilized series of chronological data of about 34 years running from 1989 to 2022 which were sourced from the secondary data sources. The data on annual GDP per capita was sourced from Word Bank Development Indicators and data on Crude oil prices were sourced from Statistical Review of World Energy, 2023. Software Package of EViews 12 was employed to analyse this time series data. The credibility of the information from World Bank statistics and Statistical Review of World Energy remains worthwhile. These sources have established numerous data collection and verification procedures over time, ensuring data is of better quality and consistence. This ensures that research findings are robust. However, the shortcomings of this data source includes but not limited to data gaps particularly countries with low and medium incomes (Utouh et al., 2024).

3.2 Empirical model

Regression analysis involve analyzing of endogenous variable in relation to several predictor variables, also known as explanatory variables, in order to determine parameter estimates (Utouh et al., 2024). The independent variable in this study is crude oil prices (COP), and the dependent variable is economic growth (PCGDP) and the regression equation can be written:

$PCGDP_t = \hat{\propto} + \hat{\beta} COP_t + \omega_t$ ..... (1)

Where $PCGDPt$ = GDP per capita as proxied of economic growth and $COPt$ = Crude oil price and ωt = error term

In this study, Autoregressive distributed lag model (ARDL) as developed by Pesaran et al. (2001) is utilized to prevent spurious regression (Pesaran et al., 2001). ARDL is also employed due the tendency of some macroeconomic variables been correlated with its lag and the lags of its independent variables (Ogboru et al., 2017).

Dickey, D. and Fuller, W. (1981) tests for stationarity of the variables was conducted (Said & Dickey, 1984). The Phillip Perron Test (PP) was also employed to comply with the results from the ADF. The ADF & PP test is carried out to find out the unit root from equation (1) as follows:

$\sum_{i=1}^{n} \beta_i \text{Log}(PCGDP)_t = \sum \alpha_i \text{Log}(COP)_{t-i} + \sum \beta_i \text{Log}(PCGDP)_{t-i} + \varepsilon_t$ ..... (2)

$\Delta x_t = \beta_0 + \partial_t + \delta x_{t-1} + \sum_{j=1}^{s} \sigma_j \Delta x_{t-j} + \varepsilon_t$ ..... (3)

Where: $\Delta x_t = x_t - x_{t-1}$, xt: is time-series; s: is optimal lag selected by AIC criterion; εt: is disturbance term and it is Independently and Identically Distributed (IID) $\sim N(0, \sigma^2)$

From the equation three (3) introduce trend variable t time called (∂t), which has values ranging from 1 to n, where n is last observation in the data series and 1 is the initial observation in the data. Disturbance term is random, Independently and Identically Distributed (IID) $\sim N(0, \sigma^2)$. Akaike information criterion (AIC), Schwarz Information Criterion (SIC), Hanna-Quinn Criterion (HQC), and Final prediction error (FPE) were used to determine the optimal k of lags for the ARDL model.

The Johansen test for cointegration was employed to determine the existing relationship between Gross Domestic Product Per Capita (PCGDP) and Crude Oil Prices (COP) are stationary time series at their first differences. The test for cointegration through Johansen method is used to establish the substantial correlation between variables. Therefore, its necessary to determine whether variables are stationary I (0), or become stationary after the first difference I (1), which is the prerequisite for ARDL model. ARDL model of lag (p, q) is as follows:

$Y_t = BY_{t-1} + \cdots + B_n Y_{t-n} + \beta X_t + \varepsilon_t$ ..... (4)

Yt is one dimensional matrix $(m \times n)$ of (1×1) between the predictor and response variable, Xt is a known variable and ε_t is the speed of predictor variables to arrive back to equilibrium.

Employing the autoregressive distributed lag (ARDL) model, the impact of fossil oil prices on growth of Tanzania's economy was estimated. ARDL model for COP and PCGDP is as follows:

$\sum_{i=1}^{n} \beta_i \text{Log}(PCGDP)_t = \sum_{i=1}^{n} \alpha_i \text{Log}(COP)_{t-i} + \sum_{i=1}^{n} \beta_i \text{Log}(PCGDP)_{t-i} + \varepsilon_t$ ..... (5)

$\text{Log}(COP)_t = \sum_{i=1}^{n} \beta_i \text{Log}(PCGDP)_{t-i} + \sum_{i=1}^{n} \alpha_i \text{Log}(COP)_{t-i} + \varepsilon_t$ ..... (6)

The response variable in the ARDL model (p, q) is PCGDP (Y) and predictor variable is COP (X). The model is presented below:

$\text{Log}(PCGDP)_t = \alpha_{0i} + \sum_{i=1}^{q} \varphi_i \text{Log}(PCGDP)_{t-i} + \sum_{i=1}^{p} \beta_i Log(COP)_{t-i} + u_{1t}$ ..... (7)

Where PCGDP depends on crude oil price variable and it is expected to be I (0) and I (1), $\varphi i \; and \; \beta i$ represent the value of the estimates; α_0 is the constant term, i is the number of observations, (p, q) represent the desirable lag and u express unobserved variables outside the model and its white noise.

ARDL bounds test justifies the existing relationship between variables. Thus, estimating the long run form by Error Correction Model (ECM) is as follows:

$$\Delta Log(PCGDP)_t = \alpha_{0i} + \sum_{i=1}^{q} \varphi_i \, \Delta \text{Log}(PCGDP)_{t-i} + \sum_{i=1}^{p} \beta_i \, \Delta Log(COP)_{t-i} + \gamma_1 \, \text{Log}(PCGDP)_{t-i} + \gamma_2 Log(COP)_{t-i} + u_{1t} \quad (8)$$

IV. FINDINGS & DISCUSSIONS

4.1 The Description of the Variables

This section provides an overview of the variables used in this study, specifying their definitions, measurements, sources and expected signs as indicated in Table 1.

Table 1

Summary of the Variable, Measurement, Sources of Data and Expected sign

Variable	Measurement	Source	Expected sign
Economic growth (PCGDP)	Gross domestic product per capita (PCGDP) (Current US$) as a proxy of the economic growth	World Bank	NA
Crude oil price (COP)	World crude oil prices in (Current US$)	Statistical Review of World Energy, 2023	+/-

4.2 Descriptive Statistics

Table 2 portrays summarized descriptive statistics. The results shows that LogPCGDP has equal mean and median and nearly zero skewness indicating that the distribution is symmetric. It may be observed that LogCOP has also approximate equal mean and median indicating that the distribution is symmetric. The standard error is small about 0.30 for LogPCGDP and LogCOP suggesting that the data set does not deviate from their mean, data has a linear trend around their mean value. Kurtosis is positive and small indicating a mesokurtic between LogPCGDP and LogCOP and there is no presence of outliers in the data set. Jarque-Bera coefficient of 4.1016192 and 2.928053 respectively and their probabilities of 0.134244 and 0.231303 which are greater than 5% significant level, signifying that the model conforms to normality distribution assumptions.

Table 2

Summary Statistics of the Variables Used in the Regression Analysis

	LogPCGDP	LogCOP
Mean	3.330194	2.403597
Median	3.297164	2.429835
Maximum	3.764283	2.846572
Minimum	2.916361	1.902976
Standard deviation	0.301484	0.300766
Skewness	0.061520	-0.000284
Kurtosis	1.320768	1.562353
Jarque-Bera	4.1016192	2.928053
Probability	0.134244	0.231303
Sum	113.2266	81.72229
Sum sq. Dev	2.999451	2.985189
Observations	34	34

Notes: Descriptive statistics performed to analyze the data set

4.3 Test for Unit Root

Table 3 presents the summary of the results of unit root test using Augmented Dickey Fuller (ADF) test and Phillips- Perron (PP) test for unit root. The results show that LogPCGDP and LogCOP are not stationary at level. However, at first difference, I (1) variables has no unit root, supported by t-statistic which is greater than 5% critical values for both ADF test and PP test. Given that both variables are integrated of paired order I (1), the study uses Autoregressive Distributed Lags (ARDL) model and Johansen cointegration technique of estimating short run and long run relationship between variables.

Table 3

Unit Root Test Results

Augmented Dickey Fuller (ADF) test				Phillips-Perron (PP) test			
Variable	**t-statistic**	**5% critical value**	**Prob.**	**t-statistic**	**5% critical value**	**Prob.**	**Order of integration**
LogPCGDP	-4.04652	-2.95711	0.0003**	-4.11710	-2.95711	0.0031**	I (1)
LogCOP	-5.11859	-2.96041	0.0002**	-5.05481	-2.95711	0.0003**	I (1)

*Notes: Unit root test performed by using ADF and PP test at 5% significant level: Sign code ** P <0.05*

4.4 Optimal Lag Selection

Table 4 shows that optimal lag for estimation of ARDL model is lag 1 which was selected by all criteria.

Table 4

Lag Selection Criteria and Results.

Lag	**0**	**1**	**2**
LR	NA	166.9077*	5.439242
FPE	0.0001926	7.84e-06*	8.26e-06
AIC	-0.576652	-6.082090*	-6.033544
SC	-0.485043	-5.807265*	-5.575501
HQ	-0.546286	-5.990994*	-5.881716

*Notes: Lag selection Criteria performed by Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), Hanna-Quinn Criterion (HQC), Final prediction error (FPE): * = the smallest optimal lag selected by criterion.*

4.5 Test for Residual Diagnostic

The Breusch- Godfrey Serial correlation LM Test in Table 5 indicates the absence of serial correlation in residuals. Implicitly, probability of Chi-square is 0.4976 which exceeds 0.05 significant level.

Table 5

Autocorrelation Test: Breusch-Godfrey LM

F-statistic	0.570119	Prob. F (2,25)	0.5726
Obs*R-square	1.395840	Prob. Chi-Square (2)	0.4976**

*Notes: Breusch-Godfrey test for autocorrelation: **means no correlation between variables and error term at 5% significant level*

4.6 Test for Heteroscedasticity

The results of heteroscedasticity presented in Table 6 indicate that residuals are homoscedastic as specified by probability of Chi-square (0.2092) which exceeds 5% significant level.

Table 6

Heteroscedasticity Test: Breusch-Pagan- Godfrey

F-statistic	1.515699	Prob. F (4, 27)	0.2256
Obs*R-square	5.867909	Prob. Chi-Square (4)	0.2092**
Scaled explained SS	6.659914	Prob. Chi-Square (4)	0.1550

*Notes: Breusch-Godfrey test for Heteroscedasticity: **means no heteroscedasticity at 5% significant level*

4.7 Test for Normality

Table 7 shows that error term is normally distributed since the probability of Jarque-Bera test (0.0526) is greater than 0.05 significant level.

Table 7

Result of Normality

Normality test	
Jarque-Bera	5.882803
Probability	0.052592

Notes: Normality test for residuals using Jarque-Bera at 5% significance level

4.8 Long Run Relationship

From Table 8, the trace statistic is smaller than the 5% critical value. Thus, LogPCGDP and LogCOP have single cointegrating equation. Therefore, long-run equilibrium of the ARDL can be measured.

Table 8

Johansen Cointegration Test

Number of Cointegrating equation	Eigenvalue	Trace statistics	5% Critical value
None	0.337336	13.30681	15.49471
At most 1	0.004341	0.139199	3.841465

Notes: Johansen Cointegration Test

4.8 Autoregressive Distributed Lags Results

Table 9 shows ARDL short-run estimates. The Error Correction Model shows that there is moderate speed of adjustment between variables with a negative and significant value of -0.100042 (p = 0.0025). This shows that economic growth will adjust to equilibrium at a speed of 10% each year due to shocks in crude oil price.

Furthermore, in short run it is exposed that price of crude oil (COP) has significant and substantial impact on growth of Tanzania's economy. A unit increase in COP will increase the economic growth of about 0.232791 units. Also, R^2 in the model show 66.73% of the variation of economic growth (PCGDP) can be explained by the crude oil prices and 33.27% variation of economic growth (PCGDP) can be explained by other external factors. Also, F-statistic is approximately 18.72448 with its probability of 0.000001 provides us with confidence to conclude that overall results obtained from ARDL are statistically significant.

Table 9

Error Correction Model with Short-run Result.

Dependent variable LogPCGDP	Coefficient	Standard Error	Test Statistic	Probability
ECM (-1)	-0.100042	0.029973	-3.337718	0.0025**
D(LogCOP)	0.232791	0.032640	7.132161	0.0000***
R^2= 0.6673	Adj R^2= 0.6317	F-statistic =18.72448	Prob. F-statistic= 0.000001	

*Notes: Short run results and ECM from ARDL: *** and ** significant at 1% and 5% level of significance respectively*

Table 10 shows the long run coefficient of ARDL model with optimum of lag is ARDL (1, 2). This optimal lag is selected by AIC. This shows that crude oil price has negative and significant effects on economic growth of Tanzania of approximately -1.15772 percent for each 1 dollar rise in crude oil price.

Table 10

ARDL Long Run Relationship Result

Dependent variable LogPCGDP	Coefficient	Standard Error	test-statistic	Probability
LogCOP	-1.157722	0.029973	-3.337718	0.0000***
C	0.076454	0.014798	5.166443	0.0000***

*Notes: ARDL estimation of the long run relationship: ***significant at 1% significance level*

4.9 Granger Causation Test

The null hypothesis of LogCOP forecast LogPCGDP and LogPCGDP forecast LogCOP at 5% significant level is rejected. Thus, no bidirectional relationship between PCGDP and COP.

Table 11

Granger Causation Test

	Observation	F-statistic	Probability
LogCOP has no Granger Causation to LogPCGDP	32	0.43981	0.6487
LogPCGDP has Granger Causation to LogCOP		1.60813	0.2189

Notes: Granger Causation test: at 5% significance level

4.10 Model Stability

Figure 1 shows that CUSUM and CUSUMSQ curves remain within 5% range (bound), which justifies the long run connection between variables. Therefore, the model is stable and errors generated from all-time series were relatively stable and suggests that no structural break.

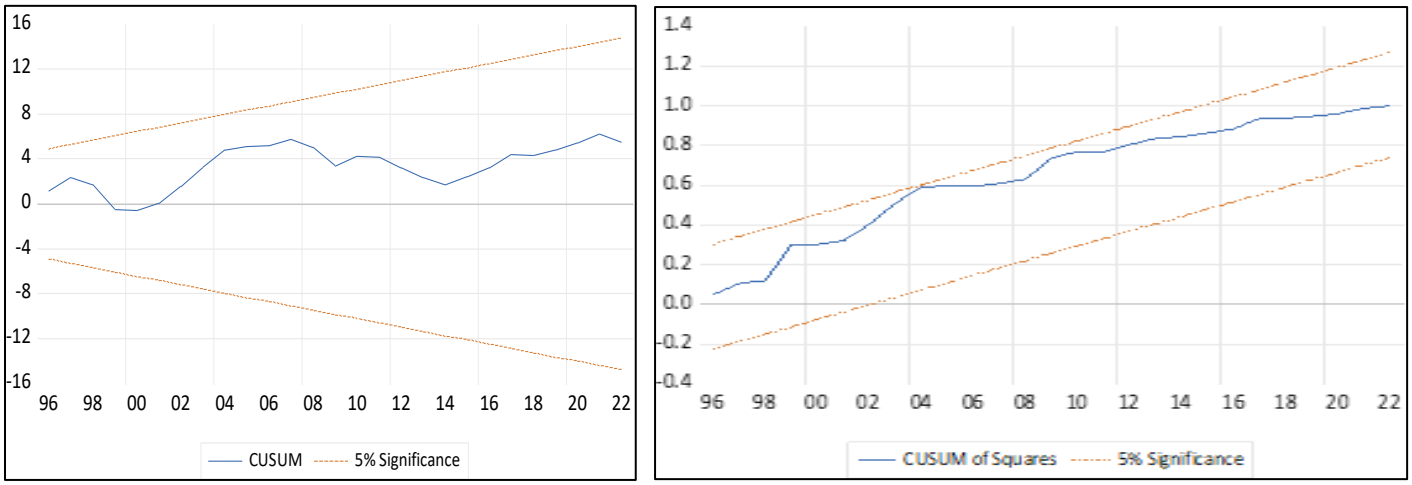

Figure 1
CUSUM and CUSMSQ at 5% Significance Level.

4.11 Discussions

The study on the influence of fluctuations in price of crude oil on economic growth has revealed that changes in price of crude oil has a significant and productive impact on Tanzania's economic upswing. One-dollar rise in price of crude oil brings about 0.232791unit rise in level of Tanzania's economic growth in short run. This result is conforming with Ogboru et al.(2017), Dinh (2022), Ftiti et al.(2016), Odhiambo & Nyasha (2019), and (Kibunyi and Wanjala, 2018). This can be caused by an increase in revenues from port charges and import levies from Dar es Salaam, Tanga, and Mtwara ports through transportation of oil tankers for both domestic and transit goods for other landlocked countries like Congo, Zambia, and Malawi (Gasper & Mbwambo, 2023). This increases revenue to the country, which could be invested in production units, resulting in a boost in the economic growth of Tanzania (Kibunyi and Wanjala, 2018). Additionally, increases in income through employment in the transport sector and fuel stations also increase income investment and hence economic growth (Utouh et al., 2024). The study further showed that in the long run, crude oil prices have a negative and significant impact on Tanzania's economic growth. The investigation showed that, one percent increase in crude oil prices brings about -1.15772 percent fall in economic growth of Tanzania. The ARDL results also shows that error correction model (ECM) of -0.1000 (p= 0.0025) is significant with its value, suggesting a moderate speed of convergence to equilibrium after shock. This is concurrent to Adam and Pasrun (2021), Dinh (2022), Ftiti et al. (2016) and Gasper and Mbwambo (2023) who found that as crude oil price increases consequently, the importation of crude oil becomes very expensive, leading people to find alternative sources of energy like electric cars and motorcycles. This reduces revenue to the government, lowers employment in the energy sector and other linked sectors. Further, it reduces consumption, pushes up overall prices, and eventually slows down the rate of economic expansion.

V. CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

The purpose of this study is to explore the effect of crude oil price volatility and economic growth of Tanzania running from 1989 to 2022. An autoregressive distributed lag model is employed because it can estimate both short-term and long-term relationships between variables while still providing unbiased estimates and significant t-statistic regardless whether some independent variables are endogenous. Granger-Causality tests showed that there is unidirectional granger-causality from economic growth to crude oil prices and vice versa in the study country. Also, crude oil price volatility has been proved to have substantial influence on the economic progress in Tanzania. This implies that an increase in crude oil price will increase the government revenues and income consequently increases the

economic activities and hence economic prosperity in short-run. However, if the volatility of crude oil prices persists, it will in the long run dampen the level of economic growth due to high cost of transportation, searching for alternative source of energy which consequently, lower investment, increase general price levels, reduces consumption, and economic growth of Tanzania. Finally, this study concludes that among other thing, economic growth of Tanzania is affected by changes in crude oil prices.

5.2 Recommendations

There has been great discussion among academicians, researchers and politicians about the effect of fluctuations in fossil fuel prices on economic expansions worldwide. This study shows that changes in price of unrefined oil negatively strike economic prosperity of Tanzania in long-run. This negative effect calls for attention to policy makers and the government to ensure effective use of fiscal policies that will reduce the adverse impact of the world crude oil prices hikes, but also searching for alternative sources of energy that will reduce depending on the crude oil. Additionally, research and development should be enhanced so as to explore and harness the oil fossils that have been found in along the coast of Indian ocean. This will reduce the importation of crude oil and Tanzania will export the rest of oil to the world and hence, expand its economy.

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Effects of Value Added Tax (VAT) Knowledge on SMEs Performance and Factors Affecting SMEs Performance in Morogoro Tanzania

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ABSTRACT

Small and Medium-sized Enterprises (SMEs) play a crucial role in driving economic growth and development in Tanzania. However, the performance of SMEs is hindered by various factors, including the challenges posed by value-added tax (VAT). This study investigates the effects of VAT knowledge and other factors on the performance of SMEs in Morogoro, Tanzania. The research objective is to provide a comprehensive understanding of the key determinants influencing SME success and growth in the region. The study assesses the level of VAT knowledge among SME owners, examines the relationship between VAT knowledge and SME performance, and analyses the impact of factors such as business location, initial capital investment, years of experience, and type of business on overall SME performance. The supply and demand theory are employed as the theoretical framework to understand how changes in costs, induced by VAT, affect the supply decisions of SMEs and consumer responses to price changes. Despite government efforts to address challenges faced by SMEs through regulatory simplification, tax reforms, and digitalization, the performance of SMEs continues to be hindered by VAT-related administrative costs, pricing, and forecasting customer needs. This study aims to fill the research gap by providing valuable insights and recommendations for policymakers and business stakeholders to foster a supportive environment that promotes the sustainable growth and development of the SME sector in Tanzania.

Keywords: Economic Growth, Business Costs, Small and Medium-sized Enterprises (SMEs), Value-Added Tax (VAT)

I. INTRODUCTION

Small and Medium-sized Enterprises (SMEs) play a pivotal role in driving global economic growth, serving as catalysts for development at local, national, and international levels (Singh et al., 2019). The significance of SMEs manifests through their contributions to job creation, fostering innovation, diversifying industries, enhancing adaptability, intensifying competition, and boosting exports, all of which collectively contribute to the Gross Domestic Product (GDP) of their respective countries (Herculina et al., 2022; Kanire et al., 2024). According to the World Bank Statistics Report (2023), SMEs constitute nearly 90% of the world's businesses and provide 50% of global employment opportunities (Dimoso & Andrew, 2021; The World Bank [WB], 2023; Kitole, 2023; Kitole & Esabo, 2022). Furthermore, the WB's (2023) business report indicates the presence of approximately 44 million SMEs in Africa. In Tanzania, reports released by the National Bureau of Statistics (NBS) and the Economic and Social Research Foundation (ESRF) in 2021 reveal the existence of more than three million SMEs employing over five million individuals (Kitole & Esabo, 2024; ESRF, 2016).

Despite the significant contributions of SMEs to Tanzania's economy, value-added tax (VAT) poses a challenge that, if not properly regulated, can discourage the emergence of new SMEs and lead to the decline of many existing ones (Kitole & Utouh, 2023; Debala, 2023). The NBS (2020) integrated labor survey, conducted in 2015, revealed that out of 154,618 surveyed SMEs, only 82,238 (53.19%) remained in business for no more than five years (Fumbwe et al., 2021). This finding indicates that over half of the enterprises experience a decline and ceasing operations. One reason cited for this phenomenon is that VAT increases the prices of commodities, consequently reducing the market share of SMEs' products.

In response to these challenges, the government of Tanzania has made notable efforts to address the situations that hinder the performance of SMEs. These efforts include simplifying regulatory procedures, implementing tax reforms, and promoting digitalization to enhance the efficiency of VAT payments (United Republic of Tanzania (URT), 2023. However, despite these initiatives, the performance of SMEs continues to be impeded by increased business operation costs, such as compliance expenses and day-to-day cash flow obstacles arising from the requirement to pay VAT prior to receiving customer payments (Sidek, 2022). The administrative costs associated with VAT regulations, particularly those related to pricing and forecasting customer needs, cause distress for most SMEs (Mohammed and Abdulraqeeb, 2022; Kanire et al., 2024). This study examines the effects of VAT knowledge and other factors on SME performance in Morogoro, Tanzania. By understanding the impact of VAT and identifying the key determinants of SME performance, this study provides valuable insights for policymakers and business stakeholders to support the growth and development of the SME sector.

1.1 Problem Statement

Despite the efforts made by the government to address the challenges faced by SMEs in Tanzania, such as simplifying regulatory procedures, tax reforms, and promoting digitalization, the performance of SMEs continues to be hindered by factors related to value-added tax (VAT). The administrative costs associated with VAT compliance, pricing, and forecasting customer needs pose significant obstacles for SMEs. There is a lack of a comprehensive understanding of how VAT knowledge and other factors impact the performance of SMEs in Tanzania, particularly in the context of Morogoro. This study aims to fill this gap by investigating the effects of VAT knowledge and identifying key determinants of SME performance.

1.2 Research Objective

The primary objective of this study was to investigate the impact of value-added tax (VAT) knowledge and various other factors on the performance of Small and Medium-sized Enterprises (SMEs) in Morogoro, Tanzania. This study aims to provide a holistic understanding of the key determinants that influence the success and growth of SMEs in the region. To achieve this, the study assessed the level of VAT knowledge among SME owners in Morogoro and examined the relationship between VAT knowledge and SME performance. Additionally, the study identified and analyzed the effects of crucial factors such as business location, initial capital investment, years of experience, and type of business on the overall performance of SMEs. Through a thorough investigation and analysis of these aspects, the study pursued to provide valuable insights and recommendations for policymakers and business stakeholders to foster a supportive environment that promotes the sustainable growth and development of the SME sector in Tanzania.

II. LITERATURE REVIEW

2.1 Theoretical literature review

The supply and demand theory are traced back to several economics' pioneers. However, it gained prominence in the late 18th century with the works of Adam Smith, who developed this theory in his seminal work 'The Wealth of Nations,' published in 1776."Supply and demand theory posits that in a competitive market, the price and quantity of goods are determined by the intersection of the supply and demand forces, which are stated as the law of demand. It states that as the price of a good decreases, the quantity demanded increases, and vice versa at ceteris paribus. Conversely, the law of supply asserts that as the price of a good increases, the quantity supplied increases, and as the price decreases, the quantity supplied decreases, ceteris paribus (Msacky & Mmassy, 2022). The supply and demand theory is highly relevant in this study in studying the effect of Value Added Tax (VAT) on Small and Medium-sized Enterprises (SMEs) performance. The theory helps to understand how changes in costs, induced by VAT, affect the supply decisions of SMEs and how consumers respond to changes in prices. Using this theory, the dependent variable is the quantity of demanded and supplied goods and services which measures the performance of SMEs while the independent variable is the cost of goods and services which is affected by the imposition of VAT which accelerates the production costs.

The demand and supply theory in this theory is strengthened due to its predictive power as the theory provides the robust framework on examining how the prices of the goods and services change due to imposition of VAT and how this is going to affect the supply and demand decisions of the SMEs and customers respectively. However, despite the strengths, this theory is also attacked due to the application of ceteris paribus condition which paves the way of neglecting other variables which might affect the quantity demanded and supplied. Also, demand and supply theory assume that the SMEs have to operate under perfect competitive markets which is characterized by the presence of many

buyers and sellers, no government intervention, no alternation of prices since both buyers and sellers are taken as price takers as they take the price as given as determined by the market forces of demand and supply and symmetric information among the buyers and sellers.

Generally, despite the weaknesses of this theory, the theory is a powerful tool in explaining how the prices of the goods and services change due to imposition of VAT and how this is going to affect the supply and demand decisions of the SMEs and customers respectively.

2.2 Empirical Literature Review

According to Singh (2019) who examined the Impact of Value Added Tax on Business Enterprises in Mettu Townin Ethiopia, a study which has the main objective of examining the impact of value addition tax on business enterprises in Ethiopia particularly in Mettu Town of Oromia region. In this study both primary and secondary data were used in which primary data were collected from the 350 business owners who were obtained by judgmental sampling technique and secondary data were obtained from State Bureau Authority. Data analysis was done using descriptive analysis because qualitative research design was employed and therefore most of the data collected were qualitative in nature. This study came to find a contradictory results on impact of VAT on business entrepreneurship in Ethiopia since the findings show that VAT is not a sole factor that impact the performance of SMEs in Ethiopia. Some other factors such as nature of business, location where business is positioned, business experience and number of employees also impact the performance of SMEs in Ethiopia in which when the business with food related goods, located in urban areas with large number of people, more business experience and large number of trained employees is likely to perform better.

Similarly, another study written by Debala (2023) called dynamic effects of VAT collection on SMEs performance and its determinants in Benishangul-Gumuz region Ethiopia which used mixed explanatory cross sectional survey research design with both primary and secondary data in which 281 sample size was used out of which 261 were tax paying SMEs and 21 tax officers who are directors and supervisors. The employed proportional and purposive sampling method. The study adopted binary logistic regression analysis to find the VAT negatively affect the performance of SMEs as it was used in Ethiopia while the factors which determine the payments of VAT are tax payers, knowledge on tax, tax law enforcement and tax staff knowledge on better ways to collect tax.

Furthermore, Chindengwike (2022) conducted a study titled 'The Influence of VAT on SMEs Performance in Developing Countries,' which employed secondary data with a time series design in Tanzania from 1998 to 2020."The data were taken from Tanzania Revenue Authority (TRA) and Bank of Tanzania (BOT) as well as World Bank (WB). In this study cointegrations method was used and came to find that VAT has negative correlations with the growth of business sector in Tanzania.

In contrast, Kuria (2018) conducted a study titled 'The Effect of VAT Incentives on the Performance of SMEs in Kenya.' The main objective was to determine the effect of VAT incentives on the performance of SMEs in Kenya. The study employed a correlation research design with a sample size of 86 registered SMEs." In this study questionnaire was used as a tool of primary data collection, while secondary data such as Returns on Investments (ROA), number and value of jobs and the number of years that an SMEs stays in the business. Then, finally, using both descriptive and inferential statistics, this study revealed that VAT at 5% significance level had a positive and positive correlation with SMEs performance and number of jobs and value which was measured using ROA.

Furthermore, in Kenya, as outlined by Kamar (2015), SMEs provide job opportunities, foster innovation, competition, and economic growth."This study which was done in Kenya with the main objective of examining the effects of government taxation policy on performance of SME in Kenya and particularly Uasin Gishu County. In order to achieve the purpose of this study, the specific research objective was addressed: to find out the effects of government taxation policy on sales revenue of SME. In this study both primary and secondary data were used making the total sample of 180 individuals who were selected using stratified sampling technique. Using explanatory research design and application of questionnaire, interview and document analysis for data collection, this study found that government VAT taxation policy has significant effect on the performance of SMEs. The researcher added that, higher VAT rates significantly lead to poor performance of SMEs due to lack of customers who avoid higher prices. Concurrent to this study, Ma (2018) on his study called Effective Tax Planning of Value - Added Tax under the Background of Small and Medium Enterprises Business Tax Change to Value-added Tax which was conducted in Shandong using paper review analysis, the study came to find that Small and Medium Enterprises are very important component of the local economy as they create employment opportunities which has a pivotal role for local, national and global economy, however, the setting of VAT and Pay As You Earn (PAYE) is retarding the performance of SMEs since higher tax rates reduce their profit margins and therefore the ROA gets reduced.

The study called The Impacts of Value-Added Tax Audit on Tax Revenue Performance: The Mediating Role of Electronics Tax System, Evidence from the Amhara Region, Ethiopia which was done by Mu (2022) using primary data sources from 377 VAT registered taxpayers in Amhara Region which applied Ability to Pay taxation theory, structural equation model, path diagram, and multiple regression with SPSS software for data analysis to identify the relationship between VAT rates and audit and tax revenue performance. This study revealed that inefficient tax rates and audits associated with poor tax education to taxpayers significantly reduce the performance of SMEs in Amhara region.

III. METHODOLOGY

This study employed cross sectional research design, since the data was collected from 240 SMEs who were purposively chosen from 600 SMEs in Morogoro urban to form a sample at one point in time. Therefore, the study used primary data from SMEs. SMEs have been chosen in this study using purposive sampling technique since the researcher ought to get necessary information from the sample which has information about the effects of VAT on the performance of SMEs and therefore the tax paying SMEs became the best sample in this study. Further, descriptive, and linear regression model analysis were employed to examine the effects of VAT on the sales of SMEs. Moreover, variable used in this study have been explained at Table 1.

Table 1

Variables, Descriptions, Measurements, Category and Expected Relationship

S/N	Variables Description	What does it measure	How to measure it	Type of variable measurement	Expected sign
1	Number of sales	Number of sales measure the number of supplies that SME sells.	This measures the quantity of goods supplied by SMEs.	Descriptive	+
2	Type of goods and services sold	This measures the category that goods and services belong	This was measured by considering if goods and services measured are food related or non-food related goods	Categorical	+
3	VAT education	Measures VAT education that SME seller has	This was measured by considering if SME received any VAT education/training or not	Categorical	+
4	Years of experience	Measures the number of years SME has been operating	This was measured by the number years SME has been operating	Continuous	+
5	Capital	Measures the amount of capital invested in business	This was measured by considering the capital invested by SME in business	Continuous	+
6	Number of workers in SME	Measures the number of workers in SME	This was measured by considering the number of workers in SME	Continuous	+
7	Sex of SME's owner	Measures sex of SME' s owner	This was measured by the sex of SMEs owner	Categorical	+
8	Location	Measures the area where SME is operating	The residing area was the location of a SME, whether the SME is in central district business area or not	Categorical	+

IV. FINDINGS & DISCUSSIONS

4.1 Response Rate

Results in Tabe 2 show that out of the total, 190 (79.17%) were male, while 90 (20.83%) were female. This information indicates that the majority of the SMEs owners were males. Moreover, 200 SMEs equivalent to 83.3% of all SMEs were located in central business areas while 40 SMEs which is equivalent to 16.7% were located in periphery region of the district.

Table 2
Description of SMEs Characteristics

Variables	Frequency	Percent
Male	190	79.17
Female	50	20.83
Total	**240**	**100**
Central Business Area	200	83.33
Periphery Business Area	40	16.77
Total	**240**	**100**
Received	60	25
Not received	180	75
Total	**240**	**100**
Food related business	110	45.83
Non-food related business	130	54.17
Total	**240**	**100**

Additionally, regarding the reception of VAT education or training to SMEs owners, results in Table 2 indicates that among the total sample of 240 SMEs owners, only 60 SMEs (25%) had VAT education/training, while 180(75%) which is majority of SMEs had never received VAT education or training. Results on the type of business that SMEs are doing indicate that out of the total sample population of 240 SMEs, 110 SMEs (45.83%) deals with food related business, while 130 (54.17%) deals with non-food related business.

Table 3
Descriptive Statistics for Continuous Variables

Variable	Observation	Mean	Standard error	Minimum	Maximum
Number of workers	240	6	1.932235	1	16
Years of experience	240	4.5	1.09391	1	20
Capital invested	240	500,000	15,000.37	150,000	10,000,000

Table 3 provides the descriptive results for the number of workers, years of experience and initial capital invested in business. The table includes mean, standard error, minimum, and maximum values for each variable. As shown in Table 3 the mean number of workers in all 240 surveyed SMEs is 6 meaning that on average, the SMEs in the sample have around 6 workers with a standard error of 1.932235 which represents the variability associated with the estimate of the mean. It indicates the average deviation of the individual observations from the mean. In this case, a standard error of 1.932235 suggests that the number of workers for SMEs in the sample vary, on average, by approximately 1.93 workers. The minimum observed number of workers is 1, while the maximum observed number of workers is 16. In addition to that results show that the mean years of experience of SMEs in business in the sample is approximately 4.5 which suggests that, on average, the years of experience in business of SMEs in the sample population is 4.5, with a standard error of 1.09391 suggesting that the actual years of experience mean varies by approximately plus or minus 109391 years from the estimated mean of 4.5. The minimum observed year is 1 year, while the maximum observed years are 20 years.

Furthermore results in Table 3 depicts the mean initial capital invested in business for SMEs in the sample is approximately 500,000TZS which suggests that, on average, the initial capital invested in business by SMEs in the sample population is 500,000 TZS with a standard error of approximately 150,000 TZS suggesting that the mean initial capital invested in business varies by approximately plus or minus 150,000 TZS from the estimated mean of 500,000 TZS. The minimum initial capital invested is 150,000 TZS, while the maximum initial capital invested is 10,000,000 TZS.

Table 4

Multiple Linear Regression Model Results

Quantity sold	Coefficient	Standard errors	t value	P>t	95% confidence interval	
Sex of SME owner	0.3459112	1.328677	0.26	0.795	-2.271905	2.963727
SME location	3.296835	1.802436	1.83	0.069	-0.2544008	6.848071
Number of workers	-0.0055337	0.0112915	-0.49	0.625	-0.0277807	0.0167133
Initial capital invested	0.8415011	0.2086486	4.03	0.000	0.4304129	1.252589
Years of experience	-0.022397	0.0015372	14.57	0.000	0.0193681	0.0254253
VAT education	0.7360704	0.154635	4.76	0.000	0.4314019	1.040739
Type of business	0.7799661	0.6546765	1.19	0.235	-0.509905	2.069837

The multiple regression model in Table 4 aims to analyze the relationship between quantity sold (the dependent variable) and independent variables: sex, SME location, number of workers, initial capital invested, years of experience, VAT education and type of business. The coefficient for sex of SME owner is 0.3459112. It suggests that, holding all other variables constant, when SME owner is a male, quantity sold of the goods is 0.3459112 higher than that of female. The result was not statistically significant at the conventional 1%, 5% and 10% level of significance (p-value = 0.795). The coefficient for location of SME is 3.296835. It suggests that, holding all other variables constant, when SME is located in central business area, quantity sold of the goods is 3.296835 higher than that of periphery located SME. The result was found to be statistically significant at the conventional 10% level of significance (p-value = 0.069).

The coefficient for number of workers in SME is -0.0055337. It suggests that, holding all other variables constant, one more worker in SME is decreases the quantity sold of goods by -0.0055337. This result was however found to be statistically insignificant at the conventional 10% level of significance (p-value = 0.625). The coefficient for initial capital invested in business by SME is 0.8415011. It suggests that, holding all other variables constant, a TZS increase in initial capital invested by SMEs increases the quantity sold of goods by 0.8415011 This result was however found to be statistically insignificant at the conventional 10% level of significance (p-value = 0.000).

The coefficient for years of experience in business of SME is -0.022397. It implies that, while controlling for other variables, a year of experience increase decreases the quantity sold of goods by SME by -0.022397. The effect is statistically significant at 1% significance level (p-value = 0.000). The coefficient for VAT education of SME owner is 0.7360704. It suggests that, holding all other variables constant, when SME owner has tax education, quantity sold of the goods is 0.7360704 higher compared to SME owner who has no tax education. The result was found to be statistically significant at the conventional 1% level of significance (p-value = 0.000). The coefficient for the type of business that SME is doing is 0.7799661. It suggests that, holding all other variables constant, when SME is doing food related business, quantity sold of the goods is 0.7360704 higher compared to SME which is dealing with non-food related business. The result was found to be statistically not significant at the conventional 1%clevel of significance (p-value = 0.235).

4.2 Discussion

This study aimed to examine the effects of VAT knowledge and other factors on the performance of SMEs. The descriptive analysis found that most SMEs are owned by males, have no VAT education, are located in central business areas, and are engaged in non-food-related businesses. These findings are consistent with the study by Kamar (2015), which also found that the majority of SMEs are owned by individuals lacking VAT education and are concentrated in central business districts, primarily in non-food-related businesses. Interestingly, this study's findings contradict Kamar's (2015) results regarding the gender of SME owners, as this study found that most currently operating SMEs are owned by females instead of males.

The empirical results of this study reveal a positive relationship between SME performance and the location of the business, where SMEs located in urban areas tend to have higher sales. This aligns with the findings of Alotaibi (2021), who observed that SMEs in urban areas have larger sales volumes despite the presence of VAT, compared to those in rural areas.

Additionally, the study found that as the initial capital invested in the business increases, the performance of SMEs also improves, which is consistent with the results of Obafemi et al., 2021. They suggest that larger initial capital investments indicate SMEs' better understanding of the business and its implications, which in turn accelerates their performance as they strive to maximize sales and returns.

Interestingly, this study uncovered that while years of experience negatively affect SME performance, VAT education has a positive impact. The findings regarding years of experience align with the existing literature, which highlights the potential drawbacks of prolonged operational experience, such as complacency and resistance to change. Conversely, the positive influence of VAT education underscores the importance of financial literacy for effective business management, as SMEs with a strong grasp of VAT principles can benefit from enhanced financial practices, accurate reporting, and reduced legal risks related to tax compliance.

V. CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusion

In conclusion, the primary objective of this research was to investigate the impact of value-added tax (VAT) knowledge and various other factors on the performance of small and medium enterprises (SMEs). The thorough analysis results this study provided valuable insights into the key characteristics of SMEs.

The majority of SMEs are owned by males, lack sufficient knowledge about VAT, are situated in central business areas, and predominantly operate in non-food-related industries. The empirical findings of the study indicated a negative relationship between SME performance and their location in rural areas. Additionally, the results emphasized the importance of initial capital investment in improving the performance of small and medium-sized enterprises.

5.2 Recommendations

Based on these findings, the study offers several recommendations. Firstly, it is imperative for policymakers and business stakeholders to prioritize the provision of VAT education to SME owners, particularly those with limited knowledge in this domain, as this education can have a positive impact on their performance.

Secondly, efforts must be undertaken to boost the growth of SMEs in urban areas and promote higher levels of initial investment, as these aspects may contribute to an overall enhancement in performance. Recognizing the negative influence of extensive business experience, it is advisable for SMEs to strike a balance between knowledge accumulation and adaptability through continuous learning. This will ensure that they remain agile in evolving markets.

Furthermore, business visionaries and entrepreneurs should consider expanding their ventures beyond non-food-related businesses, as suggested by the findings of this examination. This diversification may offer new opportunities for growth and improved performance.

Despite the valuable information obtained from this examination, it is imperative to acknowledge its restrictions. It is crucial to recognize that the research is based on a specific context, and therefore, generalizing the findings to different regions or industries may not be entirely appropriate. Furthermore, the cross-sectional design of the study limits its ability to establish causal relationships, warranting caution when drawing definitive conclusions about the impact of variables on SME performance. These additional studies would provide a more comprehensive understanding of the factors that influence SME performance and allow for more robust conclusions to be drawn.

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Forecasting the National Health Insurance Fund Membership Enrolment in Tanzania Using the SARIMA Model

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ABSTRACT

This paper aimed at forecasting membership enrolment in the National Health Insurance Fund (NHIF) in Tanzania using quarterly time series data. This study used 88 time series data to fit the seasonal Autoregressive Integrated Moving Average model (SARIMA). ARIMA (3,1,1) (0,1,0)[4] model was built and used for forecasting. The results show that there will be an increasing membership enrolment overtime over the years and no signs of decreasing. Thus, the government, apart from continuing subsidizing the cost of accessing health insurance services, should also improve the National Health Insurance (NHI) coverage to accommodate the increased enrolment and discourage dropouts. In turn, this will help to achieve the Universal Health Coverage (UHC) ultimate aim of ensuring equitable access to essential and manageable healthcare services, regardless of individuals' financial situations, their location, and personality.

Key words: ARIMA, Box- Jenkins, National Health Insurance, SARIMA, Seasonal

I. INTRODUCTION

Health insurance (HI) schemes in developing countries generally do not reach the targeted populations to ensure access to essential quality health services (Osei Afriyie et al., 2022). In recent years, enrolment into Health Insurance Scheme (HIS) as a strategy to attain the Universal Health Coverage (UHC) has become a global discussion. Numerous studies including those of Boateng (2024); Yego et al., (2023); Ng'ang'a, (2021); Alesane and Anang, (2018) have documented efforts taken by individual countries to promote the use of HI as a means to achieve a (UHC). However, according to Kathrin et al. (2021), enrolment rates of many low to middle income countries, especially those located in the Sub-Saharan Africa (SSA) region are still low and rarely exceed 10%. Compared to the rural, urban regions experience a remarkably higher number of new entrants to the health insurance scheme (Sharma, 2023). On the other hand, the urban and peri- urban areas have also experiencing a significant number of renewals compared to other areas (Nsiah-Boateng, and Aikins, 2018). This trend has been related to the affordability of the premium, and contribution (Yego et al., 2023; Kusi et al. 2015). Further, there is a direct relationship between indigenous health equity and enrolment in the health insurance scheme, an important aspect to consider in addressing the disparities faced by indigenous populations in accessing healthcare. Either way, enrolment in HI is a viable means of improving the sustainability of the schemes thereby enabling access and utilization of healthcare services towards UHC.

Studies exist in determining and assessing factors that hinder enrolment to the NHIS (Boateng, 2024; Afriyie et al., 2023; Yego et al., 2023; Ng'ang'a, 2021; Ghimire et al., 2019; Alesane & Anang, 2018; Kotoh et al., 2018) and none was able to predict future enrolment. Few studies (Marinova & Todorova, 2023; Yego et al., 2023, 2021; Kathrin et al., 2021) attempted to predict health insurance uptake using machine learning techniques, nevertheless, none of them considered time series forecasting. This study brings a new dimension of predicting future membership enrolment using historical data on enrolment. Given the seasonality nature of the membership enrolment time series data, this paper used the strengths of time series analysis, particularly SARIMA model. The model is appropriate for predicting quarterly, monthly, and yearly membership enrolment into NHIF in Tanzania but also its ability to show good prediction performance as suggested by Wiah et al. (2022). This paper is organized as follows, section 2 presents the literature review, section 3 presents the methodology. Findings and discussion is on section 4 and the paper concludes in section 5 on conclusions and policy recommendations.

II. LITERATURE REVIEW

2.1 Conventional Theory of Health Insurance

According to Nyman (2001), conventional theory holds that people purchase insurance because they prefer the certainty of paying a small premium to the risk of getting sick and paying a large medical bill. Conventional theory also holds that any additional health care that consumers purchase because they have insurance is not worth the cost of producing it. Therefore, economists have promoted policies-copayments and managed care-to reduce consumption of this additional, seemingly low-value care. This presents a new theory of consumer demand for health insurance which holds that people purchase insurance to obtain additional income when they become ill. Thus, additional income generates purchases of additional high-value care, often allowing sick persons to obtain life-saving care that they could not otherwise afford. Regarding risk, the new theory relies on empirical studies showing that consumers actually prefer risk of large risk to incurring a smaller loss with certainty. It is further argued that, if consumers purchase insurance, it is not because they desire to avoid risk. Instead, the new theory suggests consumers simply pay a premium when healthy in exchange for a claim on additional income (effected when insurance pays for the medical care) if they become ill. Health insurance is substantially more valuable to the consumer under the new theory. The new theory moreover implies that copayments and managed care-central health policies of the last 30 years-were directed at solving problems that largely did not exist. Because these policies either reduced the amount of income transferred to ill persons or limited access to valuable health care, they may have done more harm than good. The new theory also provides a solid theoretical justification for insuring the uninsured and for implementing national health insurance. It is from this theoretical background that this paper is set to forecast membership enrolment in the NHIF Tanzania using Seasonal Autoregressive Moving Average methodology of time series analysis.

2.2 Empirical Literature Review

The approaches to study the status of membership enrolment in the health insurance schemes has been the use of binary regression models and in many cases, the analysis was driven by the uptake of health insurance than predicting membership enrolment (Yego et al., 2023; Ng'ang'a, 2021; Alesane & Anang, 2018). At least for Boateng (2024), Marinova and Todorova (2023), and Yego et al. (2021), the analysis on predicting membership enrolment is seen, though all the studies do not consider time series as the tool for predictions and rather use selected families of regressions such as logit and probit because of the nature of the dependent variable. Regressions may not be sufficient to predict membership enrolment given time series data which are seasonal as they do not provide robust analysis to carter for the seasonal nature of the enrolment (quarterly, monthly and yearly data). As a result, the seasonal autoregressive moving average (SARIMA) becomes appropriate to perform the forecast. In other health related forecasts, time series analysis has also gained popularity. Kathrin et al. (2021) forecasted distribution and pattern of a health claim system whose aim was to analyze the distribution and future pattern of insurance health claim system using time series approach. Akaike information criterion and Schwarz Bayesian criterion were used to select the adequate model through maximum likelihood estimation methods. ARIMA (0, 0, 0) (1, 0, 1) [12] model was chosen to forecast claim amounts. Existing studies in forecasting in health dominates the literature (Putri et al., 2023; Morgan et al., 2022; Jalalpour et al., 2015; Soyiri & Reidpath, 2012, 2013), predominantly on membership enrolment, claims payments and health expenditure but have been using Autoregressive Integrated Moving Average (ARIMA). This study employs the SARIMA which is capable of handling non stationary data but also good for seasonal data such as membership enrolment to a health insurance scheme. Subsequently, the study identifies an appropriate type of model based on the Box-Jenkins methodology for making the forecasts. The paper applies the static one step ahead forecasting method to the annual data over the period 1970-2015.

III. METHODOLOGY

3.1 Data Source and Model Justification

The study used secondary univariate quarterly time series data obtained from the NHIF in Tanzania past 22 years and contained 88 enrolee observations a minimum requirement of the generalization ability of time series analysis (Lwaho & Ilembo, 2023). SARIMA$(p, d, q)(P, D, Q)[4]$ model was used in the analyzing membership enrolment. To achieve this the function auto. arima () in R software was used to select the best model for forecasting membership enrolment based on the minimum value of Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) and Maximum log-likelihood. According to Wiah et al. (2022), the SARIMA model shows good prediction performance when used to forecast seasonal data compared to ARIMA models which are not capable of capturing seasonal patterns. Therefore, this study used the strengths of the SARIMA model, which is capable of capturing seasonality (Box and Jenkins, 1970).

3.2 The Conceptual Framework

The analysis in this paper is guided by the Box- Jenkins methodology of forecasting (Box & Jenkins, 1970). This model involves three major steps namely identification, estimation and forecasting. In such fields such as health, business and economics, usually time series contain a seasonal periodic component which repeats every "s" observations. This study considered quarterly observations where the $s = 4$, but the application to other values of s is straightforward. According to Box and Jenkins (1970) the generalized ARIMA model to deal with seasonality and defined general multiplicative seasonal model is given by:

$$\vartheta_p(B)\varphi_p(B^4)w_t = \theta_q(B)\xi_Q(B^4)a_t \quad \ldots \; 3.1$$

Whereby B = backward shift operator

$\vartheta_p, \varphi_p, \theta_q, \xi_Q$ = polynomials of order p, P, q, Q respectively and

$\{a_t\}$ = independent random variables with zero mean and variance σ_a^2

The shift operator $(B^4)w_t$ is such that $(B^4)w_t = w_{t-4}$. Therefore, equation (3.1) defines a stationary model provided that the roots of $\vartheta_p(B)\varphi_p(B^4) = 0$ lie outside the unit circle. In order to fit the model to a non- stationary series, Box and Jenkins (1970) again, suggest differencing the original series to remove both trend and seasonality following the procedure underneath:

$$w_t = \nabla^d \nabla_4^D x_t$$

Whereby $\nabla_4 x_t = x_t - x_{t-4}$ and that:

$$\nabla^{/}\nabla_4 x_t = \nabla_4 x_t - \nabla_4 x_{t-1} = x_t - x_{t-1} - x_{t-4} + x_{t-5}.$$

The values of the integers d and D do not usually need to exceed a unit. Details that describe the Box and Jenkins procedure to forecasting can also be found in the old literature by Naylor et al. (1972), Chatfield and Prothero (1973), and Thompson and Tiao (1971). However, this paper provides the detailing procedure described by Box and Jenkins in their popular work of 1970.

The procedure consists of fitting a mixed autoregressive integrated moving average (ARIMA) model to a given set of time series data and then taking conditional expectations. The main stages in setting up a Box-Jenkins forecasting model are as outlined below:

Model identification: In this step, the aim is to examine the data to reveal which member of the class of ARIMA processes appears to be the most appropriate. Estimation*:* In this step, the aim is to estimate the parameters of the chosen model by least squares. Diagnostic checking*:* In this step, the aim is to examine the residuals from the fitted model to see if it is adequate. Alternative model consideration*:* if the first model appears to be inadequate for some reason, then consideration of an alternative model other than ARIMA models may be tried until a satisfactory model is found. For non-seasonal data, first-order differencing is usually appropriate. Moreover, for seasonal data of period 4, the operator $\nabla\nabla_4$ is often used if the seasonal effect is additive, while the operator ∇_4^2 maybe used if the seasonal effect is multiplicative. In our case, the general SARIMA model is defined as $\vartheta_p(B)\varphi_p(B^4)w_t = \theta_q(B)\xi_Q(B^4)a_t$ has been used.

3.2.1 Stationarity Test

To test for stationarity, the Augmented Dickey fuller (ADF) test was used. The null hypothesis that there is a unit root will not be accepted at a given level of significance. The Augmented Dickey-Fuller test is a unit root based on stationarity (Dickey & Fuller, 1979). The unit-root based test is associated with the first lag of the time series variable. If the coefficient $(\gamma = 1)$ has a unit root, the time series behaves similarly to the random walk model which is non-stationary, and if the coefficient $|\gamma| < 1$ then, there is no unit root. Hence, we can test statistically whether the coefficient (γ) is equal to one or not. The Dickey-Fuller test adopts this procedure by carefully manipulating the equation, given as:

$$y_t = \alpha + \beta t + \emptyset\, y_{t-1} + e_t \quad \ldots \; 3.2$$

Also, written as

$$\Delta y_t = y_t - y_{t-1} = \alpha + \beta t + \gamma\, y_{t-1} + e_t \quad \ldots \; 3.3$$

In Dickey-Fuller test, we test the hypothesis

$H_0: \emptyset = 1$

$H_1: \emptyset \neq 1$

Correlograms

ACF and PACF are statistical measures that help to analyze the relationship between a time series and its lagged values. They are generally producing plots that are very important in finding the order of Autoregressive (AR) and Moving Average (MA) models.

Autocorrelation Function (ACF)

ACF measures the linear relationship between a time series and its lagged values. It assesses how much the current value of a time series depends on its past values. Autocorrelation is fundamental in time series analysis, helping identify patterns and dependencies within the data. The correlation between the current observation (y_t) and the previous observation (y_{t-k}) is given as:

$$\rho_k = \text{corr}(y_t, y_{t-k}) = \frac{\text{Cov}(y_t, y_{t-k})}{\sqrt{\text{Var}(y_t) \cdot \text{Var}(y_{t-k})}} = \frac{\gamma_k}{\gamma_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.4$$

Where, k = 1, 2, …..

Partial Autocorrelation Function (PACF)

PACF removes the influence of intermediate lags, providing a clearer picture of the direct relationship between a variable and its past values. Unlike Autocorrelation, partial Autocorrelation focuses on the direct correlation at each lag. The partial Autocorrelation function at lag k for time series is given as:

$$\emptyset_{11} = \text{Corr}(Y_{t+1}, Y_t) = \rho_1$$

$$\emptyset_{kk} = \text{Corr}(Y_{t+k} - \hat{Y}_{t+k}, Y_t - \hat{Y}_t), \text{k} \geq 2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.5$$

The suitable values of p and q will be selected by observing the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the time series data. The appropriate ARIMA models will be selected by observing the behaviour of ACF and PACF spikes based on the order identified (Hyndman & Athanasopoulos, 2018).

Seasonality Test

An approach to seasonality testing is to conduct seasonal decomposition using an additive model. The model decomposes time series into its trend, seasonal, cyclical, and regular components by assuming time series can be modeled by using those components. Visual examination can detect the seasonal component for regular patterns that repeat at a fixed interval. If the seasonal component exhibits regular patterns, this indicates the presence of seasonality in the time series.

3.3 Model Estimation

The parameters of the selected seasonal ARIMA (SARIMA) model with the specific values of (p, d, q) × (P, D, Q) $_S$ needs to be estimated. The maximum likelihood estimation (MLE) estimates the coefficients of the suggested models at the identification stage. Selection of the best model was based on AIC and BIC.

Akaike Information Criterion (AIC)

Kullback et al. (1951) developed a measure to capture the information that is lost when approximating reality. Kullback and Leibler measure is a criterion for a good model that minimizes the loss of information. Two decades later, Akaike established a relationship between the Kullback-Leibler measure and the maximum likelihood estimation (MLE) method that was used in many statistical analyses for model selection (Akaike, 1974). This criterion referred to as AIC, is generally considered the first model selection criterion that should be used in practice. The AIC is given as:

$$\text{AIC} = -\log(\hat{\theta}) + 2k \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.6$$

Where; θ is the set of model parameters, L ($\hat{\theta}$) is the likelihood of the candidate model given the data when evaluated at the maximum likelihood estimate of θ and k is the number of estimated parameters in the candidate model. Since AIC does not consider the effect of sample size, for small sample sizes, the second-order equation of the Akaike information criterion (AIC_C) is defined as:

$$\text{AICc} = -2\text{logL}(\hat{\theta}) + 2\text{k} + \frac{(2\text{k} + 1)}{(\text{n} - \text{k} - 1)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7$$

where n denotes the total number of observations.

A small sample size is when $^{n}/_{k}$ less than 40, also that when the number of observations increases, the third term in AIC_C approaches zero and will therefore give the same result as AIC in the equation 3.6

Bayesian information criterion (BIC)

Bayesian information criterion is another model selection criterion based on information theory but set within a Bayesian context. The difference between the BIC and AIC is the greater penalty imposed for the number of parameters

$$\text{BIC} = -2\log L(\hat{\theta}) + k log\, n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.8$$

where n denotes the total number of observations.

The BIC strongly penalizes the number of involved parameters. High values of AIC mean that the observed data does fit the model, while lower values indicate strong evidence that the observed data fit the models. Similarly, lower values of BIC indicate better fitting of the models.

3.3.3 Diagnostic Checking

Diagnostic check evaluating the acceptance of the fitted SARIMA model by examining the residuals, which are the difference between observed and predicted values. The aim is to ensure the residuals are random and do not contain any patterns or structures. The diagnostic checks involved the use of the Ljung-Box test statistic and the forecasting accuracy.

Ljung-Box Test

The Ljung-Box test helps to check whether the errors or residuals in our model have any pattern or correlation. The hypotheses under the Ljung-Box test is defined as:

H_0 = Residuals are independently distributed, correlation in the population from which the sample is taken is 0

H_1 = Residuals are not independently distributed; they exhibit serial correlation.

The Ljung-Box test statistic is given as;

$$Q = \mathrm{n(n+2)} \sum_{k=1}^{h} \frac{\hat{\rho}_k^2}{n-k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.9$$

Where n is the sample size, $\hat{\rho}_k^2$ is the sample autocorrelation at lag k, and h is the number of lags being tested. Under the null hypothesis, the test statistic Q asymptotically follows a $\mathcal{X}_{(h)}^2$, for the significant level α, the critical region is rejected if;

$$\mathrm{Q} > \mathcal{X}_{(1-\alpha),\mathrm{h}}^2$$

Where $\mathcal{X}_{(1-\alpha),h}^2$ is the $(1-\alpha)$ quantile of Chi-square distribution with h degrees of freedom.

After diagnostic checking, the fitted model will used in forecasting future values if the model is adequate. Otherwise, we need to repeat the selection and estimation method. Try with another potential candidate model (Ramasubramanian, 2007).

Forecasting and Forecasting Accuracy

Once the selected model has been verified, the model will be then used to predict membership enrolment in the next 24 months. After the forecasting, this study employs the Mean Percentage Error (MAE) and Mean Absolute Percentage Error (MAPE) to evaluate the forecasting accuracy of the selected model. The MAE and MAPE are given by:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|Y_i - \hat{Y}_i\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.10$$

$$\text{and } MAPE = \frac{100}{n}\sum\left|\frac{(p_i - o_j}{o_j}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.11$$

Where p_i is the predicted value for the i^{th} observations, o_i is the observed value for the j^{th} observation, n is the number of non-missing residuals.

IV. FINDINGS & DISCUSSIONS

4.1 Model Identification Process

Figure 1 shows the time series plots for the membership enrolment from 2002 Q1 to 2023 Q4. The plot **s**hows an unpredictable pattern in the long-term and the series serves to be seasonally increasing upward movement and nonstationary. However, the stationarity of the time series is confirmed by using the ADF test.

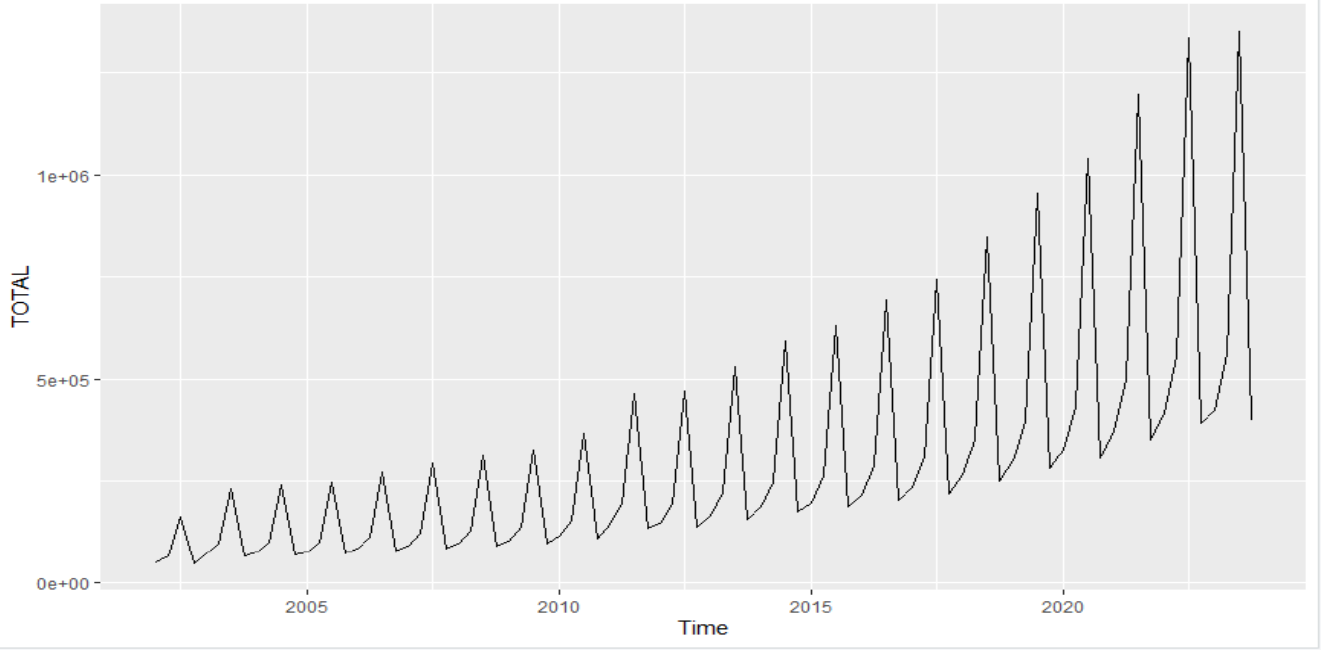

Figure 1
Time Series Plot of the Quaterly NHIF Membership Enrolment in Tanzania, 2002-2023

It was observed that there was an increasing upward but the peaks were not repeated with the same interval of time and not repeated with the same intensity rather than peaks increasing with increasing time. This shows that the series was not stationary. Since the original data show that they were nonstationary, the transformation was taken by natural logarithm and first differencing (d = 1) to make them stationary. Further checks were performed including the Dickey-Fuller test.

4.2 Augmented Dickey-Fuller (ADF) test

Table 1 shows that the results of the ADF test indicating that the null hypothesis is not rejected at 5% level of significance. Therefore, the series contains a unit root and it confirms that the data is not stationary. This result can be also inferred by examining correlograms in Figure 2 below.

Table 1
The Result of the ADF Test

Augmented Dickey-Fuller Test		
Dickey-Fuller = −0.963	Lag order = 4	P-value = 0.9389

Alternative hypothesis: Stationary

It was observed that there was an increasing membership enrolment upward but the peaks were not repeated with the same interval of time and not repeated with the same intensity rather than peaks increasing with increasing time. This shows that the membership enrolment was not stationary. Since the original data show that they were nonstationary, the transformation was taken by natural logarithm and first differencing (d = 1) to make them stationary.

4.2.1 Time Series Plot of Residuals

The adequacy of the model selected for forecasting is checked at this stage through the residuals concerning the specific variable. The residuals plot of ACF and PACF are given in Figure 2 below.

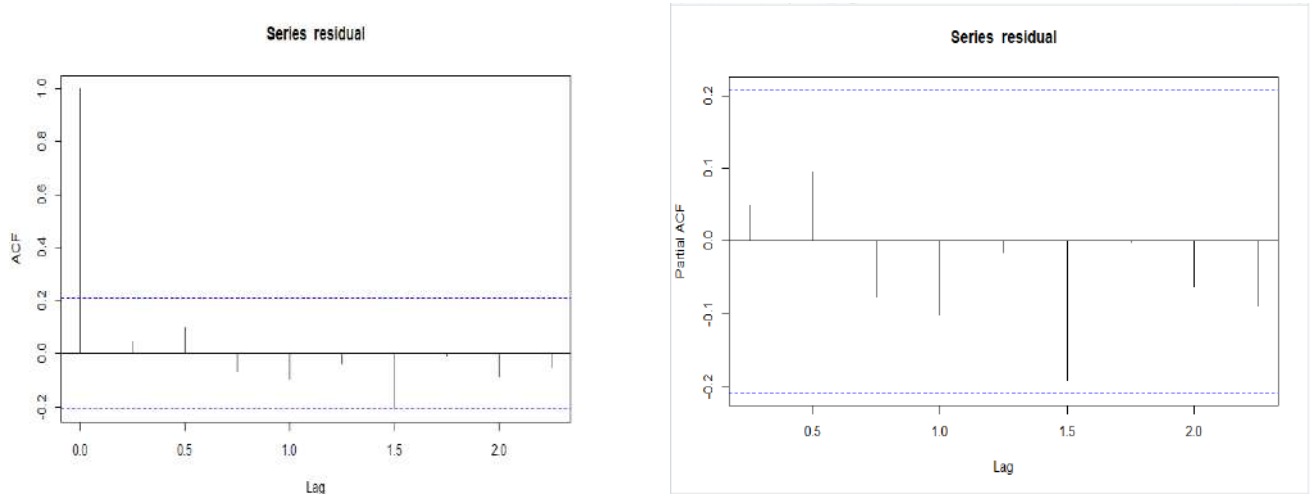

Figure 2
The ACF and PACF Residuals of the Fitted Model Total Seasonal ARIMA Model

The graphs of ACF and PACF in Figure 2 of the residual shows that most of the lags are within the limit which shows that the information was well captured through the model used and thus the model suggested is good for fitting the specific variable.

The seasonal ARIMA (3, 1, 1) (0, 1, 0) [4] was selected because of the lowest Akaike Information Criterion Corrected of 1896.91 and the largest log-likelihood of -943.46 among other models, and it was considered the best model for forecasting. The R-Software confirms the required model automatically. The model that was identified and selected was seasonal ARIMA (3, 1, 1) (0, 1, 0) [4]

4.2.2 Plots of the Residuals

Residuals of the variable are required to be normally distributed, the histogram of the residual of the total variable of the study showed the normality to prove that the model selected was the best fit. The Q-Q plot shows some data was out of the straight line so it shows that some data are not well fit. This is shown in Figure 3.

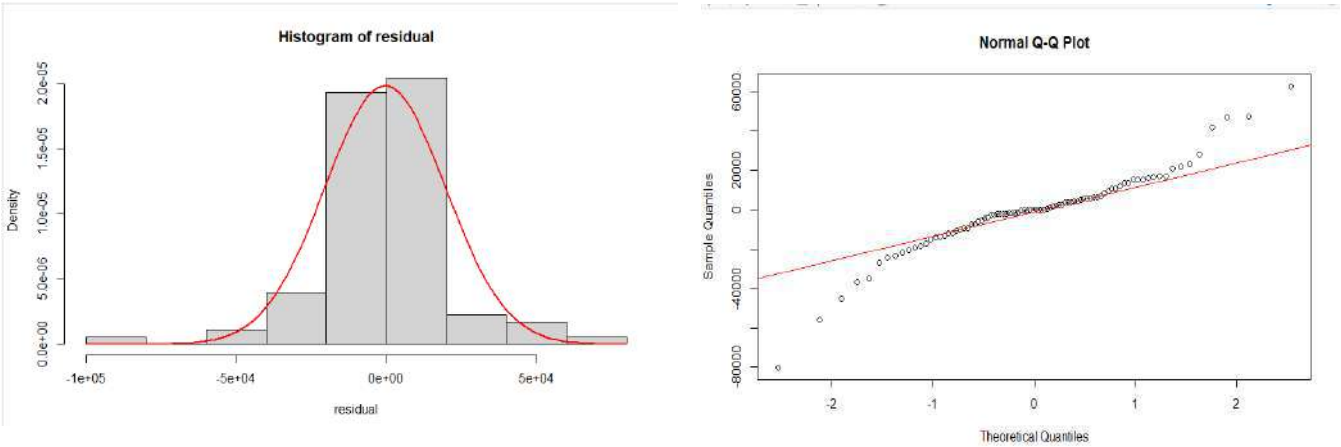

Figure 3
The histogram and Q-Q Plot of the Residuals

The time plot shows evidence of constant variance, the ACF plot shows that the errors are approximately uncorrelated, and the histogram shows that the errors are approximately normally distributed with mean zero. The adequacy test result is also confirmed by the formal test Ljung-Box in section 4.2.3 underneath.

4.2.3 Ljung-Box Test

The Ljung-Box test was performed on the residuals of the model and the results showed the p-value obtained was 0.2989 which is greater than 0.05 significance level (test value), indicating that residuals are uncorrelated and they are pure randomly.

4.3 Seasonality Test

Figure 4 shows the result of seasonal decomposition using an additive model conducted to detect seasonality. It observed that the seasonal component exhibits regular patterns or cycles, which indicate the presence of seasonality in time series. This is confirmed by the seasonality test which proves the presence of seasonality. This means that the SARIMA model could be an appropriate forecasting model to be used in predicting yearly membership enrolment to the National Health Insurance Fund.

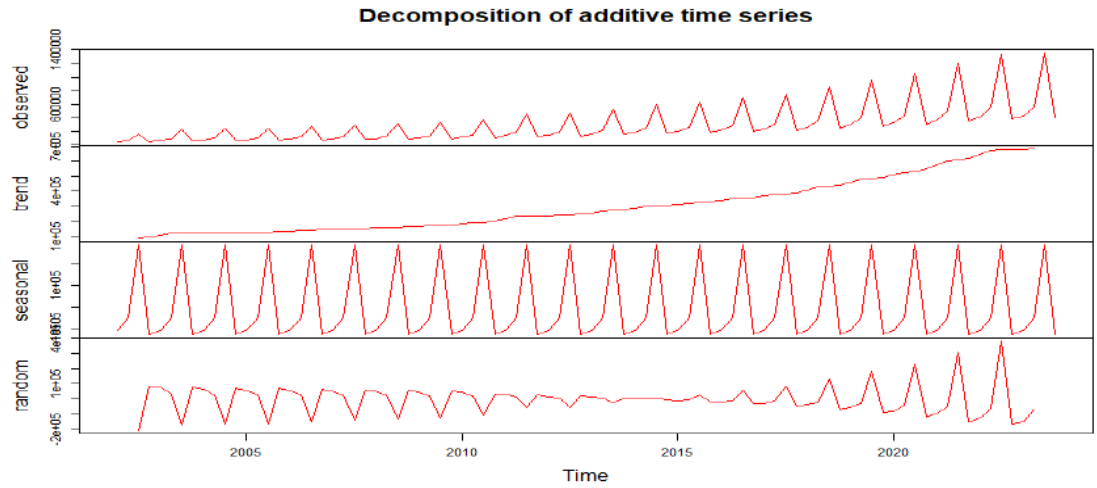

Figure 4
Time Series Decomposition

4.4 Model Selection and Estimation

Since the time series tends to be stationary, we have to identify the order of Seasonal ARIMA$(p, d, q)(P, D, Q)[4]$. To achieve this the function auto. arima () in R software was used to select the required model for forecasting membership enrolment with the minimum value of AIC and BIC and Maximum log-likelihood. The study arrived at a SARIMA model which is the Seasonal ARIMA (3,1,1) (0,1,0)[4] and was selected due to the lowest AIC of 1896.91, BIC of 1909.01 and the largest log-likelihood of -943.46 among the other models and it was considered as the best model for forecasting membership enrolment in the NHIF.

4.5 Validating the Model

The model validation is usually done to assess the precision of the model fit in estimating the observed values. The forecasted predictions for the validation set are plotted against the observed values as seen in Figure 5. The predicted values are fitted using the original data. It can be concluded that the model is best for the series.

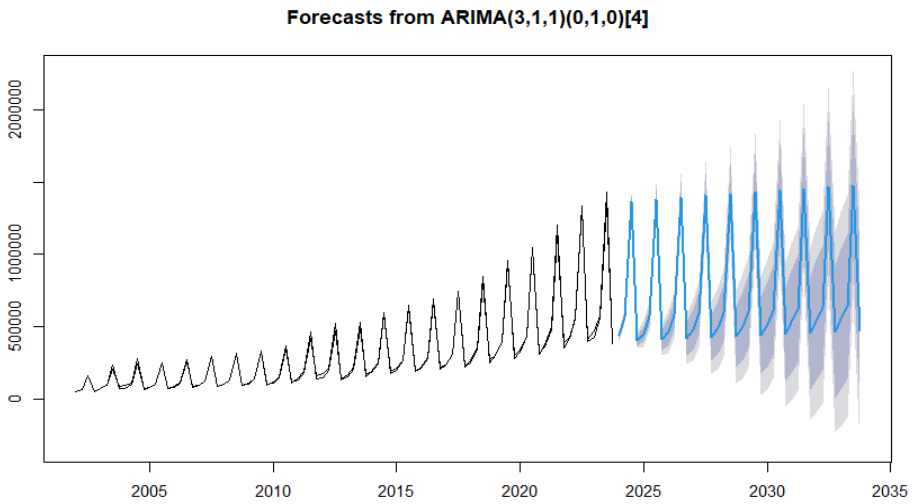

Figure 5
Model Validation of Membership Enrolment Using SARIMA (3,1,1) (0,1,0)[4]

4.6 Forecasting

Based on the SARIMA (3,1,1) (0,1,0)[4] model, the membership enrolment for the next 28 quarters is provided in Table 2, and Figures 6 and 7 show the trend of forecasted membership enrolment.

Table 2

Forecasted Values for the TOTAL Variable

Point	Forecast	Point	Forecast
2024 Q1	437937.6	2027 Q3	1407652.8
2024 Q2	568403.0	2027 Q4	423210.6
2024 Q3	1367749.9	2028 Q1	491066.5
2024 Q4	403209.4	2028 Q2	596782.6
2025 Q1	452009.3	2028 Q3	1420140.7
2025 Q2	575001.9	2028 Q4	430828.2
2025 Q3	1381497.4	2029 Q1	503319.4
2025 Q4	409295.3	2029 Q2	604665.4
2026 Q1	465482.1	2029 Q3	1432296.9
2026 Q2	581932.0	2029 Q4	438786.1
2026 Q3	1394786.7	2030 Q1	515279.7
2026 Q4	416002.7	2030 Q2	612817.6
2027 Q1	478474.1	2030 Q3	1444164.9
2027 Q2	589196.9	2030 Q4	447030.0

In Table 2, the forecasted enrollment numbers are increasing and others remain stable over time, so this indicates growing participation in the NHIF. The increase in registration membership for the coming years is shown evidently in Figures 6 and 7.

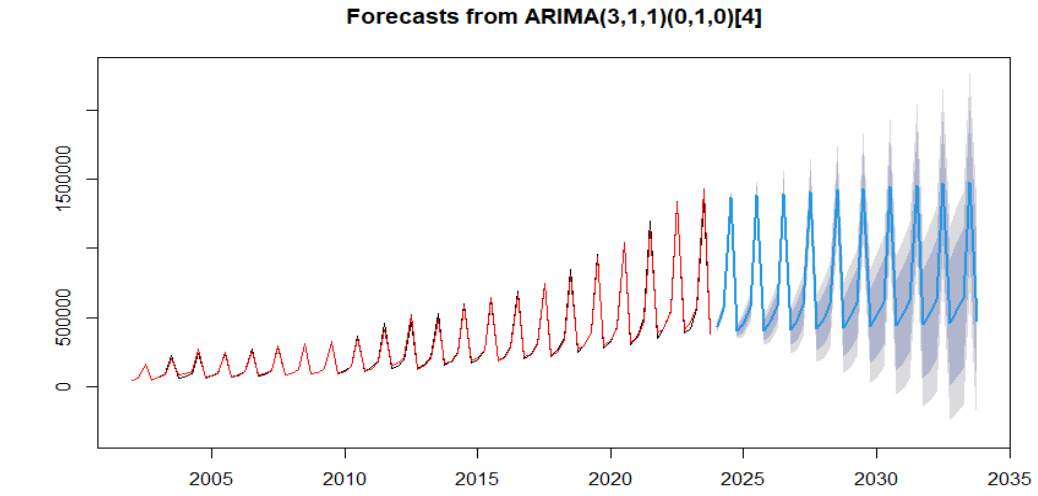

Figure 6

The Time Series Plot of the Forecasted Variable

Figure 7 presents the plot of total enrolment members against year (quarterly). The results show that there is a small upward increase in the enrolment members from 2024 to 2030, which is also a linear trend. Despite the fluctuations through increasing or decreasing from one year to another, this shows that through the line, there is an increasing trend of total enrolment of members over the years. Because the years before forecasting in 2022 (Q1, Q2, Q3, Q4) were (418516, 556034,1338918, 393610) and 2023 (Q1, Q2, Q3, Q4) were (423084, 562104,1353538, 397908) and remaining years were recorded respectively, so through the trend analysis of forecasted trend it shows there is an upward increase of enrolment for the years or quarterly. The trend line in the graph described below shows a positive slope most likely indicating an upward trend, signifying an increase in enrolment over time. This means the

data points show a general increase in total enrollment over time, The line slants upwards from left to right. This also signifies that as the x-axis value (time) increases, the y-axis value (enrolment) also increases.

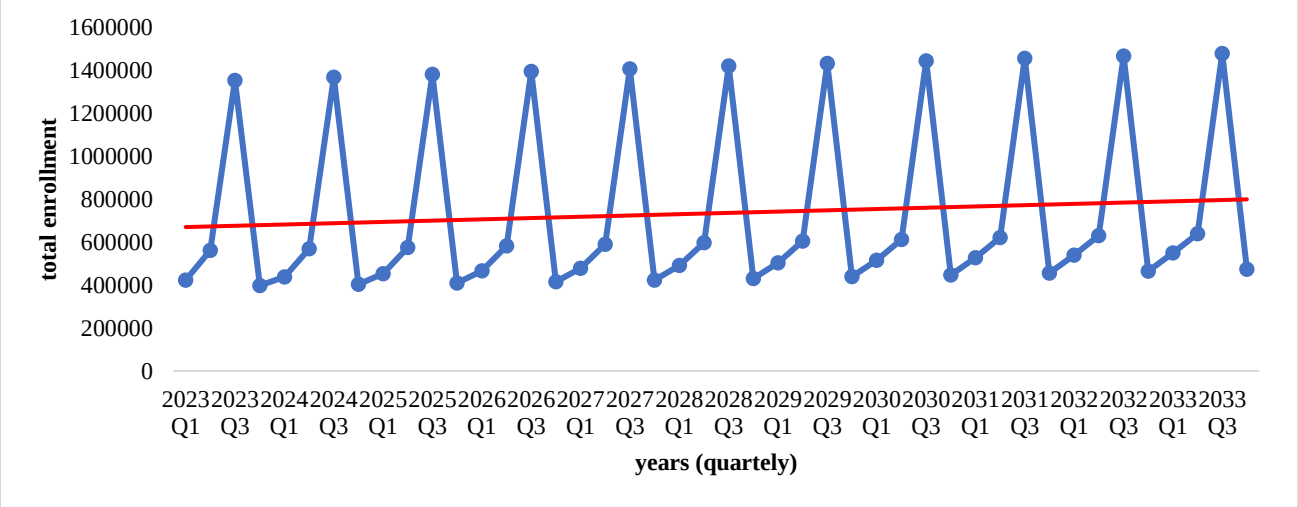

Figure 7
Trend Analysis of Forecasted NHIF Membership

V. CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

The general objective of the study was to forecast membership enrolment to the NHIF in Tanzania using the Seasonal autoregressive integrated moving average (SARIMA) model. SARIMA (3,1,1) (0,1,0)[4] model was used and was built following Box and Jenkins methodology for handling seasonal data. The results show that there will be an increasing membership enrolment overtime over the years and no signs of decreasing. The SARIMA model used, helped in determining future membership enrolment to the fund in the country. However, due to the fluctuation in the data series, this research needs to be extended by applying other methodologies such as the Autoregressive Integrated Moving Average with exogenous variable (SARIMAX), Simple Exponential Smoothing (SES) or the Holt-Winters Exponential Smoothing (HWES). These methodologies may improve the results from this study and widening the scope on how the forecasting of the membership enrolment to the country's national health insurance funds can be best handled.

5.2 Policy Recommendations

Findings from this study will enable policy makers in Tanzania and government officials responsible for the health sector, especially those responsible for the National Health Insurance Fund to make a well- informed decision in matters pertaining to NHIF enrolment, access and management of the insurance fund in general. Also, the government, should improve the NHI coverage to accommodate the increased enrolment and discourage dropouts. An increased enrolment when coupled with assured accessibility to the health services, will help to achieve the Universal Health Coverage (UHC) aim of ensuring equitable access to essential and manageable healthcare services which is inclusive, regardless of individuals' financial situations, their setting, and personality.

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The Influence of School Curriculum Support Materials on Academic Self-Concept of Pupils in Primary Schools in Informal Settlements in Kibra Sub-County, Kenya

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ABSTRACT

This study investigates the influence of curriculum support materials on pupils' academic self-concept in primary schools located in informal settlements in Kibra sub-county, Nairobi County, Kenya. Self Determination Theory guided this research. The study employed a survey research design. The target population comprised 138 head teachers, 203 teacher counsellors, and 9536 learners in Class VII, totaling 9877 respondents, from which a sample of 385 respondents was determined using Yamane's formula. Questionnaires were used to collect data from learners in Class VII, as were interview guides for head teachers and teacher-counsellors. Piloting was conducted among 38 respondents from primary schools in informal settlements in Kibra sub-county to establish validity and reliability. The opinions of educational psychology professionals were used to confirm the study's validity. The split-half method was used to calculate reliability, and the reliability coefficient ($r = 0.728$; $p<0.05$) at the 0.05 level of significance revealed a high degree of internal dependability. The findings indicated that there was a significant statistical relationship between curriculum support materials and academic self-concept of $r=0.539$, $p=0.002<0.05$. The study concluded that levels of pupils' academic self-concept in primary schools in informal settlements can be enhanced through provision of curriculum support materials. The Ministry of Education should enforce a policy that all schools in informal settlements should comply with standards for child-friendly school environment. The main beneficiaries of this study will be learners in the primary schools in the informal settlements when the policy makers improve in the provision of curriculum support materials hence their academic self-concept will be enhanced.

Keywords: Academic Self-Concept, Curriculum Support Materials, Informal Settlements, Primary Schools, Pupils

I INTRODUCTION

The efficacy of the teacher in effectively imparting information to primary school learners is contingent upon the presence of curriculum support materials of superior quality. In their transition from home to school, children bring their naiveté and innocence to elementary schools, as stated by Patterson and Fleet (2014). Teaching and learning resources that provide children with new experiences are among the factors cited by Paterson and Fleet (2014) as increasing the likelihood that students in elementary school would develop a positive attitude toward school. In other words, these resources help children open up in a variety of ways that promote their overall development as learners and people. Resource-based learning refers to a pedagogical approach that incorporates existing materials and tools into the classroom setting (Nyorere et al., 2022). The writers here emphasize the role that learning materials play in supplementing more methods of instruction. Each learner is drawn to the learning resource and material that is most suitable to their own information processing talents and styles, as stated by Ladan and Yabo (2023) as the foundation of resource-based learning.

Nyorere et al. (2022) nailed the value of educational materials perfectly. They claim that when students make use of these tools, they become more independent in their learning. This is due to their ability to think critically, ask insightful questions, analyze complex problems, and make sense of the data at hand. According to Kabwos et al. (2020) learning resources consist of all tools and methods used in the process of imparting the necessary understanding, competence, character traits, and habits of mind to students. In order to facilitate the transfer of knowledge to students, Munene (2021) divided instructional materials into two broad categories: printed and digital. According to Kenya Institute of Curriculum Development (KICD) (2019), educational materials may take several forms, including but not limited to manuals, videos, books, worksheets, games, quizzes, and exams. The textbook is the most widely used, examined, and recorded kind of classroom teaching input, according to KICD (2019). In the realm of education, the value of educational materials is widely acknowledged. According to the United Nations Educational, Scientific, and Cultural Organization (UNESCO) (2020), it is asserted that the significance of instructional materials in relation to student accomplishment is equivalent to that of teacher competency and school conditions. Thus, we can conclude that curriculum support materials are important in enhancing a learner's academic self-concept.

1.1 Statement of the Problem

The selection and development of new teaching resources is an essential part of developing any new curriculum, as pointed out by Ngwacho (2020). The incorporation of various learning materials indicates their value in the classroom. According to UNESCO (2020), ample quantities of high-quality learning materials should be made available to instructors and students in order to effectively support teaching and learning, which lends credence to this position.

The Kenya Institute of Curriculum Development (KICD) (2019) reports that teachers in Kenya have access to a wealth of materials. Examples include visual and auditory media like movies and TV shows, written materials like flash cards, and tactile tools like dolls and toys. All of these repurposed resources for education and training could prove useful for the kid. Visual aids, such as charts and pictures, may significantly improve a child's capacity for learning and memory. The classroom should be well-organized and spacious so that the teacher may easily circulate among the pupils and their belongings to check on their progress and offer words of encouragement. KICD (2019) recommends keeping learning resources in a visible and accessible spot in the home.

While the government of Kenya provides all primary schools with teaching and learning materials, a lack of these supplies makes it difficult for more students to enter kindergarten. According to Wanjohi et al. (2017), primary school students who are taught without the aid of teaching and learning tools are more likely to forget the content presented than those who are provided with appropriate instructional resources. If students in elementary school fully grasp the material, the instructors have succeeded in their pedagogical goals (Wanjohi et al, 2017). The quality of education is diminished when teachers make do with subpar resources because students simply cannot learn from them.

In Kibra Sub- County, the issue is the similar with numerous elementary schools complaining of low provision of curricular support resources. at a study conducted at a kindergarten in Kibra Sub-County, Wakahiu (2015) found that many students in primary schools located in informal settlements struggle to reach curriculum objectives because of a lack of appropriate educational components, namely, textbooks and other forms of instructional materials. This would imply that when curricular resources fall short, both educators and students face difficulties keeping up with the rest of the country. Inadequate curricular resources leave students feeling isolated, which in turn dampens their motivation to study and self-confidence. It's important to remember that even with the supply of curricular support materials, more study is needed to determine how the available resources are utilized by educators and students. The study attempted to fill a void created by Wakahiu (2015) and other empirical researchers by investigating whether or not primary schools in informal settlements would benefit from the simple provision and use of curricular support tools to guarantee the development of students' academic self-concept.

1.2 Specific Objective

To examine how curriculum support materials influence academic self-concept among pupils in primary schools in informal settlements in Kibra Sub- County, Nairobi County, Kenya.

1.3 Research Hypothesis

H_{01}: There is no significant influence of provision of curriculum support materials on development of academic self-concept among pupils in primary schools in informal settlements, in Kibra Sub- County, Nairobi County, Kenya.

II. LITERATURE REVIEW

2.1 Theoretical Review

2.1.1 Theory of Self Determination

This theory was postulated by Edward L. Deci and Richard M. Ryan in the early1980s. It alludes that motivation of human beings is guided by the need to satisfy 3 main psychological needs. According to Deci and Ryan (1985) they entail autonomy, competence, and relatedness. This concept depicts that meeting these needs results to enhanced self-motivation and mental wellness, which are important facets for the growth of an individual's self-concept more so in educational context.

A core assumption of SDT is that the surrounding environment considerably influences satisfaction of these psychological requirements. In learning contexts, curriculum support materials represent key functions in forming students' perceptions of competence and autonomy and in this regard, influences their academic self-concept. This theory posits that when pupils see themselves as adequate and capable of attaining their educational objectives, their inherent motivation improves. This results to a better and positive academic self-concept (Ryan & Deci, 2000). This correlation between environmental elements and psychological requirements proves SDT an applicable model for

assessing how curriculum support resources can influence students' self-concept in challenging settings e.g. informal settlements.

Self-Determination Theory is often critiqued for its wide applicability and the constraint of empirically evaluating the contentment of psychological needs across diverse settings (Vansteenkiste et al., 2005). A number of scholars debate that the theory's stresses on personal autonomy may not conform well to collectivist cultures. This is where communal objectives are often preferred over personal autonomy (Iyengar & De Voe, 2003). Nonetheless, SDT remains relevant in educational psychology for its extensive approach to gaining insight on motivation and self-concept.

2.2 Empirical Review

Eighty certified elementary and secondary school educators in Ghana participated in research by Quansah et al. (2019). The results indicated that although all of the participants had some level of professional education, only 12 had received training in the creation and use of learning tools for efficient curriculum delivery. The textbook, despite the significance of a variety of learning resources, becomes the most significant, if not the only, vehicle for delivering the curriculum in impoverished nations with unskilled instructors (UNESCO, 2013). Textbooks and other learning resources have a direct effect on what instructors teach and how they teach it, therefore the creation of curricula and the materials used in education are not just delicate issues, but also crucial to students' success in school. Therefore, legislators, principals, parents, and donors must create learning environments where teachers have access to the resources they need to effectively teach the curriculum in order to raise the quality of education. Donors and the government could rest easy knowing their large investments would not go to waste. Therefore, research is needed to explore the impact of instructors' and students' access to a wide range of high-quality learning materials on student accomplishment.

Hidayati's (2019) investigation of contributing factors of learning styles, teacher perspectives, and the availability of learning resources indicated that primary schools in Padang, West Sumatra, lacked access to suitable learning materials. His research suggests that while though many schools offer kids access to a wide range of audio, visual, and audio-visual resources, this isn't enough to satisfy their varied learning needs. The study's findings showed a correlation between students' chosen learning strategies and their efficient application of curricular resources.

Curriculum support resources, also known as educational inputs, are vital to the teaching of any subject at the primary school level, as they determine which topics are taught and in what order, as was found by Petriwskyj (2010) in research conducted in India. Books, instructor manuals, dictionaries, wall maps, atlases, writing implements, electronic devices (computers, projectors, DVD players, etc.), and more are all fair game and radios.

According to Riordan and Noyce (2013), textbooks and supplemental reading resources like dictionaries and encyclopedias play a key role in assisting elementary school children in their preparation for and performance on standardized examinations. This demonstrates the significance of primary schools' access to and use of curricular support materials in determining the quality of education provided; which in turn attracts more students and elevates their sense of self-worth. To back up these claims, Peters (2010) conducted a study in Russia among 23 primary schools about resources and excellence, and he discovered that students at schools with sufficient supplies of curriculum support materials had higher rates of regular school attendance and higher overall performance. Peters (2010) found that when students in elementary schools are exposed to novel pedagogical techniques using media like audio-visual presentations, they are more motivated to study.

The government of Nigeria, for instance, mandates that all citizens attend basic and secondary school. Baker et al. (2015) state that problems with mathematics education stem from a shortage of trained math instructors, bad teaching practices, and inadequate and incorrect utilization of teaching resources, and the perception of mathematics as an abstract and difficult subject are all to blame for students' lackluster performance in basic numeracy. Many students have dropped out of elementary schools as a result of situations like these.

A lack of resources and students' weak command of English were shown to significantly impact teaching and studying Integrated Science at rural Junior High Schools (JHSs) in the Effutu Municipality of Ghana's central region (Quansah et al.2019). The study concluded that Integrated Science education in rural JHSs would thrive if these issues were resolved. Integrated Science educators should be adept at improvising lessons using any materials they can get their hands on. Through practical application, students would have a deeper understanding of scientific concepts. The importance of pupils developing their English language skills in order to participate fully in the integrated science curriculum cannot be overstated.

Chepkemei et al. (2022) conducted a study to determine how well-prepared primary schools in Kenya were to apply the Competency Based Curriculum in the midst of the COVID-19 pandemic. According to the results of the study, in order for schools to efficiently implement their curricula despite the crisis brought on by the pandemic, they will need to increase and upgrade their physical resources. The study thus proposes that it is advisable for the

government to ensure the availability of appropriate teaching and learning resources in all schools to support the proper execution of the curriculum. According to the findings of this research, students are more motivated to put forth effort and attain high levels of performance when they have access to curriculum support materials that facilitate their learning and instill in them a sense of self-efficacy.

Mupa and Chinooneka (2015) research set out to identify and analyze primary school teaching and learning elements. Zero percent of seventh graders have passed their exams since 2013, which sparked the research. The researchers wanted to know why the country's educational institutions were in such disrepair. According to the results of the research, educators seldom utilize several strategies while instructing their students and rarely prepare many types of material for classroom usage. Textbooks and curricula represent the extent of teachers' resources for teaching. Inadequate textbooks, revision guides, and reference books were also identified as a problem in the survey.

Ngwacho's (2020) research in Nairobi schools confirmed the importance of instructional resources to student achievement. He argues that given the variety of accessible learning resources, instructors should use appropriate tools and strategies in the classroom. The author chimes on to say that instructional materials make it easier for instructors to convey information to students in a way that is correct, appropriate, clear, and intelligible. One way to do this is to simplify and concretize difficult concepts so that students may better grasp them.

The Florida Department of Education (2017) agrees with Baker et al. (2015) that using instructional tools in a strategic and creative way can significantly decrease apathy, pique the curiosity of elementary students by giving them something tangible to handle, and foster the development of their ability to think independently.

According to studies by Chetty (2019), the success of a program's implementation can hinge on whether or not enough textbooks and classroom space are available. Standa (1980), whose seminar article was cited by Chepkemei et al. (2022), argues that educators should have more opportunities to collaborate on the identification and application of appropriate online resources to address pressing pedagogical issues.

UNESCO (2020) found that Kenya has made great strides toward its goal to "develop a knowledge-based society" and meet the increasing regional and global competition for jobs thanks to the implementation of a competency-based curriculum in public primary schools. Almost everyone who participated in the survey (98.8%) felt unprepared to implement the competency-based curriculum; this was especially true for the study's new topics, which lacked appropriate teaching resources as compared to the study's older subjects (math and languages). The study found that providing schools with curriculum based on the idea of competences would be beneficial.

Ngwacho's (2020) investigation into the impact of the COVID-19 epidemic on the Kenyan educational system reveals that many low-income families in Kenya have suffered the effects of the shocks caused by the pandemic. School food and textbooks have become unaffordable due to the pandemic. The government's use of remote teaching to encourage distance learning and online education offered by radio, television, and the internet has resulted in an even greater gap in educational opportunity and quality for students from low-income, disadvantaged, and at-risk backgrounds who lack access to these resources.

Defending the rights of kids with disabilities to take part in mainstream classes, Kabwos et al. (2020) looked into the availability and suitability of learning tools for implementing inclusive education in public preschools in the Belgut Sub-County of Kenya. The study found that public preschools in Belgut Sub-County are unable to adopt inclusive education due to a lack of resources. The study found that public preschools need more money from the Ministry of Education so that teachers can afford more advanced resources. This emphasizes the significance of providing resources for education.

Research by Munene (2021) suggests that using animation to teach Kenya's secondary school students about political and historical concepts could be beneficial. According to the available data, educational documentaries and videos are being used in the classroom more frequently. This research set out to determine if showing pupils animated versions of military concepts helped them retain such learning later on.

III. METHODOLOGY

3.1. Research Design

The study employed a survey research design as it allows for the investigation of the current status of the phenomena in question (Creswell, 2014). Given the research focus on role of curriculum support materials on the academic self-concept of learners, data collection involved both quantitative and qualitative information, where questionnaires were used for quantitative data, and interviews for qualitative insights. This study employed concurrent triangulation, combining quantitative and qualitative data collection and analysis methods to provide a comprehensive view of curriculum support materials role in shaping academic self-concept.

3.2 Location of Study

The study was conducted in Kibra Sub-County, an informal settlement located five kilometres from Nairobi's Central Business District, with a population of 185,777 people. This research focused on primary schools in this region, which rely heavily on non-formal schooling. Given the recent effort to register such schools with the Ministry of Education and the documented challenges in education, including student disengagement and low self-concept, this area was chosen as the study location.

3.3 Target Population

In Kibra Sub-County, there are 138 non-formal primary schools that meet the Ministry of Education's registration guidelines, providing an education based on the regular curriculum. The target population for this study consisted of 138 headteachers, 203 teacher-counsellors, and 9,536 Class VII learners, totalling 9,877 potential respondents.

3.4 Sampling Techniques and Sample Size

The study employed stratified sampling, dividing Kibra Sub-County into seven zones. To ensure representative findings and evaluate the influence of curriculum support materials, purposive sampling was used to select 35 principals and 35 teacher-counsellors, while 315 students in Grade VII were randomly chosen from the sampling schools. The sample size was determined using Yamane's Formula, resulting in a desired sample size of 385 respondents, focusing on the role of teacher-learner interactions.

3.5 Data Collection Instruments

Data collection instruments included questionnaires for Class VII students with Likert Scale questions and interview guides for headteachers and teacher-counsellors. The questionnaires assessed aspects related to curriculum support materials and their influence on academic self-concept.

3.6 Piloting of Research Instruments

The research instruments were pilot-tested with 38 respondents from informal primary schools in Kibra Sub-County, ensuring their relevance and clarity. This process was essential for assessing instrument validity and trustworthiness.

3.7 Data Collection Procedures

The researcher, with the necessary permits and authorization, conducted data collection through visits to the schools. Research assistants were trained for questionnaire administration. Interviews were conducted with head teachers, teacher-counsellors, and students in Class VII.

3.8 Data Analysis and Presentation

Data analysis involved coding, frequency counts, and thematic analysis to examine the influence of curriculum support materials on academic self-concept. Descriptive and inferential statistics were conducted using statistical software. The study's quantitative results were presented through tables, charts, and narrative formats, focusing on curriculum support materials and their role in shaping academic self-concept.

IV. FINDINGS & DISCUSSION

4.1 Provision of Curriculum Support Materials and Pupils' Academic Self-concept

The goal of the study was to evaluate how curriculum support materials supplied in primary schools located in informal settlements affect students' assessments of their academic abilities. Information was received from pupils in class VII. Results are shown in Figure 1.

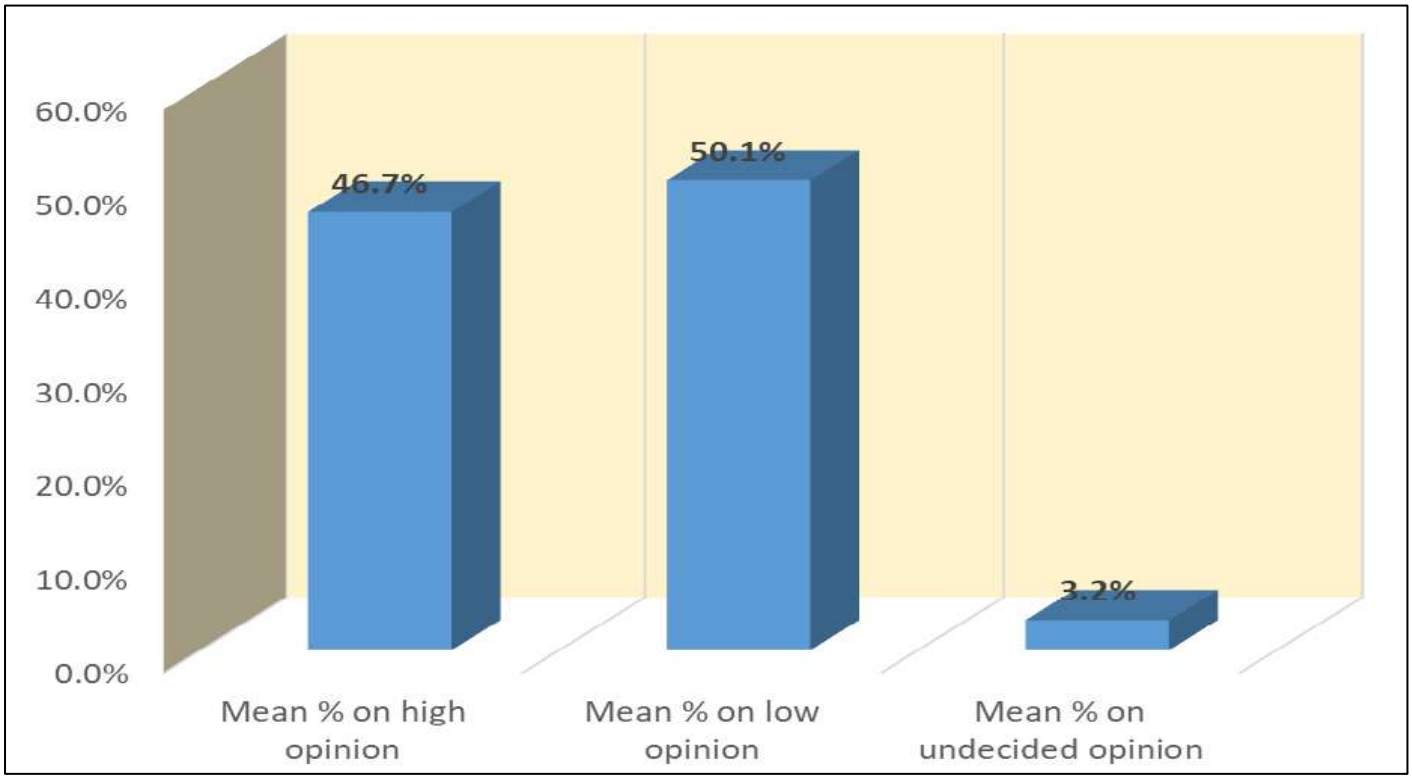

Figure 1
Views of Learners in Class VII on the Influence of Curriculum Support Materials on Academic Self-Concept.

According to Figure 1, a considerable percentage of students in primary schools located in informal settlements (46.7% to be exact) believe that the availability of curriculum support resources has a significant influence on their perceptions of their own academic talents. However, half of the respondents (50.10 percent) had a different opinion. It was agreed upon that primary schools in slum areas suffer from an insufficient supply of engaging textbooks covering a wide range of disciplines.

School administrators and guidance counselors concur with students that meeting the demand for textbooks while maintaining the ideal student-to-book ratio is difficult. H8, the school principal, delivered an announcement.

> *In my primary school, the learner-book ratio is still low and the textbooks we have cannot cater for the interests of all learners. In many cases, pupils share textbooks for critical subjects such as mathematics, sciences and languages.*

Teacher-counsellors have also voiced their endorsement of the viewpoint that the present accessibility of textbooks for primary school students residing in informal settlements is inadequate. TC7, the teacher-counselor, was being monitored.

> *In my class, many pupils do not complete their assignments due to lack of adequate textbooks for critical subjects.*

These findings corroborate the results of a research conducted by Wakahiu (2015) in Kibra Sub-County, which revealed that a significant proportion of students in primary schools located in informal settlements struggle to achieve the objectives outlined in the curriculum due to insufficient availability of instructional materials.

During the course of the interviews, it was observed by the headteachers and teacher-counsellors that the lack of resource centers, libraries, and essential reference books remains a significant obstacle in numerous primary schools located within informal settlements. The headteacher of H9 made a statement.

> *In my primary school, we have tried to ensure that learners have resource centers and critical learning materials. This has enhanced the desire of learners to attend academic activities.*

The statements were corroborated by the guidance counselors and educators. This study's findings corroborate those of Wakahiu (2015), who found that inadequate funding poses a serious challenge to students' ability to learn and succeed in elementary schools located in informal settlements. Both principals and teachers-in-charge said that arranging field trips as a means of instruction has proven challenging. H10, the school principal, remarked as follows:

> *In my primary school, the cost of taking pupils out for field trips and participate in outdoor learning activities has often made it difficult to engage in such programmes.*

The research showed that seventh graders felt that educational field trips outside of the classroom would be helpful. This result is consistent with research by Nyorere et al. (2022) that looked at how participating in educational field trips affected students' perceptions of their own abilities in the classroom.

It has been noted by teacher-counselors working in informal settlements that primary schools have a hard time arranging instructional field trips due to budgetary constraints and a lack of resources. This study's findings go counter to those of Paterson & Fleet (2014), who claim that educational resources like field trips improve students' dispositions toward school. As a result, students show more competence in developing a wide range of skills and benefit from smoother transitions between disciplines. In conclusion, the results show that, despite their limited use, study excursions serve as a significant teaching strategy by adding variety to the typical classroom setting.

4.2 Inferential Findings on the Influence of Provision of Curriculum Support Materials on Pupils' Academic Self-concept in Primary Schools in Informal Settlements in Kibra Sub-County, Nairobi County.

To test the null hypothesis, **H_{01}:** *There is no statistically significant influence of provision of curriculum support materials on development of academic self-concept among pupils in primary schools in informal settlements*, data were collected from the 30 teacher-counsellors on levels of adequacy (Adequate = 3, Not adequate = 2 and Not Sure = 1) of curriculum support materials and number of learners sampled in class VII who set academic goals as shown in Figure 2.

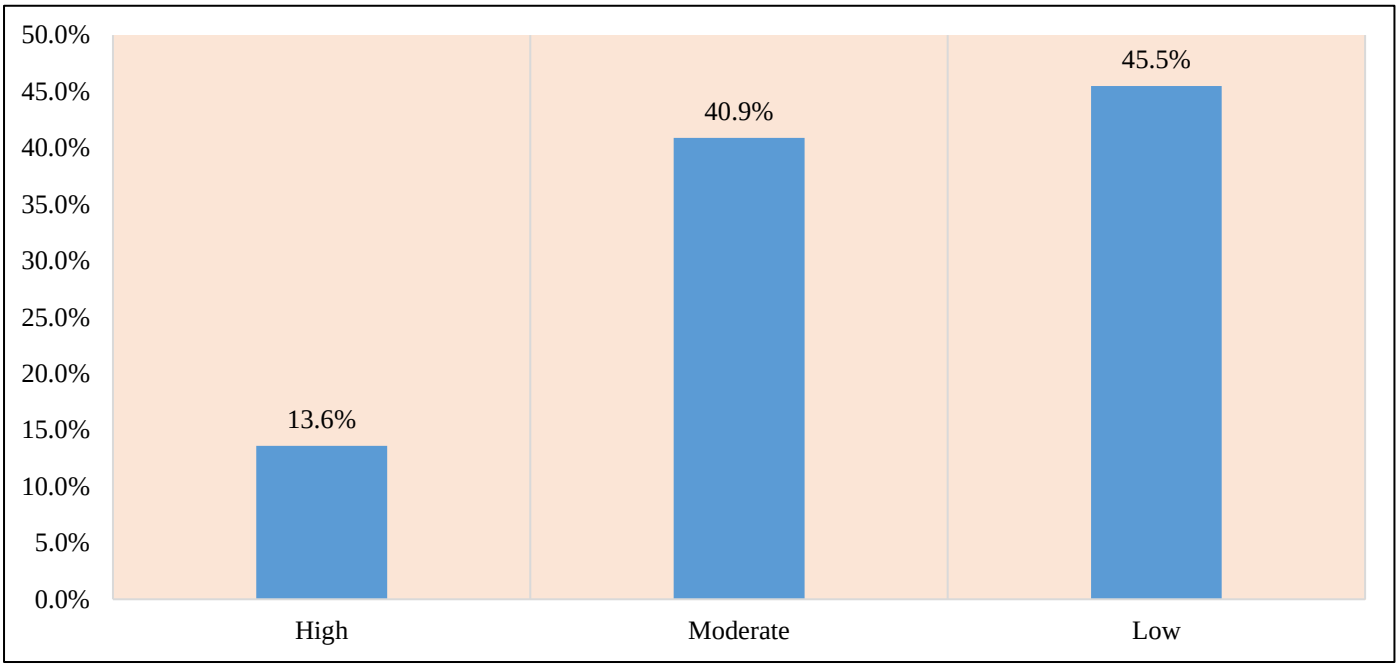

Figure 2

Levels of Adequacy of Curriculum Support Materials and the Number of Learners Who Set Academic Goals

Figure 2 shows how having access to extra-curricular materials might boost students' self-efficacy in the classroom. This suggests that students who are provided with better curricular materials will have a more positive view of themselves as students and will be more motivated to strive for more rigorous academic accomplishments. Pearson's Product Moment Correlation Analysis was applied to the data, and the results are shown in Table 1.

Table 1

Pearson's Product Moment Correlation Analysis Showing Relationship Between Levels of Adequacy of Curriculum Support Materials and Pupils' Academic Self-Concept

		Pupils' Academic Self-concept
Levels of Adequacy of Curriculum Support Materials	Pearson Correlation	.539**
	Sig. (2-tailed)	0.002
	N	30

**. Correlation is significant at the 0.001 level (2-tailed).

Table 1 displays the results of a Pearson's Product-Moment Correlation Analysis, which examines the connection between students' perceptions of their own academic aptitude and the quality of the supplementary materials they were given. A p-value of 0.002 indicates statistical significance, meaning the result is more likely to be true than not (p-value 0.05). The results showed a correlation with a coefficient of r = 0.539. As a result, we can't start with H02 as our foundation. This study adds to the growing body of evidence suggesting that exposing elementary school students to a variety of curriculum options outside of the classroom can have a positive effect on their confidence in the classroom.

This confirms the findings of Chetty (2019), who studied the lives of teachers of reading and writing in South Africa's Western Cape's poorest communities. According to the results, several factors have been identified as major contributors to underachievement in literacy. These factors include a lack of resources, family support, teacher expertise, curriculum changes, cognitive activities, and the complex social dynamics associated with poverty.

V. CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

The study's quantitative and qualitative results both confirmed that curriculum support materials play a significant impact in enhancing the academic self-concept of primary school students in informal settlements. Nevertheless, the study unveiled that primary schools situated in informal settlements suffer from a scarcity of textbooks, reference books, and narrative books across all disciplines, hence lacking the necessary resources to foster student motivation towards attending school. To put it another, a considerable proportion of students attending primary schools in informal settlements experience difficulties in achieving curricular objectives as a result of insufficient provision of instructional materials.

In contrast, a significant number of primary schools lack library facilities, while those that do possess libraries often lack additional reading or reference resources to cultivate students' enthusiasm for attending school, as well as lacking access to ICT devices. This observation highlights the ongoing difficulty in providing curriculum support resources in primary schools located in informal settlements, resulting in learners facing challenges in meeting their curriculum objectives. The study additionally indicated that primary schools located in informal settlements exhibit a limited occurrence of study excursions beyond the school premises, and the learners in these schools demonstrate a lack of utilization of diverse learning methodologies for acquiring new knowledge.

The findings were substantiated by the rejection of the null hypothesis, which posited that there would be no alteration in students' academic self-perceptions due to the implementation of curriculum support materials in primary schools situated in informal settlements. This conclusion was reached with a 95.0% confidence interval and a p-value of 0.002. This suggests that when elementary schools provide learners with sufficient curriculum support materials, it leads to the establishment of academic goals by many learners.

5.2 Recommendations

On provision of curriculum support materials and pupils' academic self-concept, the study recommends that schools should collaborate with donors and other well-wishers to support the primary schools with adequate scholastic materials to attain the standard learner-book ratio for quality education.

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On Some Aspects Of Degenerated Cyclic Codes

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ABSTRACT

Degenerated cyclic codes constitute a fascinating area of study within Coding Theory, offering profound insights into the realm of algebraic structures and their applications in error detection and correction. In this work, we delve into various aspects of degenerated cyclic codes, aiming to provide a comprehensive understanding of their properties and significance. We begin by elucidating the fundamental concepts underlying cyclic codes and their degeneration, establishing mathematical framework for analysis. Subsequently, we explore the algebraic structure of degener-ated cyclic codes, investigating their generator and parity-check matrices, as well as their relation-ships with conventional cyclic codes. Moreover, we investigate the decoding algorithms tailored for degenerated cyclic codes, evaluating their efficiency and performance under different error conditions. Furthermore, we examine the applications of degenerated cyclic codes in practical scenarios, highlighting their utility in diverse domains such as telecommunications, storage systems, and cryptography. Through theoretical analysis and numerical simulations, we demonstrate the efficacy and versatility of degenerated cyclic codes, thereby emphasizing their significance in modern information theory. Overall, this study contributes to the advancement of coding theory by shedding light on the intricacies of degenerated cyclic codes and paving the way for future research endeavors in this burgeoning field.

Keywords: Cyclic codes, linear block codes, algebraic coding, generator polynomials.

Mathematics Subject Classification: Primary 11H71; Secondary 14G50.

1 Introduction

Degenerate cyclic codes are subset of cyclic codes,[1, 4, 8, 12, 13, 16, 18, 20, 22] which are linear block codes defined by shifts of their codeswwords. Mathematically, a cyclic code of length n is generated by a polynomial $g(y)$ of degree r, where r is the dimension of the code. The codewords are obtained by polynomial multiplication of the message polynomial by $g(y)$ modulo $< y^n - 1 >$.
In the case of degenerate cyclic codes[22] , we intentionally introduce irregularities into the code by modifying certain coefficients of the generator polynomial $g(y)$. This modification results in a generator polynomial that may not conform to the standard form of a cyclic code. One common way to achieve degeneracy is by setting specific co-efficients of $g(y)$ to zero or adjusting them from their typical values. Mathematically, let $g(y) = g_0 \oplus g_y \oplus ... \oplus g_r y^r$ be the generator polynomial as defined in [2] of the cyclic code where $g_0, g_1, ..., g_r$ are coefficients in some finite field. To introduce degeneracy, we may set certain coefficients to zero, such as g_0 or g_r or modify them from their usual values. For instance, we could intentionally make $g(y)$ not monic, that is, $(g_r \neq 1)$, which leads to departure from the standard cyclic code.
Despite those modifications, degenerate cyclic codes still retain some crucial properties. For example, they remain cyclic, meaning that cyclic shifts of code words are still code words. This property facilitates efficient encoding and decoding algorithms, levaraging the structure inherent in cyclic codes thus contributing to coding theory [2, 5, 6, 7, 14, 19, 23].
Understanding Mathematical intricacies of degenerate cyclic codes is crucial for analyzing their properties, designing specific applications including specific applications and unique characteristics. Researchers often explore various methods for constructing and analyzing degenerate cyclic codes to unlock their potential advantages in error control, cryptography and their areas of information theory.

Departure of degenerated cyclic codes:
Degenerate cyclic codes, a subset of cyclic codes, have been studied for their structural properties and practical applications in error correction. Here are some works that delve into these codes and highlight the point of departure from degenerate cyclic codes to more general or different types of codes.

Firstly, the research in [9] provides a comprehensive overview of various error-correcting codes, including cyclic codes and discusses the properties of degenerate cyclic codes and transitions into the study of more general cyclic codes, BCH codes, and Reed-Solomon codes. Secondly, San and Chaoping [11] discuss cyclic codes and then explores the limitations of degenerate cyclic codes, moving on to more robust coding schemes such as BCH codes and Goppa codes. In [15], Mattson provides a detailed examination of error control coding techniques including cyclic codes. The research highlights the characteristics of degenerate cyclic codes, and then transitions to more powerful error-correcting codes like LDPC codes and turbo codes. Also, Ron [17], explains the concept of degenerate cyclic codes and their limitations, before introducing more advanced topics such as algebraic geometry codes and convolutional codes. In [3], Blahut provides a thorough understanding of algebraic codes, including cyclic codes. The research discusses the departure from degenerate cyclic codes to more efficient

codes like Reed-Solomon codes, focusing on their application in data transmission.

These works collectively illustrate the evolution of coding theory from the study of degenerate cyclic codes to the development and application of more advanced and efficient error-correcting codes. They highlight the limitations of degenerate cyclic codes and the need for more powerful coding schemes in practical applications.

2 Preliminaries

Degenerate cyclic codes are a subset of cyclic codes characterized by repeated or linearly dependent code words due to the generator polynomial having repeated roots, resulting in reduced error-correcting capabilities and lower minimum distances. They illustrate the importance of polynomial selection in the design of cyclic codes. By studying these degenerate cases, we gain insights into the properties that make certain cyclic codes more effective for error correction, guiding the development of more powerful and reliable coding schemes.

Cyclic codes A *cyclic code* of length [1, 4, 8, 12, 18, 20, 22] n over a finite field F_q is a linear code such that if $c = (c_0, c_1, \ldots, c_{n-1})$ is a code word, then the cyclic shift $(c_{n-1}, c_0, c_1, \ldots, c_{n-2})$ is also a code word. Mathematically, this can be expressed as follows:

Theorem 2.1. *Let C be a linear code of length n over F_q. Then C is a cyclic code if and only if for every code word $c = (c_0, c_1, \ldots, c_{n-1}) \in C$, the code word $(c_{n-1}, c_0, c_1, \ldots, c_{n-2})$ also belongs to C.*

Generator polynomial: Given a cyclic code of length n over a finite field F_q, the generator polynomial $g(x)$ is a polynomial of degree k that divides $x^n - 1$ in $F_q[x]$. The code consists of all multiples of $g(x)$ modulo $x^n - 1$, and can be expressed as:

$$\mathcal{C} = \{c(x) = g(x)q(x)(x^n - 1) \mid q(x) \in F_q[x], \deg(q(x)) < n - k\}$$

Parity Check Matrix: The parity check matrix H of a cyclic code can be constructed using the generator polynomial g(y). It has dimensions $(n - r) \times n$ and it is derived from coefficients of $g(y)$ using certain algebraic structure.

Degenerated Cyclic Codes: A degenerate cyclic code of length n over a finite field F_q is a linear code characterized by the following properties:

i. It is a subset of the cyclic code, where the code words exhibit redundancy or linear dependence.

ii. The generator polynomial $g(x)$ of the code has at least one repeated root over F_q, causing the code to contain repeated or linearly dependent code words.

iii. Formally, if $g(x)$ is the generator polynomial of degree k, then the code can be described as:

$$\mathcal{C} = \{c(x) = g(x)q(x)(x^n - 1) \mid q(x) \in F_q[x], \deg(q(x)) < n - k\}$$

where $\mathcal{C}$ denotes the set of all code words of the degenerate cyclic code.

Relaxed ConStraints: Degenerated cyclic codes involve relaxing some of the constraints imposed on standard cyclic codes, such as the requirement for the generator polynomial to divide $< y^n - 1 >$ exactly. This relaxation allows for a border range of polynomial structures, potentially leading to improves code properties or easier encoding-decoding procedures.

Algebraic Operations: Analysis of cyclic and degenerated cyclic codes involves algebraic operations in polynomial rings and quotient rings. Addition and scalar multiplication of polynomials modulo $< y^n - 1 >$ are fundamental operations used in code construction and manipulation.

Ring Theory: The study of quotient rings and ideals provide rigorous mathematical foundation for understanding the structure and properties of cyclic and degenerated cyclic codes. Concepts from Ring Theory, such as factorization and prime ideals, play a crucial role in code analysis.

Code distance and error correction: Analysing the distance properties both standard and degenerated cyclic codes involving examining the algebraic relationships between code elements and their implications for error detection and correction algorithms [10, 21].

Magma Algorithms for constructing the degenerated cyclic codes:

Example in Magma

```
n := 7;
F := GF(2);
R<x> := PolynomialRing(F);
g := (x - 1)^2 * (x^3 + x + 1);

C := CyclicCode(n, g);
Codewords := { c : c in C };
"Minimum Distance:", MinimumDistance(C);
"Weight Distribution:", WeightDistribution(C);

Cyclic Code: [7, 4, 3] Cyclic Linear Code over GF(2)
Generator matrix:
[1 0 0 0 1 1 0]
[0 1 0 0 0 1 1]
[0 0 1 0 1 1 1]
[0 0 0 1 1 0 1]
Minimum Distance: 3
Weight Distribution: [ <0, 1>, <3, 7>, <4, 7>, <7, 1> ]
```

3 Main Results

In the research work of the paper we assume $HCF(n, q) = 1$, hence give some aspects of degenerated cyclic codes of length n over F_q.

Theorem 3.1. *Let $HCF(n,q)=1$. These statements are equivalent. [20, 22]*

i. **The generator polynomial $g(y)$ has no repeated roots modulo y^n-1:** This asserts that the generator polynomial of the cyclic code, $g(y)$, does not have any repeated roots when considered modulo y^n-1, meaning that every root has multiplicity 1.

ii. **The code has no repeated code words:** This implies that the cyclic code does not contain any repeated code words. Each distinct message corresponds to unique code word in the code word. Now, lets establish the equivalence between these statements in the context where $gcd(n,q)=1$. Firstly, for cyclic codes [1, 4, 8, 12, 13, 16, 18, 20, 22] over $GF(q)$, where q is the size of the finite field. If the $gcd(n,q)=1$, then the order of any non-zero element in $GF(q)$ is co-prime to n. This implies that the polynomial y^n-1 has distinct roots in the field $GF(q)$, as every non-zero element generates a distinct root. Consequently, any polynomial $g(y)$ with distinct roots will also have distinct roots modulo y^n-1. Since the roots of $g(x)$ are distinct modulo y^n-1, it follows that the corresponding code words will also be distinct, ensuring that $gcd(n,q)=1$.
Thus, under the condition $gcd(n,q)=1$, the statements, "the generator polynomial $g(y)$ has no repeated roots modulo $<y^n-1>$," and, "the code has no repeated code words," are equivalent in the context of degenerated cyclic codes.

Proof. A cyclic code C of length n over F_q is degenerate:

i **Degeneracy of cyclic code:** Degeneracy of cyclic codes arises when certain coefficients of its generator polynomial manipulated to deviate from the typical form. This manipulation introduces irregulatories into the into the code's structure, leading to the departure from the standard cyclic geometry.

ii **Mathematical elaboration:** Let $g(y)$ be the generator polynomial of the cyclic code C. It is typically of the form $g(y)=g_0+g_1y+...+g_ry^r$ where $g_0,g_1,...,g_r$ are coefficients in F_q.
In the case of a degenerate cyclic code, certain coefficients of $g(y)$ are modified from their standard values. This modification could involve setting specific coefficients to zero, adjusting them arbitrarily, or making the polynomial non-monic, that is, leading coefficients g_r is not necessarily 1.

Mathematically, the manipulation of coefficients might be represented as $g(x)=g_0'+g_1'y+...+g_r'y^r$ where $g(x)=g_0'+g_1'+...+g_r'$ are the altered coefficients. □

Implication of degeneracy: The introduction of irregularities into the generator polynomial alters the algebraic structure of the code. This deviation from the standard cyclic symmetry can affect the properties such as minimum distance, error-correction, and decoding complexity.
Degenerate cyclic codes may exhibit unique characteristics that make them suitable for specific applications, for example, intentional degeneracy might enhance the code's ability to correct certain types of errors to improve performance under particular channel conditions.

In summary, the degeneracy of cyclic codes over F_q is manifested through intentional modifications to its generator polynomial, leading to deviations from the standard cyclic structures. Understanding the mathematical implications of degeneracy is essential for analyzing the properties and applications of such codes in various communication and storage systems.

Theorem 3.2. *There exist integers* $r, 1 < r < n$, *and* $s, 1 < s < n$, *such that* $n = rs$ *and* $1 + y^s + ... + y^{(r-2)s} + y^{(r-1)s}$ *divides* $g_c(y)$.

Proof. Given $n = rs$, where $1 < r < n$ and $1 < s < n$, we can express $gC(y)$ as $gC(y) = (1 + y^s + ... + y^{(r-2)s} + y^{(r-1)s})Q(y)$ mod rs since $n = rs$, we can rewrite the expression as: $gC(y)$ mod n $= (1 + y^s + ... + y^{(r-2)s} + y^{(r-1)s})Q(y)$ mod n.

Now, we can see that each term of the $(1+y^s+...+y^{(r-2)s}+y^{(r-1)s})$polynomial will be congruent to zero modulo n because each term is divisible by s, (which divides n), modulo n, implying that n divides $g(y)$. So, $n = rs$ divides $gC(y)$, content... □

Theorem 3.3. *There exists integer* $r, 1 < r < n$, *and* $s, 1 < s < n$, *such that* $n = rs$ *and* $g_C^{\perp}(y)$ *divides* $y^s - 1$.

Proof. Given $n = rs$, where $1 < r < n$ and $1 < s < n$, we show that $g_C^{\perp}(y)$ divides $y^s - 1$. First, lets express $y^s - 1$ in terms of its factors.
By using the difference of squares formula, we have $y^s - 1 = (y^{\frac{s}{2}} - 1)(y^{\frac{s}{2}} + 1)$.
Now, we want to show that $g_C^{\perp}$ divides $y^s - 1$, or equivalently, that, $y^s - 1$ is congruent to zero modulo $g_C^{\perp}(y)$.

Lets express this mathemayically:
$y^s - 1 \equiv 0 \text{mod} g_C^{\perp}(y)$
This means that there exists some $P(y)$ such that $y^s - 1 = p(y).g_C^{\perp}(y)$
Now we need to show that such a polynomial $P(y)$ exists:
Given that $n = rs$ divides $gC(y)$ according to the definition of $gC(y)$ it follows that $g_C^{\perp}(y)$ must divide $y^s - 1$ because $y^s - 1$ is a factor of $y^r - 1$ (by setting $r = \frac{s}{2}$).
Therefore, we have shown that there exist integer r and s, such that $n = rs$ and $g_C^{\perp}(y)$ divides $y^r - 1$. □

Theorem 3.4. *Let* $m > 1$. *Let* C' *be a cyclic code of length* n'. *Let* $C = R_m(C')$ *be a degenerate cyclic code. Then given:*

i) $m > 1$

ii) C_1 *is a cyclic code of length* n_1

iii) $C = R_m(C_1)$ *is a degenerate cyclic code, where* R_m *denotes the* m^{th} *repeated concatenation operation.*

Proof. The repeated concatenation operation R_m takes cyclic code C_1 and replicates m times.
Now, we want to understand the properties of the degenerate cyclic code C obtained by $m-$fold repetition of C_1
Lets denote the generator polynomial of C_1 as $g_1(y)$. Since C_1 is cyclic, $g_1(y)$ generates C_1 and divide $(y^{n_1} - 1)$.

□

Now, to understand C mathematically, we need to analyze its properties:

i) **Linearity:** C is still a linear code because repetition does not affect linearity of the code, [19].

ii) **Cyclic property:** C inherits the property from C_1 because the cyclic shifts of codewords in C_1 will result in cyclic shifts of codewords in C, [1, 4, 8, 12, 18, 20] .

iii) **Generator polynomial:** The generator polynomial of C is $g(y) = [g_1(y)]^m$,[2].

iv) **Minimum distance:** The minimum distance of C may change depending on the property of C_1 and m. If C_1 has a minimum distance d_1, then C will have a minimum distance atleast d_1 (but it could be higher depending on specific codewords resulting from the repetition), [20].

v) **Encoding and decoding:**Encoding and decoding for C can be derived from these of C_1, possibly with some modifications due to repetition, [6, 10, 21] .
In summary, a degenerate cyclic code obtained by repeating a cyclic code with modified parameters, while retaining many of the properties of the original code

From $g_C(y) = g_{C'}(y)(1 + y^{n'} + y^{2n'} + ... + y^{n-n'})$, then:

i) $gC(y)$ is the generator polynomial of a cyclic code C of length n.

ii) $gC_1(y)$ is the generator polynomial of a cyclic code C_1 of length n_1.

iii) The expression $(1 + y^{n'} + y^{2n'} + ... + y^{n-n'})$ represents the polynomial factor.

Thus the equation $g_C(y) = g_{C'}(y)(1 + y^{n'} + y^{2n'} + ... + y^{n-n'})$ states that the generator polynomial of C can be obtained by multiplying the generator polynomial of C_1 with a polynomial factor that accounts for certain cyclic shifts.

Hence elaborating Mathematically:

i. **Generator polynomial of C:** The generator polynomial $gC(y)$ of code C represent all the code words of C which generates the cyclic code C that has a length of n.

ii. **Generator polynomial of C_1:** The generator polynomial $gC_1(y)$ of code C_1 represent all the code words of C_1 which generates the cyclic code C_1 that has a length of n_1.

iii. **Polynomial factor:** The polynomial factor $(1 + y^{n'} + y^{2n'} + ... + y^{n-n'})$ represents a polynomial that includes terms corresponding to the cyclic shifts of the code words of C_1 to form code words of C. The terms account for the cyclic nature of the code.

iv. **Multiplication:** Multiplying $gC_1(y)$ by the polynomial factor results to a new polynomial, $gC(y)$, which includes all the terms needed to generate the codewords of C based on the codewords of C_1, and their cyclic shifts.

In summary, the equation $g_C(y) = g_{C'}(y)(1 + y^{n'} + y^{2n'} + ... + y^{n-n'})$ mathematically expresses how the generator polynomial of a cyclic code C can be constructed from the generator polynomial of a cyclic code C_1 by inco-orperating a polynomial factor that accounts for the cyclic shifts. Generally, see [2]. Given ${g_C}^{\perp}(y) = {g_C}^{\perp}(y)$ suggests that the dual generator polynomial of a cyclic code C is equal to the dual generator polynomial of another cyclic code C_1, that is:

i. $gC^{\perp}(y)$ is the dual generator polynomial of a cyclic code C.

ii. ${gC_1}^{\perp}(y)$ is the dual generator polynomial of another cyclic code C_1.

The dual generator polynomial represents the polynomial whose roots correspond to the non-zero elements of the dual code. The dual code of a cyclic code is also cyclic on $gC^{\perp}(y) = gC_1{}^{\perp}(y)$ mathematically, thus we can consider the properties of the dual codes:

i. **Generator polynomial of the dual code:** The dual generator polynomial $gC^{\perp}(y)$ of a code C generates a dual code, whose codewords are orthogonal to the codewords of C.

ii. **Generator polynomial of the dual code C_1:**
Similarly, $gC_1{}^{\perp}(y)$ generates the dual cyclic code $gC_1{}^{\perp}$ which is orthogonal to the codewords of C_1.

iii. **Equivalence of the dual codes:** The equation $gC^{\perp}(y) = gC_1{}^{\perp}(y)$ implies the dual cyclic code $C^{\perp}$ and $C_1{}^{\perp}$ polynomial. This means that the structure of the orthogonal codewords of C and C_1 is the same.

iv. **Orthogonal preservation:** Since the dual generator polynomial determines the structure of the orthogonal codewords, the equation suggests that orthogonality properties are preserved between $C^{\perp}$ and $C_1{}^{\perp}$.

In summary, the equation $gC^{\perp}(y) = gC_1{}^{\perp}(y)$ mathematically expresses that the dual generator polynomial of a cyclic code is equal to the dual generator polynomial of another cyclic code C_1, indicating that their dual codes have same structure and orthogonal properties.

Theorem 3.5. *Let $gcd(n, g) = 1$. Let $n = p_1^{e_1}, ..., p_t^{e_t}$ be the prime decomposition of n, let $N(d)$ be the number of the divisors of $X^d - 1$ over F_q. Then the number of the degenerate cyclic codes of length n over F_q is $\sum_{l=1}^{t}(-1)^{l+1} \sum_{\{i_1,...,i_l\}\in\{1,...,t\}} N(\frac{n}{p_{i_1},...,p_{i_l}})$*

Proof. The proof can be found in [20] □

This theorem addresses the enumeration of degenerate cyclic codes over a finite field F_q of the length n, under the condition that the gcd of n and q is 1. Lets break down the theorem and its mathematical implications:

3.0.1 Degenerate cyclic codes

Cyclic codes are subclass with additional properties.

Definition 3.1. *N(d)is the number of divisors of $y^d - 1$ over F_q, In other words, it presents the number of elements in F_q that are roots of the polynomial $y^d - 1$.*

Cyclic codes are subclass with additional properties
Implications:
The theorem suggests a connection between the structure of cyclic codes and the roots of certain polynomials over the finite field F_q.
The prime factorization of n plays a crucial role in determining the number of degenerate cyclic codes of length n over F_q.
It involves the summation over subsets $\{i_1, ..., i_l\}$ of the subset $\{1, ..., t\}$, where t is the number of the distinct primr factors in the prime factorization of n.
For each subset, the product $n = p_1^{e_1}, ..., p_t^{e_t}$ is calculated, representing a divisor of n obtained by

selecting certain prime factors and their correspondiiing exponents.
The function N comes into play to count the number of roots of $y^d - 1$ over F_q where d is the divisor obtained from the current subset **Mathematical Elaboration:**
The theorem involves iterating over all possibilities subsets of prime factors of n, each time calculating a divisor d of n and finding the number of roots of $y^d - 1$ over F_q
These counts are then combined using a summation formula with alternating signs $(-1)^{l+1}$ where l is the size of the current subset being considered.
The result of this computation gives the number of degenerate cyclic codes lengthn n over F_q. In summary, the above Theorem provides a Mathematical relationship between the structure of cyclic codes and properties of certain polynomials over finite finite fields, specifically in terms of their roots and the prime factorization of the code length.

Theorem 3.6. *Let $m > 1$, let $C = R_m(C')$ degenerate cyclic code. Then $\mathcal{H}(C) = \mathcal{R}_m(\mathcal{H}(C'))$.*

Proof. The proof can be found in [20]. □

Definition 3.2. *Let C be a degenerate cyclic code over F_q, where $C = \mathcal{R}_m(C)$. Here, $\mathcal{R}_m$ denotes the ring formed by polynomials of degree less than m over F_q.*
Let $\mathcal{H}(C)$ be the parity-check matrix.
Let $C^\perp$ denote the dual code of C.

Theorem 3.7. $\mathcal{H}(C) = \mathcal{R}_m(\mathcal{H}^\perp_{C^\perp})$.

Proof. Let $\{g_1, g_2, ..., g_k\}$ be the basis for C, then, $\mathcal{H}(C)$ is formed by taking two row vectors corresponding to the orthogonal complements of $g_1, g_2, ..., g_k$.

The dual code $C^\perp$ consist of all vectors v such that $< v, c >= 0$ for all $c \in C$, where $< ., . >$ denote the dot product.
Let $\{h_1, h_2, ..., h_k\}$ be a basis for $C^\perp$.
Then, $\mathcal{H}^\perp_{C^\perp}$ is formed by taking the row vectors corresponding to $\{h_1, h_2, ..., h_k\}$.
Since C is degenerate cyclic code, its generator polynomial can be represented as $g(y) = yhy$, where $h(x)$ is a polynomial degree $k - m$
$\mathcal{H}(C)$ be represented as $\mathcal{H}^\perp_{C^\perp}$ due to specific structure induced by degenerate by cyclic code of C.
Thus, theorem establishes a specific relationship between the parity-check matrix of a degenerate cyclic code and the parity-check matrix of its dual code. □

Corollary 1. *Let $m > 1$, let $C = R_m(C')$ degenerate cyclic code. Then $\mathcal{H}(C) = \mathcal{R}_m(\mathcal{H}(C'))$, See [20].*

In this corollary, C' represents the generator polynomial of the cyclic code C. The statement suggests the Hamming Weight of a code C is euivalent to the hamming weight of its generator polynomial C'.
Elaborating mathematically, we can explain it as follows:

a **Degenerate cyclic codes:** A cyclic code C is called degenerate if its generator polynomial has roots in common, $y^m - 1$ for $m > 1$.

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b **Generator polynomial:** Let C' be the generator polynomial of C. Since C is a cyclic code, C' is a divisor of $y^m - 1$, that is, $y^m - 1 = g(y).C'(y)$ for a polynomial $g(y)$, [22].

c **Hamming Weight of C:** The Hamming Weight of code C is the minimum weight among all non-zero codewords in C denoted by $dim_H(C)$. This represents the number of non-zero elements in the smallest non-zero codeword.

d **Hamming Weight of C':** Similarly, the Hamming Weight of C', denoted by $dim_H(C')$, represents the number of non-zero co-efficients in C'.

Given the degeneracy property, the number of non-zero co-efficients in C' is the same as the minimum weight of C, which is the number of non-zero elements in the smallest non-zero codeword in C. Thus $dim_H(C) = dim_H(C')$.

Definition 3.3. *Degenerated cyclic codes, also known as degenerate cyclic codes, are a type of a linear code where some codewords are repeated or redundant, leading to reduced effective capability*

Here is a perfect example to illustrate this concept:
Consider a binary cyclic code with a generator polynomial $g(y)$. In degenerated cyclic codes, the generator polynomial $g(y)$ is such that the contains repeated codewords. This can occur when $g(y)$ is not irreducible or when the code length is not relatively to the field size.
Example of a degenerated cyclic code:

i **Field:** Lets work over the binary field $GF(2)$.

ii **Code length:** Consider a code of length 4.

iii **Generator polynomial:** Let $g(y) = y^2 + 1$

Step-by-step construction:

i **Generating the code:**
The generator polynomial $g(y) = y^2 + 1$ is used to generate codewords. The codewords are obtained by multiplying $g(y)$ by all polynomials of degree less than $n - k$, where n is the code length and k is the degree of the generator polynomial.

ii **Codewords:**
-Multiply $g(y)$ by 1: $g(y) \times 1 = y^2 + 1 \rightarrow 1100$
-Multiply $g(y)$ by y: $g(y) \times y = y^3 + y \rightarrow 0110$
-Multiply g(y) by y^2: $g(y) \times y^2 = y^4 + y^2 = y^4 + y^2 \equiv y^2$ (Since y^4 in a binary field of length 4 cycles back to $x^0 \rightarrow 0011$
-Multiplying $g(y)$ by $y^3 : g(y) \times y = y^5 + y^3 \Longrightarrow y^5 + y \equiv 0$ (Since y^5 in the binary field of length 4 cycles back to y^1 and adding y in binary results in o) $\rightarrow 0000$

Thus, here the codewords are:

- 1100
- 0110
- 0011
- 0000

Notice that 0000 is the repeated codeword that reduces the effective error-correcting capability of the code. This repetition makes the code degenerate or degenerated.

The above example shows a degenerated cyclic code where the presence of all-zero codeword 0000 (resulting from the polynomial multiplication) indicates redundancy and a reduction in the effective error-correcting power of the code. This is a typical characteristic of degenerated cyclic codes.

4 Conclusion

Let C be a cyclic code over F_q of length n and dimension k. The cyclic code C is defined by a generator polynomial $g(y)$ and its elements are multiples of this polynomial in $F_q[y]/< y^n - 1 >$.
For $m > 1$, the code $R^m(C)$ is formed by taking all multiples of $g(y^m)$ in $\frac{F_q[y]}{y^n-1}$.
These multiples form a cyclic code. Thus, researchers can further explore on circulant bases for degenerated cyclic codes. The condition likely effects the properties and behaviour of cyclic codes, but its specific implications need to be explored further. It may influence the structure of the code, the existence of certain types of codewords, or other properties to their relevant study. To elaborate further mathematically, one could delve into specific algebraic structures and properties of cyclic codes. Additionally, exploring the impact of the condition $gcd \neq 1$ on algebraic properties of cyclic codes would be crucial for comprehensive understanding.

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Analysis of Adjacency, Laplacian and Distance Matrices of Zero Divisor Graphs of 4-Radical Zero Completely Primary Finite Rings

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ABSTRACT

This study is an extension of our study on matrices of zero divisor graphs of classes of 3-radical zero completely primary finite rings. It focusses on Matrices of a class of finite rings R whose subset of the zero divisors $Z(R)$ satisfies the condition $(Z(R))^4 = (0)$ and $(Z(R))^3 \neq (0)$ for all characteristics of R that is; $p,\ p^2,\ p^3$ and p^4. We have formulated the zero divisor graphs $\Gamma(R)$ of R and associated them with three classes of matrices, namely, the Adjacency matrix $[A]$, the Laplacian matrix $[L]$ and the Distance matrix $[d_{ij}]$. The study has further characterized the properties of the graphs $\Gamma(R)$ and the matrices mentioned.

Mathematics Subject Classification: Primary 13A70; Secondary 13A18.
Keywords: Completely Primary Finite Rings, Matrices of Zero Divisor Graphs

1 Introduction

The study of various classes of the zero divisor graphs obtained from finite rings has been very active since its inception by Beck [2] in an investigation of graph colourings of commutative finite rings. Perhaps, further and simpler developments in this endeavour can be attributed to Anderson and Livingston and Mulay in [1] and [8] respectively who investigated various properties other than colouring. In particular, Mulay in [8] characterized the cycles and symmetries associated with a class of zero divisor graph obtained from finite rings. The mentioned studies in [1, 2, 8] were all aimed at determining the classification of classes of finite rings using the properties of their graphs. The structures of unit groups, zero divisor graphs and the associated adjacency matrices of Galois rings, square radical and classes of cube radical zero completely primary finite rings are well understood (see for example [5, 6, 7, 9, 10]). In particular, Lao *et al* in [5, 6, 7], considered the automorphism groups of the zero divisor graphs of Galois rings, 2-radical zero and 3-radical radical zero completely primary finite rings, while Ndago *et al* in [9] obtained the properties of the Adjacency and Incidence Matrices from the zero divisor graphs of the 2-radical zero finite rings. Most recently, the authors in [10] extended the study of 3-radical zero finite rings covering the algebraic properties of the Ajacency, Laplacian and Distance Matrices associated with the graphs $\Gamma(R)$ of the 3-radical zero finite completely primary rings. Closely related works can be found in [14, 15] where $R = \mathbb{Z}_p \times \mathbb{Z}_p$ for $p = 2, 3$ and 5, $\mathbb{Z}_p[i] \times \mathbb{Z}_p[i]$ for $p = 2, 3$ and 5. In each case, an analysis of the determinant, trace, rank and the symmetry of the matrices was done. Further, a research on the adjacency universal spectrum of $\Gamma(R)$ on the ring $\mathbb{Z}_n$ with its compliment was done in [16]. In the study, an investigation on the loopless graph G with matrices $[A]$ and $[D]$ was performed by choosing a universal matrix $U(G)$ whose computation algorithm was $\beta D + \gamma l + \eta j + \alpha A$ with $\alpha(\neq 0), \gamma, \beta, \eta \in R, l$ being the matrix identity and j having entries of 1.

The Adjacency matrix $[A]$, the Laplacian matrix $[L]$ and Distance matrix $[d_{ij}]$ have inherent structural algebraic relationships which give the matrix representation of the zero divisor graphs $\Gamma(R)$ for ease of their algebraic and geometric analyses. Consider the Adjacency matrix $[A]$ and the degree matrix $[D]$ of $\Gamma(R)$, the Laplacian matrix is a square matrix computed through the relation, $[L] = [D] - [A]$. Bilal in [3] investigated the eigenvalues of Laplacian matrix of $\Gamma(R)$ associated with $\mathbb{Z}_n$. The research showed that the Euler's totient function Φ satisfies the relation $\Phi(qp) = \Phi(q)\Phi(p)$ for relatively prime integers p and q in the ring $\mathbb{Z}_p \times \mathbb{Z}_q$. Also, in [13] and [17], signless Laplacian spectrum and Laplacian Eigenvalues of zero divisor graphs of the ring $\mathbb{Z}_n$ were investigated.

Let R be a completely primary finite ring whose subset of zero divisors $Z(R)$ satisfy the condition $(Z(R))^4 = (0)$ and $(Z(R))^3 \neq (0)$. Then it is well known that the characteristic of R is $p,\ p^2,\ p^3$ or p^4. For certain classes of R, the unit groups R^*, the automorphism group $aut(R^*)$, the zero divisor graphs $\Gamma(R)$ as well as the $aut(\Gamma(R))$ are well known. This paper focusses on the adjacency, Laplacian and distance Matrices of the zero divisor graphs of the classes of R.
Throughout the paper, $R, \Gamma(R), deg(v), V(\Gamma(R)), [A]_{ij}, [L]_{ij}$ and $[d_{ij}]$ are used to denote the completely primary finite ring, the zero divisor graph of R, the degree of a vertex in $\Gamma(R)$, a vertex set of the zero divisor graph and adjacency, Laplacian and distance matrices respectively.

2 4-Radical Zero Completely Primary finite Rings of Characteristic p

The following construction can be obtained from [11].

2.1 Construction I

Let $R' = GR(p^r, p)$ be a Galois ring of order p^r and characteristic p. Consider finitely generated R'-modules $U,\ V$, and W such that $dim_{R'}U = s,\ dim_{R'}V = t$ and $dim_{R'}W = \lambda$ and $s + t + \lambda = h$. Let the R' modules be generated by $\{u_1, u_2 \cdots, u_s\}, \{v_1, v_2, \cdots, v_t\}$ and $\{w_1, w_2, \cdots, w_\lambda\}$ respectively so that $R = R' \oplus U \oplus V \oplus W$ is an additive abelian group. Suppose $s = 1, t = 1$ and $\lambda = h-2$, then $R = R' \oplus R'u \oplus R'v \oplus \sum_{k=1}^{h-2} R'w_k$ where $pu = 0, pv = 0, pw_k = 0$ such that $1 \leq k \leq h-2$ for any prime integer p. We define multiplication on R as follows;

$$(a_\circ, a_1, a_2, \cdots, a_h)(b_\circ, b_1, b_2, \cdots, b_h) =$$

$$(a_\circ b_\circ, a_\circ b_1 + a_1 b_\circ, a_\circ b_2 + a_2 b_\circ + a_1 b_1, a_\circ b_3 + a_3 b_\circ + a_1 b_2 + a_2 b_1, \cdots, a_\circ b_h + a_h b_\circ + a_1 b_2 + a_2 b_1).$$

As established in [11], R is turned by this multiplication into a commutative ring with identity $(1, 0, 0, \cdots, 0)$ and further, the set $Z(R)$ of zero divisors of R satisfy the following properties:
$Z(R) = R'u \oplus R'v \oplus \sum_{k=1}^{\lambda} R'w_k,\ (Z(R))^2 = R'v \oplus \sum_{k=1}^{\lambda} R'w_k,\ (Z(R))^3 = \sum_{k=1}^{\lambda} R'w_k, (Z(R))^4 = (0)$.
As a consequence, the next result in the sequel holds for $\Gamma(R)$.

Proposition 2.1. *Let R be a ring of Construction I. Then the zero divisor graph $\Gamma(R)$ satisfy the following properties:*

(i) The cardinality of the vertices, $\mid V(\Gamma(R)) \mid = p^{hr} - 1.$

(ii) Minimum degree, $\delta(\Gamma(R)) = p^r - 1.$

(iii) Maximum degree, $\Delta(\Gamma(R)) = p^{hr} - 2.$

(iv) $\Gamma(R)$ is incomplete.

Proof. (i) Since $char(R) = char(R') = p$ and $pu_i = pv_j = pw_k = 0$,
$\mid R'u_i \mid = p^{sr}, \mid R'v_j \mid = p^{tr}, \mid R'w_k \mid = p^{\lambda r} \Longrightarrow \mid Z(R) \mid = p^{sr}.p^{tr}.p^{\lambda r} = p^{(s+t+\lambda)r} = p^{hr}$
but $\mid Z(R)^* \mid = \mid V(\Gamma(R)) \mid = p^{hr} - 1.$
(ii) With the multiplication described, $Ann(Z(R)) = (Z(R))^3$. Suppose the vertex set $V_1 = Ann(Z(R)) \setminus \{0\}$, we thus have that $\mid V_1 \mid = p^r - 1$. Since there are only $p^r - 1$ vertices adjacent to every vertex then the minimum degree of a vertex is $p^r - 1$.
(iii) Since the number of vertices in $\Gamma(R)$ is $p^{hr} - 1$, there exist $x \in V_1$ connected to every vertex in the graph. Therefore, the degree of $x,\ deg(x) = (p^{hr} - 1) - 1 = p^{hr} - 2 \Longrightarrow \Delta(\Gamma(R))$ for the avoidance of self loop.
(iv) Clearly, $\delta(\Gamma(R))$ is not equal to $\Delta(\Gamma(R))$ illustrating that the vertices in $\Gamma(R)$ do not have the same degree of connectedness. That is, not every pair of vertices in $\Gamma(R)$ are connected. Further, due to the fact that $(Z(R))^2 \neq (0)$, the incompleteness of $\Gamma(R)$ follows.

□

2.2 Matrices of Zero Divisor Graphs of a Ring in Construction I

Proposition 2.2. *Let R be a ring of Construction I. The Adjacency and Laplacian matrices satisfy the following properties:*

(i) $[A]_{p^{hr}-1}$ and $[L]_{p^{hr}-1}$ are singular.

(ii) $rank([A]_{p^{hr}-1}) = p^{hr} - p^{(h-1)r}.$

(iii) $rank([L]_{p^{hr}-1}) = p^{(h-1)r} + 2.$

(iv) $Tr([L]_{p^{hr}-1}) = 2p^{(h+1)r} - 3p^{hr} + p^{2(h-1)r} + 2p^r + 1.$

(v) For $[A]_{p^{hr}-1}$, *the number of real and complex eigenvalues*

are $p^{(h-1)r}$ *and* $p^{hr} - p^{(h-1)r} - 1$ *respectively.*

Indeed, the real eigenvalues $\lambda[A]_{p^{hr}-1} = \begin{cases} 0, & \textit{of multiplicity } p^{(h-1)r} - 1; \\ -1, & \end{cases}$

and the complex eigenvalues $\lambda[A]_{p^{hr}-1} = \begin{cases} (p^{(h-1)r} - 2)i, & \textit{of multiplicity } p^{hr} - p^r - 2; \\ (p^{(h-1)r} - 1)i, & \textit{of multiplicity } p^r - p^{(h-1)r} + 1. \end{cases}$

(vi) The eigenvalues $\lambda[L]_{p^{hr}-1} = \begin{cases} 0, & \\ p^{hr} - 1, & \\ p^{(h-1)r} - 1, & \\ 1, & \textit{of multiplicity } p^{hr} - 4. \end{cases}$

Proof. (i) Given the adjacency matrix $\begin{bmatrix} 0 & 1 & \cdots & \cdots & \cdots & 1 \\ 1 & 0 & 1 & \cdots & \cdots & 1 \\ 1 & 1 & 0 & 1 & \cdots & \vdots \\ 1 & 1 & 1 & 0 & \cdots & 1_{p^{hr}-p^{(h-1)r}} \\ 0 & 0 & \cdots & \cdots & \cdots & 0_{p^{hr}-p^r} \\ \vdots & \vdots & 0 & \cdots & 0 & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0_{p^{hr}-1} \end{bmatrix}$, suppose we take row 1 as the pivot row in obtaining the determinant, let $a_{11}, a_{12}, \cdots, a_{1p^{hr}-1}$ be the elements of the first row of $[A]_{p^{hr}-1}$. Expanding the minor determinants along the first row, we notice that the matrix minors of $a_{1j}, j = 1, 2, \cdots, p^{hr} - 1$ have zero determinants. That is,

$a_{11}(-1)^{1+j} \mid minor\ (a_{11}) \mid = \cdots = a_{1p^{hr}-1}(-1)^{1+(p^{hr}-1)} \mid minor\ (a_{1p^{hr}-1}) \mid = 0.$

Therefore $\sum_{j=1}^{p^{hr}-1}((-1)^{1+j}a_{1j} \mid minor\ (a_{1j}) \mid) = 0$, hence the determinant of $[A]_{p^{hr}-1}$. A similar argument can be extended for the Laplacian matrices $[L]_{p^{hr}-1}$. This proves the singularity for the matrices.

ii) Reducing the adjacency matrix to its echelon form by conducting a row operation on it, we obtain the matrix

$$\begin{bmatrix} 1 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & \vdots \\ 0 & 0 & 0 & 1 & \cdots & 1_{p^{hr}-p^{(h-1)r}} \\ 0 & 0 & \cdots & \cdots & \cdots & 0_{p^{hr}-p^r} \\ \vdots & \vdots & 0 & \cdots & 0 & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0_{p^{hr}-1} \end{bmatrix}.$$

Clearly, from this reduced echelon form, we obtain $p^{hr}-p^{(h-1)r}$ non zero rows spanning the matrix space. This leads to a rank of $p^{hr}-p^{(h-1)r}$ for the adjacency matrix $[A]_{p^{hr}-1}$.

(iii) Similar to (ii), the Laplacian matrix obtained can be reduced to an echelon form

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & -1 \\ 0 & 1 & 0 & 0 & \cdots & \cdots & \cdots & -1 \\ 0 & 0 & 1 & 0 & 0 & \cdots & \cdots & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdots & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots & -1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 & -1_{p^{(h-1)r}+2} \\ \vdots & 0 & \cdots & & & & 0 & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0_{p^{hr}-1} \end{bmatrix}$$

which is of order $(p^{hr}-1)\times(p^{hr}-1)$. This results to $p^{(h-1)r}+2$ linearly independent vectors which span the matrix row space for the Laplacian matrix $[L]_{p^{hr}-1}$, hence its rank.

(iv) Let $\gamma_1,\cdots,\gamma_r \in R'$ with $\overline{\gamma}_1,\cdots,\overline{\gamma}_r \in R'$ form a basis for R' over its prime subfield R'/pR'. From the multiplication defined on R, $Ann(Z(R)) = (Z(R))^3 = p^3R'$. Let $V_1,\ V_2$ and V_3 be the vertex sets partitioning $V(\Gamma(R))$ such that $V_1 = Ann(Z(R)^*)$. This implies that $\mid V_1 \mid = p^r - 1$. Therefore, for $x \in V_1$, $deg(x) = p^{hr}-2$.

Consider the vertex set $V_2 = \{\gamma_i v + \sum_{k=1}^{h-2} b\gamma_i w_k | b \in R'\}$. Then, $\mid V_2 \mid = p^{(h-1)r} - p^r$ and each vertex $y \in V_2$ is adjacent to a vertex of the form $\gamma_i v + \sum_{k=1}^{h-2}\gamma_i w_k$. Therefore, $deg(y) = p^{(h-1)r}-1$. Let the set $V_3 = \{\gamma_i u + a\gamma_i v + \sum_{k=1}^{h-2} c\gamma_i w_k | a, c \in R'\}$. This means that $\mid V_3 \mid = p^{hr} - p^{(h-1)r}$ and $deg(z) \in V_3 = p^r - 1$ since z is only adjacent to the vertices in the annihilator set V_1.

The trace of the Laplacian matrix is the sum of diagonal entries in the degree matrix $[D]_{p^{hr}-1}$. Thus,

$Tr([L]_{p^{hr}-1}) = (p^{hr}-2)(p^r-1)+(p^{(h-1)r}-1)(p^{(h-1)r}-p^r)+(p^{hr}-p^{(h-1)r})(p^r-1)$. Upon expansion and simplification of this expression, we obtain $Tr([L]_{p^{hr}-1}) = 2p^{(h+1)r} - 3p^{hr} + p^{2(h-1)r} + 2p^r + 1$.
(v) Solving the equation $\mid \lambda I - A \mid = 0$, we obtain the characteristic polynomial equation $\lambda^{p^{hr}-1} - (p^{hr}-1)\lambda^{p^{hr}-p^r-1} - p^r\lambda^{p^{(h-1)r}} + p^{(h-1)r}\lambda^{p^{(h-1)r}-1} = 0$ which can be expressed in factor form as $\lambda^{p^{(h-1)r}-1}(1+\lambda)(\lambda^{p^{(h-1)r}-1} - \lambda^{p^r} - (p^{hr}-p^{(h-1)r})\lambda + p^{(h-1)r}) = 0$. Finding λ, we solve $\lambda^{p^{(h-1)r}-1} = 0 \Longrightarrow \lambda = 0$ of multiplicity $p^{(h-1)r}-1$, $(1+\lambda) = 0 \Longrightarrow \lambda = -1$. The order of the real eigenvalues is obtained by adding the multiplicities $(p^{(h-1)r}-1)+1 = p^{(h-1)r}$.
The equation $(\lambda^{p^{(h-1)r}-1} - \lambda^{p^r} - (p^{hr}-p^{(h-1)r})\lambda + p^{hr}) = 0$ yields the complex eigenvalues as $(p^{(h-1)r}-2)i$ of multiplicity $p^{hr}-p^r-2$ and $(p^{(h-1)r}-1)i$ of multiplicity $p^r - p^{(h-1)r}+1$. Therefore, the sum of multiplicities of complex eigenvalues are $(p^{hr}-p^r-2)+p^r - p^{(h-1)r}+1 = p^{hr}-p^{(h-1)r}-1$.
(vi) For the Laplacian matrix $[L]_{p^{hr}-1}$, we evaluate $\mid \lambda I - [L]_{p^{hr}-1} \mid = 0$ to obtain the characteristic polynomial equation $-\lambda((-(p^{hr}-1)+\lambda)(-(p^{(h-1)r}-1)+\lambda)(-1+\lambda)^{p^{hr}-4}) = 0$. Finding the values of λ in each factor, we have $-\lambda = 0 \Longrightarrow \lambda = 0$. Next, $-(p^{hr}-1)+\lambda = 0 \Longrightarrow \lambda = p^{hr}-1$ and further $-(p^{(h-1)r}-1)+\lambda = 0 \Longrightarrow \lambda = p^{(h-1)r}-1$. Finally, $(-1+\lambda)^{p^{hr}-4} = 0 \Longrightarrow \lambda = 1$ of multiplicity $p^{hr}-4$. □

Proposition 2.3. *Let R be a ring of Construction I and $[d_{ij}]$ be the distance matrix then:*

(i) $Tr([d_{ij}]) = 0$.

(ii) $rank([d_{ij}]) = p^{hr}-1$.

(iii) The eigenvalues $\lambda[d_{ij}] = \begin{cases} -1, & \textit{of multiplicity } p^r-1; \\ -p^r, & \textit{of multiplicity } p^{hr}-2p^r+1; \\ -(p^r-1)i, & \textit{of multiplicity } p^r-1, \textit{ where } \lambda \in \mathbb{C}. \end{cases}$

(iv) $Det([d_{ij}]) = p^{(2hr+1)r}$.

Proof. (i) Since the minimum distance between a vertex and its self $d(v_i, v_i) = 0$, it means that every entry d_{ii} of $[d_{ij}]$ is zero and thus $\sum_{i=1}^{p^{hr}-1} d_{ii} = 0$. Hence the trace, $Tr([d_{ij}]) = 0$.

(ii) We carry out an elementary row operation on $[d_{ij}]$ to obtain a row reduced matrix of the form

$$\begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & 0 & \cdots & \vdots \\ \vdots & 0 & \ddots & 0 & \\ \vdots & & & \ddots & 0 \\ 0 & \cdots & \cdots & & 1_{p^{hr}-1} \end{pmatrix}.$$

Clearly there are $p^{hr}-1$ linearly independent vectors in the matrix span hence the rank.
(iii) To find the characteristic equation, we solve $\mid \lambda I - [d_{ij}] \mid = 0$ to obtain the equation

$$-(1+\lambda)^{p^r-1}(p^r+\lambda)^{(p^{hr}-2p^r+1)}(\lambda^{p^r-1} - (p^{hr}-1)\lambda^{p^r-2} - (p^{(h+2)r}-1)\lambda - p^{(h+1)r}) = 0.$$

From the equation, the real eigenvalues are $-(1+\lambda) = 0 \Longrightarrow \lambda = -1$ of multiplicity $p^r - 1$ and $(p^r+\lambda)^{(p^{hr}-2p^r+1)} = 0 \Longrightarrow \lambda = -p^r$ of multiplicity $p^{hr} - 2p^r + 1$.
Solving the equation $(\lambda^{(p^r-1)} - (-1+p^{hr})\lambda^{(p^r-2)} - (p^{(h+2)r}-1)\lambda - p^{(h+1)r}) = 0$ yields the complex eigenvalues as $-(p^r-1)i$ of multiplicity $p^r - 1$.
(iv) In obtaining the determinant we evaluate $\sum_{i,j=1}^{p^{hr}-1}(d_{ij}(-1)^{i+j} \mid minor\ (d_{ij}) \mid) = p^{hr}.p^{(h+1)r} = p^{(hr+hr+r)} = p^{(2hr+1)r}$. □

3 4-Radical Zero Finite Completely Primary Rings of Characteristic p^2

3.1 Construction II

Let $R' = GR(p^{2r}, p^2)$ be a Galois ring of order p^{2r} and characteristic p^2. Consider R' modules U, V and W which are generated finitely by $\{u_1, \cdots, u_s\}, \{v_1, v_2, \cdots, v_t\}$ and $\{w_1, w_2, \cdots, w_\lambda\}$ respectively so that $R = R' \oplus U \oplus V \oplus W$ is additive abelian group and $s + t + \lambda = h$. Assume $s = h-1, t = 1$ and $\lambda = 0$ so that $R = R' \oplus \sum_{i=1}^{h-1} R'u_i \oplus R'v$ where $pu_i \neq 0, p^2u_i = 0$ and $pv = 0$ with $1 \leq i \leq s$. The following defines multiplication on R.

$$(a_\circ, a_1, a_2, , \cdots, a_{h-1}, \bar{a}_h)(b_\circ, b_1, b_2, , \cdots, b_{h-1}, \bar{b}_h) =$$

$$(a_\circ b_\circ + p\sum_{i,j=1}^{h-1} a_i b_j, a_\circ b_1 + a_1 b_\circ, \cdots, a_\circ b_{h-1} + a_{h-1} b_\circ, a_\circ \bar{b}_h + \bar{a}_h b_\circ)$$

where $\bar{a}_h, \bar{b}_h \in R'/pR'$. The multiplication so defined turns R into a commutative finite ring of identity $(1, 0, 0, \cdots, \bar{0})$ as verified in [4].
$Z(R)$ satisfies the following properties;
$Z(R) = pR' \oplus \sum_{i=1}^{s} R'u_i \oplus R'v,\ (Z(R))^2 = pR' \oplus p\sum_{i=1}^{s} R'u_i \oplus R'v,\ (Z(R))^3 = p\sum_{i=1}^{s} R'u_i,$
$(Z(R))^4 = (0)$.
The following result describes some properties of $\Gamma(R)$ of the ring constructed in this section.

Proposition 3.1. *Let R be a ring of Construction II. Then:*

(i) The cardinality, $\mid V(\Gamma(R)) \mid = p^{2hr} - 1$.

(ii) The maximumn degree, $\Delta(\Gamma(R)) = p^{2hr} - 2$.

(iii) $\Gamma(R)$ *is an incomplete graph.*

(iv) The minimum degree, $\delta(\Gamma(R)) = p^{hr} - 1$.

Proof. (i) Given that the structure of zero divisors is given by $Z(R) = pR' \oplus \sum_{i=1}^{s} R'u_i \oplus R'v$ and due to the fact that $pu_i \neq 0,\ p^2u_i = 0$ and $pv = 0$ with $1 \leq i \leq s,\ \mid pR' \mid = p^r,\ \mid R'u_i \mid = p^{2r}$ and $\mid R'v \mid = p^r$. Therefore, $\mid Z(R) \mid = p^r(p^{2r(h-1)})p^r = p^{2hr}$. Since $\mid Z(R)^* \mid = \mid Z(R) \setminus \{0\} \mid,\ \mid Z(R)^* \mid = p^{2hr} - 1 = \mid V(\Gamma(R)) \mid$.
(ii) Let $\gamma_1, \cdots, \gamma_r \in R'$ with $\gamma_1 = 1$ such that $\overline{\gamma}_1, \cdots, \overline{\gamma}_r \in R'$ is a basis for R' over its prime subfield R'/pR'. Let $V_1 = Ann(Z(R)) \setminus \{0\}$. From the multiplication described, $Ann(Z(R)) = \{pc_1\gamma_i u_1 + \cdots + pc_{h-1}\gamma_i u_{h-1} + b\gamma_i v | c_1, \cdots, c_{h-1}, b \in R'\}$. Vertices in V_1 are adjacent to every vertex in $\Gamma(R)$. Therefore, every $y \in V_1$ is of degree $p^{2hr} - 2$ for an avoidance of self loop. Hence the maximum degree $\Delta(\Gamma(R)) = p^{2hr} - 2$.
(iii) This is clear due to the fact that $(Z(R))^2 \neq (0)$.
(iv) Let V_1 be the set described in (ii), $deg(y) \in V_1 = p^{2hr} - 2$ and $\mid V_1 \mid = p^{hr} - 1$. Any vertex of minimum degree is not adjacent to any other vertex in $V(\Gamma(R))$ a part from the vertices in the set V_1. Since there are $p^{hr} - 1$ vertices in set V_1, it implies that $\delta(\Gamma(R)) = p^{hr} - 1$.

□

The results below describe the properties of the matrices associated with $\Gamma(R)$ of the ring constructed in this Section.

3.2 Matrices of the Zero Divisor Graph of the Ring in Construction II

Proposition 3.2. *Let R be a ring of Construction II. Suppose $[A]_{p^{2hr}-1}$ and $[L]_{p^{2hr}-1}$ are the Adjacency and Laplacian matrices respectively;*

(i) Both matrices are singular.

(ii) $rank([A]_{p^{2hr}-1}) = p^{2hr} - p^{hr}$.

(iii) $rank([L]_{p^{2hr}-1})$ *is* $p^{hr} + 2$.

(iv) The number of real and complex eigenvalues λ for $[A]_{p^{2hr}-1}$
$$= \begin{cases} p^{hr}, & \lambda \in \mathbb{R}; \\ p^{2hr} - p^{hr} - 1, & \lambda \in \mathbb{C}. \end{cases}$$

(v) The eigenvalues $\lambda[L]_{p^{2hr}-1} = \begin{cases} 0, \\ p^{2hr} - p^{hr}, \\ p^{hr} + p^{r}, \\ 1, \qquad\qquad \textit{of multiplicity } p^{hr}. \end{cases}$

(vi) $Tr([L]_{p^{2hr}-1}) = p^{2hr} + p^{hr} + p^{r}$.

Proof. The proofs for (i), (ii) and (iii) can easily be followed from proposition 2.2.
(iv). Solving the equation $\mid \lambda I - [A]_{p^{2hr}-1} \mid = 0$ results to a characteristic equation of the form $\lambda^{p^{2hr}-1} - (p^{2hr} - 1)\lambda^{p^{2hr}-p^{hr}-1} - p^{hr}\lambda^{p^{hr}} + p^{hr} = 0$ which factorizes as $\lambda^{p^{hr}-1}(1+\lambda)(\lambda^{p^{hr}-1} - \lambda^{p^{r}} - (p^{2hr} - p^{hr})\lambda + p^{hr}) = 0$. Finding the values of λ from the equation, we obtain $\lambda^{p^{hr}-1} = 0 \Longrightarrow \lambda = 0$ of multiplicity $p^{hr}-1$ and $\lambda+1 = 0 \Longrightarrow \lambda = -1$, as the real eigenvalues. Therefore, by evaluating the sum of the multiplicities of real eigenvalues, we obtain the number of real eigenvalues to be $p^{hr} - 1 + 1 = p^{hr}$.
The equation from the remaining factor, $(\lambda^{p^{hr}-1} - \lambda^{p^{r}} - (p^{2hr} - p^{hr})\lambda + p^{hr}) = 0$ yields $(p^{2hr} - 1) - p^{hr} = p^{2hr} - p^{hr} - 1$ complex eigenvalues due to the fact that the adjacency matrix $[A]_{p^{2hr}-1}$ is a square matrix with $p^{2hr} - 1$ rows and columns.
(v). For the Laplacian matrix $[L]_{p^{2hr}-1}$, the equation $\mid \lambda I - [L]_{p^{2hr}-1} \mid = 0$ results to the characteristic polynomial equation of the form $-\lambda(-(p^{2hr}-p^{hr})+\lambda)(-(p^{hr}+p^{r})+\lambda)(-1+\lambda)^{p^{hr}} = 0$. Upon solving the equation, $-\lambda = 0 \Longrightarrow \lambda = 0$, $-(p^{2hr} - p^{hr}) + \lambda = 0 \Longrightarrow \lambda = p^{2hr} - p^{hr}$ and $-(p^{hr} + p^{r}) + \lambda = 0 \Longrightarrow \lambda = p^{hr} + p^{r}$. Finally, $(-1+\lambda)^{p^{hr}} = 0 \Longrightarrow \lambda = 1$

of multiplicity p^{hr}. Hence the eigenvalues for $[L]_{p^{2hr}-1}$.
(vi). Since trace can be computed as the sum of eigenvalues, $Tr([L]_{p^{2hr}-1}) = \sum_{i=1}^{p^{2hr}-1} \lambda_i \Longrightarrow$ $Tr([L]_{p^{2hr}-1}) = 0 + p^{2hr} - p^{hr} + p^{hr} + p^r + 1(p^{hr}) = p^{2hr} + p^{hr} + p^r$ as required. □

Proposition 3.3. *Let R be a ring of Construction II and $[d_{ij}]$, the distance matrix then;*

(i) $Tr([d_{ij}]) = 0$.

(ii) $rank([d_{ij}]) = p^{2hr} - 1$.

(iii) The eigenvalues $\lambda = \begin{cases} -1, & \textit{of multiplicity } p^{(h+2)r} - 2\textit{;} \\ -p^r, & \textit{of multiplicity } p^{(h+2)r} - 1\textit{;} \\ \frac{1}{2}(\sigma \pm \sqrt{\sigma^2 - 4\tau}) & \textit{.} \end{cases}$

(iv) $Det([d_{ij}]) = p^{(2h+2)r}$.

Proof. (i) Follows from the fact that $d(v_i, v_i) = 0$, thus entries d_{ii} of the main diagonal are all 0's hence the trace.

(ii) Given the general distance matrix $[d_{ij}]_{p^{2hr}-1} = \begin{pmatrix} 0 & 1 & 1 & \cdots & \cdots & 1 \\ 1 & 0 & 1 & 1 & \cdots & 1 \\ \vdots & & \vdots & \ddots & & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0_{p^{hr}-1} \end{pmatrix}$, consider the set $V = \{v_1, \cdots, v_{p^{2hr}-1}\}$ consisting of vectors which are linearly independent from a row reduced echelon form of matrix $[d_{ij}]_{p^{2hr}-1}$ such that $v_1 = \begin{pmatrix} 1 \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ . \\ . \\ 0 \end{pmatrix}, \cdots, v_{p^{2hr}-1} = \begin{pmatrix} 0 \\ . \\ . \\ . \\ 0 \\ 1 \end{pmatrix}$. Clearly, the set V is of dimension $p^{2hr} - 1$ equivalent to the dimension of the matrix thus the matrix space is spanned by vectors in V. Therefore

the $rank([d_{ij}]) = p^{2hr} - 1$.
(iii) We solve the equation $\mid [d_{ij}] - \lambda I \mid = 0$ to obtain the characteristic polynomial $-(1+\lambda)^{p^{(h+2)r}-2}(p^r+\lambda)^{p^{(h+2)r}-1}(\lambda^2-(p^{(h-1)r}(p^{(h+2)r}-p^{hr}-1)\lambda+(2p^{(h+2)r}+2p^{hr}-4)(p^{hr}+3))$. Finding λ in each factor, we solve $(p^r+\lambda)^{p^{(h+2)r}-1} = 0 \Longrightarrow \lambda = -p^r$ of multiplicity $p^{(h+2)r}-1$. Further, $-(1+\lambda)^{p^{(h+2)r}-2} = 0 \Longrightarrow \lambda = -1$ with a multiplicity of $p^{(h+2)r}-2$. For the quadratic part, we solve $\lambda^2-(p^{(h-1)r}(p^{(h+2)r}-p^{hr}-1))\lambda+(2p^{(h+2)r}+2p^{hr}-4)(p^{hr}+3) = 0$. If we let $(p^{(h-1)r}(p^{(h+2)r}-p^{hr}-1)\lambda = \sigma$ and $(2p^{(h+2)r}+2p^{hr}-4)(p^{hr}+3) = \tau$, we obtain $\frac{1}{2}(\sigma \pm \sqrt{\sigma^2-4\tau})$.
(iv) This follows form the proof of the determinant of distance matrix in proposition 2.3. □

4 The 4-Radical Zero Finite Completely Primary Rings of Characteristic p^3

4.1 Construction III

Let $R' = GR(p^{3r}, p^3)$ be a Galois ring of characteristic p^3 and of order p^{3r}. Consider finitely generated R' modules U, V and W with dimensions s, t and λ respectively whose generating sets are $\{u_1, \cdots, u_s\}, \{v_1, \cdots, v_t\}$ and $\{w_1, \cdots, w_\lambda\}$ where $s+t+\lambda = h$ so that $R = R' \oplus U \oplus V \oplus W$ is an additive abelian group. Consider $s = h-1, t = 1$ and $\lambda = 0$ so that $R = R' \oplus \sum_{i=1}^{h-1} R'u_i \oplus R'v$ where $p^2u_i \neq 0,\ p^3u_i = 0$ where $1 \leq i \leq s$ and $pv = 0$. The following multiplication is defined on R:
$(a_\circ, \overline{a}_1, \overline{a}_2, \cdots, \overline{a}_{h-1}, \tilde{a}_h)(b_\circ, \overline{b}_1, \overline{b}_2, \cdots, \overline{b}_{h-1}, \tilde{b}_h) = (a_\circ b_\circ, a_\circ\overline{b}_1+\overline{a}_1b_\circ, \cdots, a_\circ\overline{b}_{h-1}+\overline{a}_{h-1}b_\circ, a_\circ\tilde{b}_h+\tilde{a}_hb_\circ + \sum_{i,j=1}^{h-1}\overline{a_ib_j})$ where $\overline{a}_i, \overline{b}_j \in R'/p^2R'$ and $\tilde{a}_h, \tilde{b}_h \in R'/pR'$. From [12], it is verifiable that R is turned into a commutative ring with identity $(1, \overline{0}, \cdots, \overline{0}, \tilde{0})$ by the multiplication The set of zero divisors $Z(R)$ satisfy the properties below;
$Z(R) = pR' \oplus \sum_{i=1}^{s} R'u_i \oplus R'v,\ \ (Z(R))^2 = p^2R' \oplus p\sum_{i=1}^{s} R'u_i \oplus R'v,\ \ (Z(R))^3 = pR'v,\ (Z(R))^4 = (0)$.

The results in the sequel describe some properties of $\Gamma(R)$ of the ring constructed in this Section.

Proposition 4.1. *Let R be a ring of Construction III. Then:*

(i) The cardinality, $\mid V(\Gamma(R)) \mid = p^{3hr} - 1$.

(ii) The maximum degree, $\Delta(\Gamma(R)) = p^{3hr} - 2$.

(iii) The minimum degree, $\delta(\Gamma(R)) = p^{hr} - 1$.

(iv) The graph $\Gamma(R)$ *is incomplete.*

Proof. (i) Given that $Z(R) = pR' \oplus \sum_{i=1}^{s} R'u_i \oplus R'v$ and that $p^2u_i \neq 0,\ p^3u_i = 0$ and $pv = 0$, it is easy to see that $\mid pR' \mid = p^{2r},\ \mid R'u_i \mid = p^{3r}$ and $\mid R'v \mid = p^r$. Therefore, $\mid Z(R) \mid = p^{2r}(p^{3r(h-1)})p^r = p^{3hr}$. Since $\mid Z(R) \setminus \{0\} \mid = \mid (Z(R))^* \mid = p^{3r} - 1, \Longrightarrow \mid (Z(R))^* \mid = \mid V(\Gamma(R)) \mid = p^{3r} - 1$.

The Proofs for (ii) and (iii) are described in the next Proposition. For (iv), the fact that $(Z(R))^2 \neq (0)$ explains the incompleteness of $\Gamma(R)$. □

Proposition 4.2. *Let* R *be a ring of Construction III. Suppose* V_1, V_2, V_3, V_4 *and* V_5 *are the partitions of* $V(\Gamma(R))$. *Then the degrees of vertices* $v \in V(\Gamma(R))$

$$= \begin{cases} p^{3hr} - 2, & v \in V_1 \textit{ and } \mid V_1 \mid = p^{hr} - 1; \\ p^{2hr} - 2, & v \in V_2 \textit{ and } \mid V_2 \mid = p^{2hr} - p^{hr}; \\ deg(v) \in (X \cup Y) = V_3, & v \in V_3 \textit{ and } \mid V_3 \mid = p^{(h+1)r} - p^{(h-1)r}\ ; \\ deg(v) \in (W \cup Z) = V_4, & v \in V_4 \textit{ and } \mid V_4 \mid = 2p^{(h+2)r}; \\ p^{(h+1)r} - p^{hr} + p^{(h-1)r} - 1, & v \in V_5 \textit{ and } \mid V_5 \mid = p^{3hr} - 2p^{(h+2)r} + p^{(h+1)r}. \end{cases}$$

Proof. We describe the connectedness of $\Gamma(R)$ for the ring in this section as follows: Let $\gamma_1, \cdots, \gamma_r \in R'$ with $\gamma_1 = 1$ such that $\overline{\gamma}_1, \cdots, \overline{\gamma}_r \in R'$ is the basis of R' over its prime subfield R'/pR'. From the defined multiplication, $Ann(Z(R)) = \{p^2\gamma_i u_1 + \cdots + p^2\gamma_i u_{h-1} + b\gamma_i v \mid b \in R'\}$. Let $V_1 = Ann(Z(R))^*$, therefore the order of $V_1,\ \mid V_1 \mid = p^{hr} - 1$. Every $v \in V_1$ is adjacent to each vertex in $\Gamma(R)$ and therefore the degree, $deg(v) \in V_1 = p^{3hr} - 2$. Similarly, consider set $V_2 = \{p^2r_\circ + p^2\gamma_i u_1 + \cdots + p^2\gamma_i u_{h-1} + b\gamma_i v \mid p^2r_\circ \neq 0, b \in R'\}$. Each vertex $v \in V_2$ is connected to other vertices in $\Gamma(R)$ apart from the vertices of the form $pr_\circ + \gamma_i u_1 + \cdots + \gamma_i u_{h-1} + b\gamma_i v, b \in R'$ where $r_\circ$ is not a multiple of p. Thus, $\mid V_2 \mid = p^{2hr} - p^{hr}$ and $deg(v) \in V_2 = p^{2hr} - 2$.
Next, suppose $X = \{p^2r_\circ + p\gamma_i u_1 + \cdots + p\gamma_i u_{h-1} + b\gamma_i v\} \setminus V_1 \cup V_2$. It means that the order of X, $\mid X \mid = p^{(h+1)r} - p^{hr}$. Each vertex in set X is connected to a vertex in either set $V_1,\ V_2,\ X$ or Y where $Y = \{p\gamma_i u_1 + \cdots + p\gamma_i u_{h-1} + b\gamma_i v \mid b \in R'\} \setminus V_1$. This implies that $\mid Y \mid = p^{hr} - p^{(h-1)r}$ hence, $deg(v) \in X = p^{(h-1)r} - 1 + p^{hr} - p^{(h-1)r} + p^{hr} - p^{(h-1)r} + p^{(h+1)r} - p^{hr} - 1 = p^{(h+1)r} + p^{hr} - 2p^{(h-1)r} - 2$ and each $v \in Y$ is adjacent to either a vertex in V_1, V_2, X or Y. Thus $deg(v) \in Y = p^{(h+1)r} + p^{hr} - 2p^{(h-1)r} - 2$.

Further, let $V_3 = X \cup Y$. and consider set $W = \{pr_\circ + p\gamma_i u_1 + \cdots + p\gamma_i u_{h-1} + b\gamma_i v \mid b \in R'\} \setminus V_1 \cup V_2 \cup V_3$. Therefore, the order of W, $\mid W \mid = p^{(h+2)r} - (p^{(h-1)r} + p^{(h+1)r} - p^{hr} + p^{hr} - p^{(h-1)r}) = p^{(h+2)r} - p^{(h+1)r}$. Each $v \in W$ is either adjacent to a vertex in V_1 or V_2 therefore, $deg(v) \in W = p^{(h-1)r} - 1 + p^{hr} - p^{(h-1)r} = p^{hr} - 1$.
Similarly, let $Z = \{p^2 r_\circ + \gamma_i u_1 + \cdots + \gamma_i u_{h-1} + b\gamma_i v \mid b \in R'\}$. It means that the order of Z, $\mid Z \mid = p^r(p^{hr} - p^{(h-1)r})p^r = p^{(h+2)r} - p^{(h+1)r}$. Each vertex, $v \in Z$ is either connected to a vertex in V_1 or Y. So, $deg(v) \in Z = p^{(h-1)r} - 1 + p^{hr} - p^{(h-1)r} = p^{hr} - 1$. We finally consider the set $V_4 = W \cup Z$. and let set $V_5 = \{pr_\circ + \gamma_i u_1 + \cdots + \gamma_i u_{h-1} + b\gamma_i v \mid b \in R'\} \setminus Z$. Then, $\mid V_5 \mid = p^{(h-1)r}(p^{(h-1)r})p^r - (p^{(h+2)r} - p^{(h+1)r} = p^{(h-1)r}(p^{(h+1)r} - p^{hr}) - (p^{(h+2)r} - p^{(h+1)r}) = p^{(h+3)r} - 2p^{(h+2)r} + p^{(h+1)r}$. Therefore the degree of every vertex in V_5 is $p^{(h-1)r} - 1 + (p^{(h+1)r} - p^{hr}) = p^{(h+1)r} - p^{hr} + p^{(h-1)r} - 1$.

□

4.2 Matrices of the Zero Divisor Graph of a Ring in Construction III

The following results describe some properties of the Adjacency, Laplacian and distance matrices associated with $\Gamma(R)$ of the ring described in this section.

Proposition 4.3. *Let R be a ring of Construction III. The adjacency and Laplacian matrices have the following properties;*

(i) $[A]_{p^{3hr}-1}$ *and* $[L]_{p^{3hr}-1}$ *are both singular and symmetric.*

(ii) $rank([A]_{p^{3hr}-1}) = p^{3hr} - p^{2hr} + p^r + 1.$

(iii) $rank([L]_{p^{3hr}-1}) = p^{3hr} - p^{(h-1)r}.$

(iv) The number of real and complex eigenvalues $\lambda = \begin{cases} p^{2hr} - p^{hr} + 1, & \lambda \in \mathbb{R}; \\ p^{3hr} - p^{2hr} - p^{hr}, & \lambda \in \mathbb{C}. \end{cases}$
for both the adjacency and Laplacian matrices.

Proof. The steps for the proof of (i),(ii) and (iii) are similar to the ones in proposition 2.2. We provide the proof for (iv) as follows.
Upon solving the equation $\mid \lambda I - [A]_{p^{3hr}-1} \mid = 0$, we obtain the real eigenvalues by evaluating $-\lambda^{(p^{2hr}-p^{hr}-p^r)}(1+\lambda)^{p^r+1} = 0$. This implies that $\lambda = 0$ of multiplicity $p^{2hr} -$

$p^{hr} - p^r$ and $\lambda = -1$ of multiplicity $p^r + 1$. Therefore real eigenvalues are $p^{2hr} - p^{hr} - p^r + p^r + 1 = p^{2hr} - p^{hr} + 1$ in number. The number of complex eigenvalues in $[A]_{p^{3hr}-1}$ is $(p^{3hr} - 1) - (p^{2hr} - p^{hr} + 1) = p^{3hr} - p^{2hr} - p^{hr}$.
For the Laplacian matrix, simplifying $\mid \lambda I - [L]_{p^{3hr}-1} \mid = 0$ results to the characteristic equation of the form $-(-1 + \lambda)^{(p^{2hr}-p^{hr}-p^r)}\lambda^{p^r+1} = 0$. Solving the equation yields real eigenvalues $\lambda = 0$ of multiplicity $p^r + 1$ and $(-1 + \lambda)^{(p^{2hr}-p^r-1)} = 0$ implying that $\lambda = 1$ of multiplicity $p^{2hr} - p^{hr} - p^r$. Therefore, the number of real eigenvalues are $p^{2hr} - p^{hr} - p^r + p^r + 1 = p^{2hr} - p^{hr} + 1$. From this and given that the matrix is of order $p^{3hr} - 1$, the complex eigenvalues are $p^{3hr} - p^{2hr} - p^{hr}$ in number. □

Proposition 4.4. *Let R be a ring of Construction III and $[d_{ij}]$, the distance matrix. Then;*

(i) $Tr([d_{ij}]) = 0$.

(ii) $rank([d_{ij}]) = p^{2hr} - 2$.

(iii) The eigenvalues $\lambda = \begin{cases} -1, & \text{of multiplicity } p^{2hr}; \\ -p^{2r}, & \text{of multiplicity } p^{2hr} - 1; \\ p^{2r} + 1 & . \end{cases}$

(iv) $Det([d_{ij}]) = p^{hr}$.

Proof. The steps for the proof are similar to those in propositions 3.3. □

5 4-Radical Zero Finite Completely Primary Rings of Characteristic p^4

5.1 Construction IV

Let $R' = GR(p^{4r}, p^4)$ be a Galois ring of order p^{4r} and characteristic p^4. Consider finitely generated R'-modules U, V and W generated by $\{u_1, u_2, \cdots, u_s\}, \{v_1, v_2, \cdots, v_t\}$ and $\{w_1, w_2, \cdots, w_\lambda\}$ respectively. Let $dim_{R'}U = s, dim_{R'}V = t$ and $dim_{R'}W = \lambda$, so that $R = R' \oplus U \oplus V \oplus W$ is an additive abelian group and $s+t+\lambda = h$. Assume $s = h, t = 0$ and $\lambda = 0$ so that

$R = R' \oplus \sum_{i=1}^{s} R'u_i$ with $pu_i = 0,\ 0 \leq i \leq s$. The multiplication on R is defined by;

$$(a_\circ, \overline{a}_1, \cdots, \overline{a}_h)(b_\circ, \overline{b}_1, \cdots, \overline{b}_h) =$$
$$(a_\circ b_\circ, a_\circ \overline{b}_1 + \overline{a}_1 b_\circ, \cdots, a_\circ \overline{b}_h + \overline{a}_h b_\circ)$$

where $\overline{a}_i,\ \overline{b}_j \in R'/pR'$ and $1 \leq i, j \leq s$. R is turned by this multiplication into a commutative ring with identity $(1, \overline{0}, \cdots, \overline{0})$. The set $Z(R)$ satisfy the following properties; $Z(R) = pR' \oplus \sum_{i=1}^{s} R'u_i,\ (Z(R))^2 = p^2R',\ (Z(R))^3 = p^3R',\ (Z(R))^4 = (0)$.

The following result describes the zero divisor graph $\Gamma(R)$ of the ring constructed in this Section.

Proposition 5.1. *Let R be a ring of Construction IV. Let V_1, V_2, V_3 and V_4 be the order of partitions of vertices in $V(\Gamma(R))$. Then:*

(i) The cardinality, $\mid V(\Gamma(R)) \mid = p^{(h+3)r} - 1$.

(ii) $deg(v) = \begin{cases} p^{(h+3)r} - 2, & v \in V_1 \text{ and } \mid V_1 \mid = p^{(h+2)r} - 1; \\ p^{hr} + p^{(h-1)r} + p^r, & v \in V_2 \text{ and } \mid V_2 \mid = p^{hr}; \\ p^{(h+1)r} - p^r, & v \in V_3 \text{ and } \mid V_3 \mid = p^{hr} + p^{(h-1)r}; \\ p^{hr} - p^{(h-1)r} + 1, & v \in V_4 \text{ and } \mid V_4 \mid = p^{(h+1)r} - p^{hr}. \end{cases}$

Proof. (i) Given $Z(R) = pR' \oplus \sum_{i=1}^{s} R'u_i$ and that $pu_i = 0$, then, $\mid Z(R) \mid = \mid V(\Gamma(R)) \mid$. Further, $\mid pR' \mid = p^{3r}$ and $\mid R'u \mid = p^{hr}$. Therefore, $\mid Z(R) \mid = p^{3r}(p^{hr}) = p^{(h+3)r}$ and $\mid Z(R) \setminus \{0\} \mid = p^{(h+3)r} - 1 =$
$\mid V(\Gamma(R)) \mid$.
(ii) Let $\gamma_1, \gamma_2, \cdots, \gamma_r \in R'$ with $\gamma_1 = 1$ such that $\overline{\gamma}_1, \overline{\gamma}_2, \cdots \overline{\gamma}_r \in R'$ forms a basis for R' over its prime subfield R'/pR'. From the multiplication given, $Ann(Z(R)) = \{p^3 r_\circ + b\gamma_i u_1 + \cdots + b\gamma_i u_h \mid b \in R'\}$. Let $V_1 = Ann(Z(R)) \setminus \{0\}$. This implies that $\mid V_1 \mid = p^{(h-1)r} - 1$. Each vertex $v \in V_1$ is connected to every other vertex in $V(\Gamma(R))$. Therefore, $deg(v)$ in the set V_1 is $p^{(h+3)r} - 1 - 1 = p^{(h+3)r} - 2$.
Let $V_2 = \{p^3 r_\circ + b\gamma_i u_1 + \cdots + b\gamma_i u_h \mid b \in R'\}$. Clearly $\mid V_2 \mid = p^{hr}$ and every $v \in V_2$ is adjacent to a vertex of the form $pr_\circ + b\gamma_i u_1 + \cdots + b\gamma_i u_h$ therefore, $deg(v)$ in the set V_2 is $p^{hr} + p^{(h-1)r} + p^r$.
Further, let $V_3 = \{p^2 r_\circ + b\gamma_i u_1 + \cdots + b\gamma_i u_h \mid b \in R'\}$ then $\mid V_3 \mid = p^{hr} + p^{(h-1)r}$. Each $v \in V_3$ is adjacent to the vertex of the form $p^2 r_\circ + b\gamma_i u_1 + \cdots + b\gamma_i u_h$ therefore, $deg(v)$ in V_3 is $p^{(h+1)r} - p^r$.
Finally, let $V_4 = \{pr_\circ + \gamma_i u_1 + \cdots + \gamma_i u_h\} \setminus V_1 \cup V_3$. Therefore, $\mid V_4 \mid = p^{(h+1)r} - (p^{(h-1)r} +$

$p^{hr} - p^{(h-1)r}) = p^{(h+1)r} - p^{hr}$. Each $v \in V_4$ is either adjacent to a vertex in V_1 or V_2. So, $deg(v)$ in the set V_4 is $p^{hr} - (p^{(h-1)r} - 1) = p^{hr} - p^{(h-1)r} + 1$. □

5.2 Matrices of the Zero Divisor Graph of a Ring in Construction IV

Proposition 5.2. *Let R be a ring of Construction IV. The adjacency and Laplacian matrices satisfy the following properties;*

(i) $[A]_{p^{(h+3)r}-1}$ *and* $[L]_{p^{(h+3)r}-1}$ *are both singular.*

(ii) $rank([A]_{p^{(h+3)r}-1}) = p^{hr} + p^{(h-2)r} + 1$.

(iii) $rank([L]_{p^{(h+3)r}-1}) = p^{(h+1)r} + p^{hr} + 2$.

(iv) The number of real and complex eigenvalues $\lambda = \begin{cases} p^{(h+1)r} + 2p^{(h-1)r}, & \lambda \in \mathbb{R}; \\ p^{(h+2)r} - p^{(h+1)r} - 2p^{(h-1)r} - 1, & \lambda \in \mathbb{C}. \end{cases}$

for both $[A]_{p^{(h+3)r}-1}$ *and* $[L]_{p^{(h+3)r}-1}$.

Proof. We provide a proof for (iv). The proof for (i),(ii) and (iii) are clear. Upon obtaining the characteristic polynomial for the adjacency matrix, we find the real eigenvalues from the equation $-\lambda^{(p^{(h+1)r}+p^{(h-1)r}-1)}(1+\lambda)^{(p^{(h-1)r}+1)} = 0$. The solution to this results to $\lambda = 0$ of multiplicity $p^{(h+1)r}+p^{(h-1)r}-1$ and $(1+\lambda)^{(p^{(h-1)r}+1)} = 0$ implying that $\lambda = -1$ of multiplicity $p^{(h-1)r}+1$. Therefore, the number of real eigenvalues from the characteristic polynomial equation of the adjacency matrix is $p^{(h+1)r} + p^{(h-1)r} + p^{(h-1)r} - 1 + 1 = p^{(h+1)r} + 2p^{(h-1)r}$.
Given that the adjacency matrix $[A]_{p^{(h+3)r}-1}$ is a square matrix with $p^{(h+2)r} - 1$ rows and columns and its characteristic polynomial has both real and complex parts, we have that the number of complex eigenvalues are $(p^{(h+2)r} - 1) - p^{(h+1)r} + 2p^{(h-1)r} = p^{(h+2)r} - p^{(h+1)r} - 2p^{(h-1)r} - 1$.

For the Laplacian matrix, the characteristic polynomial equation is of the form $-(\lambda^{p^{(h+2)r}-1}+\lambda p^{(h+1)r}-p^{(h-1)r})(-1+\lambda)^{(p^{(h+1)r}+p^{(h-1)r}-1)}\lambda^{(p^{(h-1)r}+1)} = 0$. From the equation, we obtain the real eigenvalues by solving the equation $(-1+\lambda)^{(p^{(h+1)r}+p^{(h-1)r}-1)}\lambda^{(p^{(h-1)r}+1)} = 0$. This implies that $\lambda = 0$ of multiplicity $p^{(h-1)r} + 1$ and $\lambda = 1$ of multiplicity $p^{(h+1)r} + p^{(h-1)r} - 1$. Similarly, we can find the values of λ in the remaining factor by solving the equation $-(\lambda^{p^{(h+2)r}-1} + \lambda p^{(h+1)r} - p^{(h-1)r}) = 0$ to obtain the complex eigenvalues. □

6 Conclusion

In this paper, we have established that the zero divisor graphs of classes of 4-Radical Zero Completely Primary Finite Rings can be expressed in terms of matrices. Therefore, this provides an illustration for better analysis of the graphs from the perspective of matrix algebraic properties. The focus of this research was on the Adjacency, Laplacian and Distance matrices associated with the zero divisor graphs of the classes of rings in constructions I to IV. A further research on other types of matrices can be explored.

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Graph Numbers and Distance Related Parameters of Zero Divisor Graphs

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ABSTRACT

Distance-related parameters have applications in the field of pharmaceutical chemistry, network discovery, robot navigation, and optimizations. Cyclic structures exhibit significant topological features that have become important research areas in the field of computer science and mathematics. Due to the inherent algebraic relationship between graph numbers and distance related parameters, this paper characterizes variants of distance related parameters and graph numbers associated with the zero divisor graphs akin to cyclic structures obtained from classes of completely primary finite rings. In particular, we investigate the local fractional metric dimension and provide certain results concerning graph indices namely the Weiner index and the Zagreb index.

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Keywords: *Cyclic structures, Weiner index, Zagreb index*

1 Introduction

The graph distance related parameters has been extensively studied by various authors. Among others, Harary and Melter [10] studied the problem of finding the metric dimension of a graph, [12] showed that the metric dimension of a graph is an NP-complete problem. Distance-based parameters for networks play a vital role in various fields including pharmaceutical chemistry [5], network discovery [4], robot navigation, and optimizations [12]. Besides, many real-life large-scale systems having substantial topological features can be modeled as complex networks such as social networks, information networks, technological networks, and biological networks. This representation has innovative impacts to information processing and co-ordination of these large-scale networks. Management of large-scale networks such as Internet with their tremendous growth and heterogeneity is a challenging mathematical problem which have profound implications for the efficient design of future communication networks. Complex networks are composed of building blocks, and if the building blocks are considered as symmetric networks, then complexity of these networks can be reduced for better analysis and interpretation. The concept of undirected zero-divisor graph of a commutative ring was first studied by Beck in [3] showed that all the elements of a ring R were the vertices of the graph, and he was mainly interested in coloring. This work was further studied by Anderson and Naseer [2]. A different approach of associating a graph $\Gamma(R)$ to R with vertices as $Z^*(R) = Z(R)\backslash\{0\}$ was given in [1]. Two vertices $x, y \in Z^*(R)$ of $\Gamma(R)$ are adjacent if and only if $xy = 0$. They believed that this better illustrates the zero-divisor structure of the ring. The zero-divisor graph of a commutative ring has also been studied in [1, 3, 6, 15] and was extended by Redmond [20] to noncommutative rings. Redmond [15] also extended the zero-divisor of a commutative ring to an ideal-based zero-divisor graph of a commutative ring. For a given ideal I of R, he defined an undirected graph $\Gamma_I(R)$ with vertex set $\{x \in R - I \mid xy \in I$ for some $y \in R - I\}$, where distinct vertices x and y are adjacent if and only if $xy \in I$.

A simple graph $G(V, E)$ consists of a finite nonempty set $V(G)$ of objects called vertices together with a set $E(G)$ of unordered pairs of distinct vertices of G called edges. A graph G is connected if there is a path between every two distinct vertices of G. The distance from a vertex v to u denoted by $d(v, u)$ is the length of the shortest path from v to $u(d(v, v) = 0$ and $d(v, u) = \infty$, if there is no such path) [13]. The diameter of G is $\mathrm{diam}(G) = \sup\{d(v, u) \mid v, u \in V(G)\}$. The neighborhood $N(v)$ of a vertex v denotes the set all vertices of G adjacent to the vertex v and $N[v] = N(v) \cup \{v\}$.

The concept of the metric representation and the metric dimension in terms of the locating number in a zero-divisor graph associated with a commutative ring with unity was introduced in [8] and had been further studied in [7]. Feng and Wang in [8] discussed various properties of the locating set and the locating number which includes the characterization of all finite commutative rings with unity, examination of two equivalence relations on the vertices of $\Gamma(R)$, relationship between the locating set and the cut vertices of $\Gamma(R)$, investigation of the locating number in $\Gamma(R)$ when R is a finite product of the integral domains and so on.

Let G_k be a graph on infinite number of vertices with vertex set $V(G_k) = \{v\} \cup \left\{v_1^{(1)}, v_2^{(1)}, \ldots, v_k^{(1)}\right\} \cup \left\{v_1^{(2)}, v_2^{(2)}, \ldots, v_k^{(2)}\right\} \cup \cdots \cup \left\{v_1^{(i)}, v_2^{(i)}, \ldots, v_k^{(i)}\right\} \cup \cdots$ for $i \geq 1$, and the edges are defined by the rule $vv_t^{(1)}, (1 \leq t \leq k)$, and $v_1^{(j)}v_1^{(j+1)}, v_2^{(j)}v_2^{(j+1)}, \ldots, v_k^{(j)}v_k^{(j+1)}$ for all $j = 1, 2, 3, \ldots$. For $k = 1, G_1$ is an infinite tree with vertex set $V(G_1) = \{v\} \cup \left\{v_1^{(1)}\right\} \cup \left\{v_1^{(2)}\right\} \cup \cdots \cup \left\{v_1^{(i)}\right\} \cup \cdots$ for $i \geq 1$, and the edges are defined by $vv_1^{(1)}$ and $v_1^{(j)}v_1^{(j+1)}$ for all $j = 1, 2, 3, \ldots$. Notice, here that the infinite tree G_1 is rooted at the vertex v.

For $k = 2, G_2$ is a graph with vertex set $V(G_2) = \{v\} \cup \left\{v_1^{(1)}, v_2^{(1)}\right\} \cup \left\{v_1^{(2)}, v_2^{(2)}\right\} \cup \cdots \cup \left\{v_1^{(i)}, v_2^{(i)}\right\} \cup \cdots$ for $i \geq 1$, and the edges are defined by $vv_t^{(1)}, (1 \leq t \leq 2)$, and $v_1^{(j)}v_1^{(j+1)}, v_2^{(j)}v_2^{(j+1)}$ for all $j = 1, 2, 3, \ldots$ [11]. Clearly, G_2 is a tree rooted at the vertex v with two infinite branches. The infinite trees G_k are often denoted by $P_{k,1}$ to indicate trees rooted in v with k infinite branches. It is straight forward to prove that $\dim_M(P_{1,1}) = 1$ and $\dim_M(P_{2,1}) = 2$.

In this paper, we characterize variants of distance related parameters and graph numbers associated with the zero divisor graphs akin to cyclic structures obtained from classes of completely primary finite rings. In particular, we compute the local fractional metric dimension and provide certain results concerning graph indices namely the Weiner index and the Zagreb index. We also explore the connection between graph number and the metric dimension of the zero divisor graph. We investigate these parameters in $R = \Gamma(R_1 \oplus R_2 \oplus \ldots \oplus R_n)$, where $R_1, R_2, \ldots, R_n$ are n finite commutative rings each having unity 1 and none of $R_i, (1 \leq i \leq n)$.

2 Preliminaries

The following results are useful in the sequel.

Theorem 2.1. *Let R be a commutative ring with unity 1 (not a domain). Then $\dim_M(\Gamma(R))$ is finite if and only if R is finite.*

Proof. Suppose R is finite. Then, it is clear that $\dim_M(\Gamma(R))$ is finite. Now, suppose $\dim_M(\Gamma(R))$ is finite. Let S be the metric basis for $\Gamma(R)$ with $|S| = k$, where k is some non-negative integer. The diameter of $\Gamma(R)$ is not more than 3 . Therefore, $d(x, y) \in \{0, 1, 2, 3\}$ for every $x, y \in Z^*(R)$. For each $x \in Z^*(R)$, the metric representation $D(x \mid S)$ is the k coordinate vector, where each coordinate is in the set $\{0, 1, 2, 3\}$. Thus there are only $(3 + 1)^k$ possibilities for $D(x \mid S)$. Since $D(x \mid S)$ is unique for each $x \in Z^*(R)$, so $|Z^*(R)| \leq 4^k$. This implies that $Z^*(R)$ is finite and hence R is finite. □

Theorem 6.1 in [14] gives the metric dimension for the zero-divisor graph $\Gamma(R_1 \times R_2 \times \ldots \times R_n)$, where $R_1, R_2, \ldots, R_n$ are integral domains, and also gives bounds for the metric dimension of the zero-divisor graph $\Gamma(\prod_{i=1}^n \mathbb{Z}_2)$. Special emphasis has been given to the graph $\Gamma(\prod_{i=1}^n \mathbb{Z}_2)$ of a finite Boolean ring, and it is shown that $\dim_M(\Gamma(\prod_{i=1}^n \mathbb{Z}_2)) \leq n$, $\dim_M(\Gamma(\prod_{i=1}^n \mathbb{Z}_2)) = n - 1$, for $n = 2, 3, 4$, and $\dim_M(\Gamma(\prod_{i=1}^n \mathbb{Z}_2)) = n$, for $n = 5$ [14]. We need to know as how the metric dimension behaves with respect to the product $R_1 \times R_2 \times \cdots \times R_n$, where $R_1, R_2, \ldots, R_n$ are n finite commutative rings with each having unity 1 .

Lemma 2.2. *A finite commutative ring R with unity 1 has exactly one unit if and only if $R \cong \prod_{i=1}^n \mathbb{Z}_2$ for some positive integer n.*

Proof. Clearly, the ring listed has only one unit. Suppose R has exactly one unit. If R is a local ring with maximal ideal M, then $|R| = p^k$ and $|M| = p^m$ for some prime p and integers $0 \leq m < k$. Then, $1 = |U(R)| = |R| - |M|$ only when $|R| = 2$ and $|M| = 1$. If R is not local, then R can be written as the finite product of local rings that is $R \cong R_1 \times R_2 \times \ldots \times R_n$, where $R_1, R_2, \ldots, R_n$ are finite local rings. If any R_i has more than one unit, then R would have more than one unit. □

Theorem 2.3. *Let $R_1, R_2, \ldots, R_n$ be n finite commutative rings (not domains) each having unity 1 with none of $R_i, 1 \leq i \leq n$, being isomorphic to $\prod_{i=1}^{n} \mathbb{Z}_2$ for any positive integer n. Then for a finite commutative ring R with unity 1 and for a finite field $\mathbb{F}_q$ on prime q number of elements,*

(a) $\dim_M(\Gamma(R_1 \times R_2 \times \cdots \times R_n)) \geq \sum_{i=1}^{n} \dim_M(\Gamma(R_i))$,

(b) $\dim_M(\Gamma(R \times \mathbb{F}_q)) = |Z^*(R \times \mathbb{F}_q)| - 2^{n+1} + 2$ *or* $|Z^*(R \times \mathbb{F}_q)| - 2$ *or at least* $|U(R)| + (|Z^*(R)| + 1)\, q - t - 3$*, where t is any positive integer.*

3 A Survey on the Distance Parameters of Cyclic Structures

Let C_n be a cyclic network with the vertex and edge set given by $V(C_n) = \{a_i \mid 1 \leq i \leq n\}$ and $E(C_n) = \{a_i a_{i+1} \mid 1 \leq i \leq n\}$, respectively, with indices taken $\mod n$.
The local fractional strong metric dimension of certain complex networks is computed.

Theorem 3.1. *For $n \geq 3$,*

$$\mathrm{lsdim}_f(C_n) = \begin{cases} 1, & \text{if } n \equiv 0(\mathrm{mod}2); \\ \frac{n}{n-1}, & \text{if } n \equiv 1(\mathrm{mod}2). \end{cases}$$

Proof. To prove the above claim, we consider the following cases: Case $1(n \equiv 0(\mathrm{mod}2))$ We take note that, $\gamma(C_n) = |V(C_n)| = n$ and $\beta(C_n) = |\cup L(C_n)| = |V(C_n)| = n$. Hence, we conclude

$$\mathrm{lsdim}_f(C_n) = \sum_{s=1}^{\beta(C_n)} \frac{1}{\gamma(C_n)} = 1.$$

Case $2(n \equiv 1(\mathrm{mod}2))$ Here, $\gamma(C_n) = (n-1)$ and $\beta(C_n) = |\cup L(C_n)| = n$. We have

$$\mathrm{lsdim}_f(C_n) = \sum_{s=1}^{\beta(C_n)} \frac{1}{\gamma(C_n)} = \frac{n}{n-1}.$$

□

Theorem 3.2. *For $n \geq 6, \mathrm{lsdim}_f(C_n(1,2)) = n/2(\lceil m + 1/2 \rceil)$.*

Proof. Clearly, $\gamma(C_n(1,2)) = |S\{a_r, a_{r+1}\}| = |S\{a_r, a_{r-1}\}| = 2(\lceil m + 1/2 \rceil)$ where $1 \leq r \leq n$ and $m = \lceil n - 5/4 \rceil$. Moreover, $\beta(C_n(1,2)) = |\cup L(C_n(1,2))| = n$. Therefore, we have

$$\mathrm{lsdim}_f(C_n(1,2)) = \sum_{s=1}^{\beta(C_n(1,2))} \frac{1}{\gamma(C_n(1,2))} = \frac{n}{2(\lceil m + 1/2 \rceil)}.$$

□

Theorem 3.3. *For* $n \geq 6$,

$$\mathrm{lsdim}_f\left(M_{2n}\right) = \begin{cases} 1, & \text{if } n \equiv 1(\mathrm{mod}2); \\ \frac{n}{n-1}, & \text{if } n \equiv 0(\mathrm{mod}2). \end{cases}$$

Proof. The proof of this theorem is subdivided into the following two cases:
Case 1 $(n \equiv 1(\mathrm{mod}2))$
We have $\gamma\left(M_{2n}\right) = |V\left(M_{2n}\right)| = 2n$ and $\beta\left(M_{2n}\right) = |\cup L\left(M_{2n}\right)| = |V\left(M_{2n}\right)| = 2n$. Hence the following can be concluded:

$$\mathrm{lsdim}_f\left(M_{2n}\right) = \sum_{s=1}^{\beta(M_{2n})} \frac{1}{\gamma\left(M_{2n}\right)} = 1.$$

Case 2 $(n \equiv 0(\mathrm{mod}2))$
In this case by considering $\gamma\left(M_{2n}\right) = 2(n-1)$ and $\beta\left(M_{2n}\right) = |\cup L\left(M_{2n}\right)| = 2n$. Hence, we have

$$\mathrm{lsdim}_f\left(M_{2n}\right) = \sum_{s=1}^{\beta(M_{2n})} \frac{1}{\gamma\left(M_{2n}\right)} = \frac{n}{n-1}.$$

□

Theorem 3.4. *For* $n \geq 6$,

$$\mathrm{lsdim}_f\left(G_m^n\right) = \begin{cases} 1, & \text{if } n \equiv 0(\mathrm{mod}2); \\ \frac{n}{n-1}, & \text{if } n \equiv 1(\mathrm{mod}2). \end{cases}$$

Proof. Proof. The proof can be segregated into the following two cases:
Case 1 $(n \equiv 1(\mathrm{mod}2))$
$\gamma\left(G_m^n\right) = m(n-1)$ and $\beta\left(G_m^n\right) = |\cup L\left(G_m^n\right)| = |V\left(G_m^n\right)| = mn$.
We have

$$\mathrm{lsdim}_f\left(G_m^n\right) = \sum_{t=1}^{\beta(G_m^n)} \frac{1}{\gamma\left(G_m^n\right)} = \frac{n}{n-1}.$$

Case 2 $(n \equiv 0(\mathrm{mod}2))$
In this , $\gamma\left(G_m^n\right) = mn$ and $\beta\left(G_m^n\right) = |\cup L\left(G_m^n\right)| = mn$. Hence we conclude that

$$\mathrm{lsdim}_f\left(G_m^n\right) = \sum_{s=1}^{\beta(G_m^n)} \frac{1}{\gamma\left(G_m^n\right)} = 1.$$

□

4 Some Cyclic Structures obtained from $\Gamma(R)$

The following graphs represent cyclic structures associated with graph networks obtained from classes of completely primary finite rings. Consider a finite ring R of idealization given by $R = R_0 \oplus U \oplus V \oplus W \oplus Y$ where U, V, W and Y are R_0-modules generated by various basis elements u_i, v_j, w_k and y_l. If R is closed under product given by

$$\big(r_\circ + \sum_{i=1}^{e} r_i u_i + \sum_{j=1}^{f} s_j v_j + \sum_{k=1}^{g} t_k w_k + \sum_{l=1}^{h} z_l y_l\big)\big(r_\circ^{'} + \sum_{i=1}^{e} r_i^{'} u_i + \sum_{j=1}^{f} s_j^{'} v_j + \sum_{k=1}^{g} t_k^{'} w_k + \sum_{l=1}^{h} z_l^{'} y_l\big)$$

$$= r_\circ r_\circ^{'} + p^a \sum_{i,m=1}^{e} \big(r_i r_m^{'} + pR_0\big) + \sum_{i=1}^{e} \big[r_\circ r_i^{'} + r_i r_\circ^{'} + pR_0\big] u_i + \sum_{j=1}^{f} \big[\big(r_\circ + pR_0\big) s_j^{'} + s_j \big(r_\circ^{'} + pR_0\big) +$$

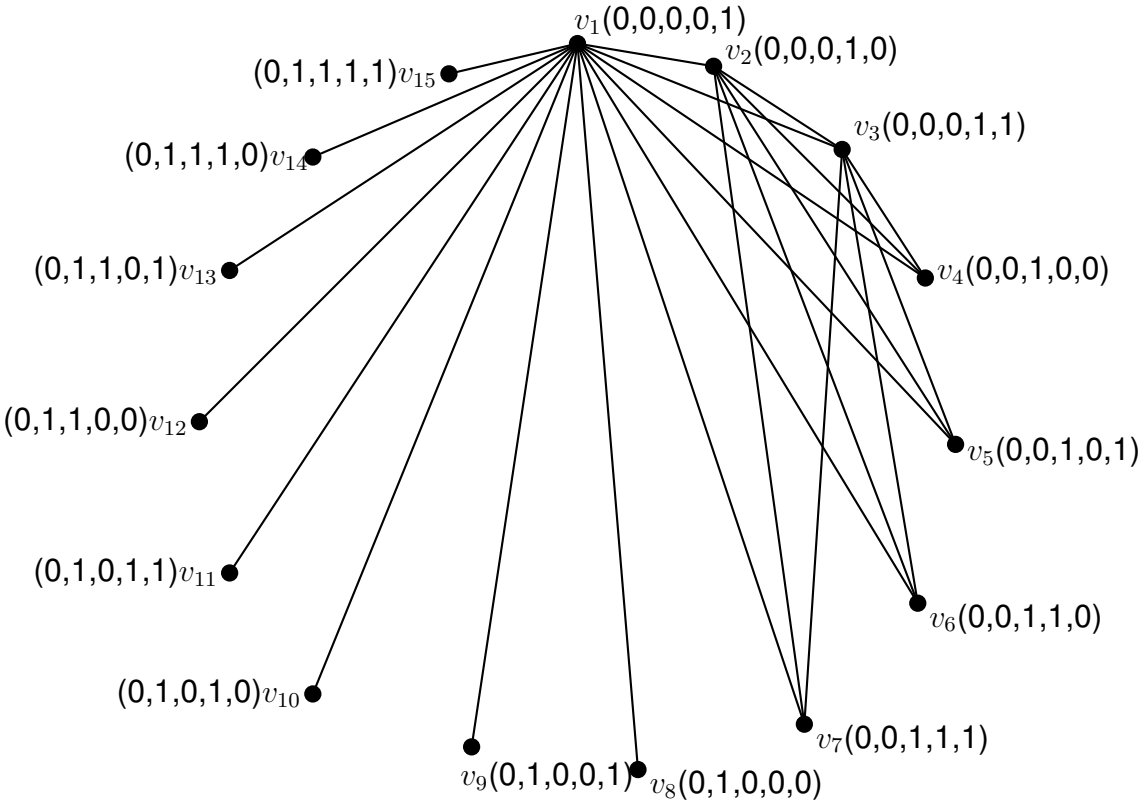
$$\sum_{\nu,\mu=1}^{e} \big(r_\nu r_\mu^{'} + pR_0\big)]v_j + \sum_{k=1}^{g} \big[(r_\circ + pR_0\big) t_k^{'} + t_k \big(r_\circ^{'} + pR_0\big) + \sum_{i,j} \big(r_i + pR_0\big) s_j^{'} + s_j \big(r_i^{'} + pR_0\big)]w_k +$$

$$\sum_{l=1}^{h} \big[\big(r_\circ + pR_0\big) z_l^{'} + z_l \big(r_\circ^{'} + pR_0\big) + \sum_{i,k} \big(r_i + pR_0\big) t_k^{'} + t_k (r_i^{'} + pR_0\big) + \sum_{\kappa,\tau=1}^{f} \big(s_\kappa s_\tau^{'} + pR_0\big] y_l$$

where $a = 1, 2, 3$ or 4 depending on whether $charR = p^2, p^3, p^4$ or p^5. This multiplication turns R into a commutative ring with identity $(1, 0, 0, 0, 0)$ [9].
Given a zero divisor graph $\Gamma(R)$ with vertex set $V(\Gamma(R))$. If $x\ and\ y$ are any two vertices of the graph, then x and y lie in the edge of the graph $E(\Gamma(R))$ if and only if $xy = 0$. Using this adjacency property, we have the following cyclic representation of the geometries of $\Gamma(R)$ for various characteristics.

Example 4.1. *If R is the ring of the construction above, where $R = R_0 \oplus U \oplus V \oplus W \oplus Y$ then for $charR = p = 2$*
$R = \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$ then the zero divisors will be
$Z(R) = \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$ In this case $p = 2$, $r = 1$, $e = 1$, $f = 1$, $g = 1$ and $h = 1$.
So the $\Gamma(R)$ is $4-$ partite with $dim(\Gamma(R)) = 2$, $gr(\Gamma(R)) = 3$ and $b(\Gamma(R)) = \frac{1}{4}$.

When $charR = p = 2$ *the graph is as follows;*

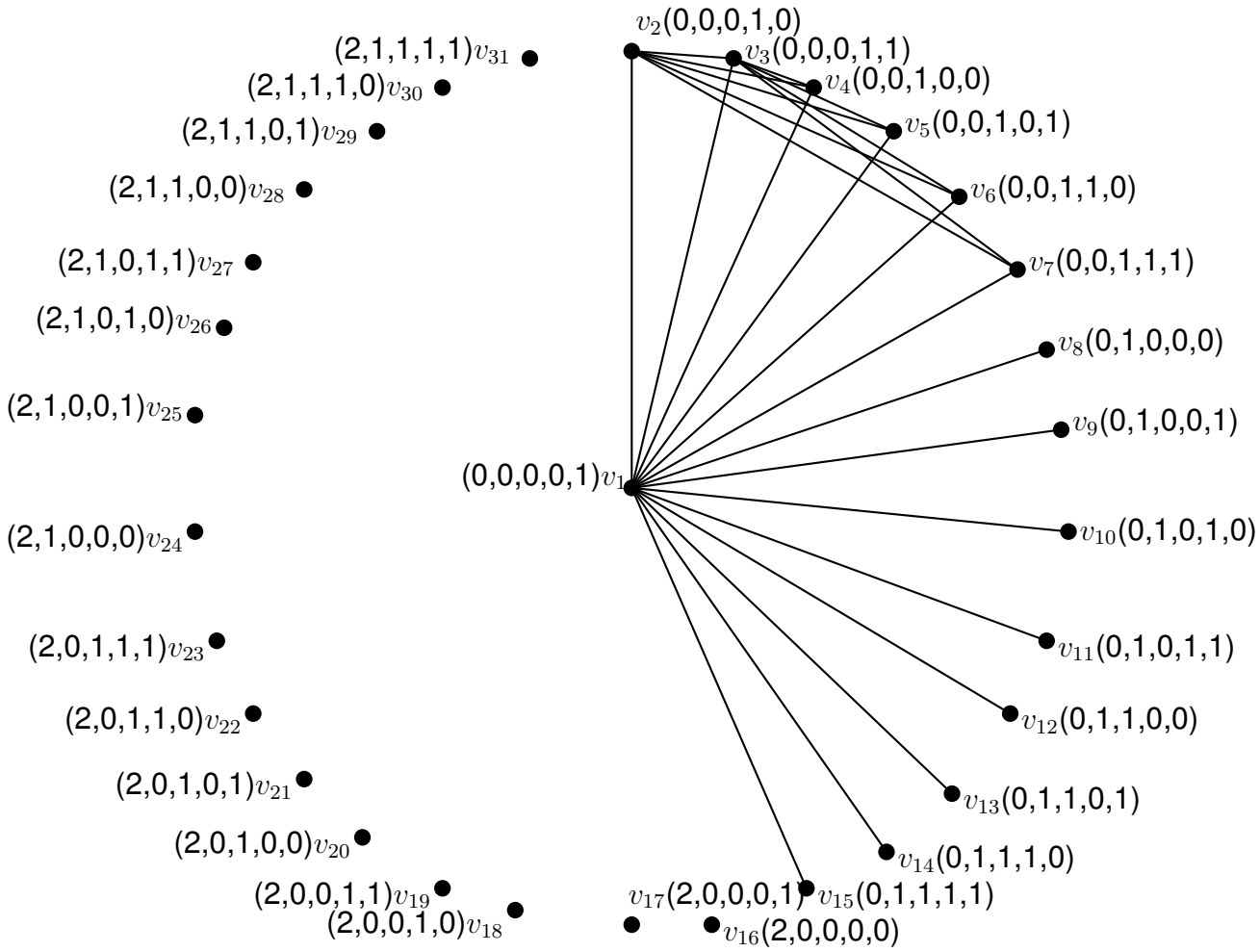


Example 4.2. *Let* $Z(R) = 2Z(R) = \mathbb{Z}_4 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus (Z)_2$. *In this case* $p = 2$, $r = 1$,$e = 1$ $f = 1$,$g = 1$, $g = 1$ *and* $h = 1$. *Then the set of vertices* $V(\Gamma(R))$ *is given by*

Let $Z(R) = 2Z(R) = \mathbb{Z}_4 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus (Z)_2$. *In this case* $p = 2$, $r = 1$,$e = 1$ $f = 1$,$g = 1$, $g = 1$ *and* $h = 1$. *Then*

$\Gamma(R)$ is $8-$ partite with $diam(\Gamma(R)) = 2$, $gr(\Gamma(R)) = 3$ and $b(\Gamma(R)) = \frac{7}{24}$.

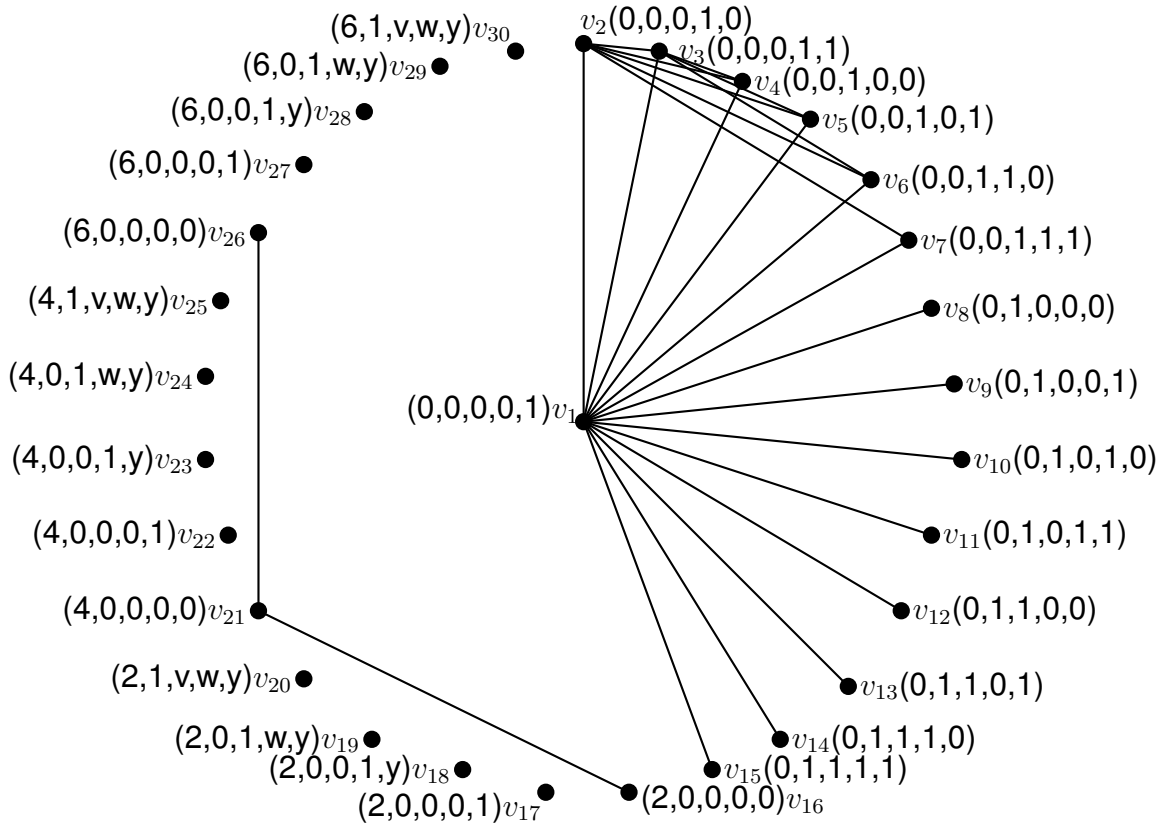
When $charR = P^2$ the graph is as follows;



Example 4.3. *Let* $Z(R) = 2Z(R) = 2\mathbb{Z}_8 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus (Z)_2$. *In this case* $p = 2$, $r = 1$,$e = 1$ $f = 1$,$g = 1$, $g = 1$ *and* $h = 1$. *Then*

$\Gamma(R)$ is $16-$ partite with $diam(\Gamma(R)) = 2$, $gr(\Gamma(R)) = 3$ and $b(\Gamma(R)) = \frac{5}{16}$.

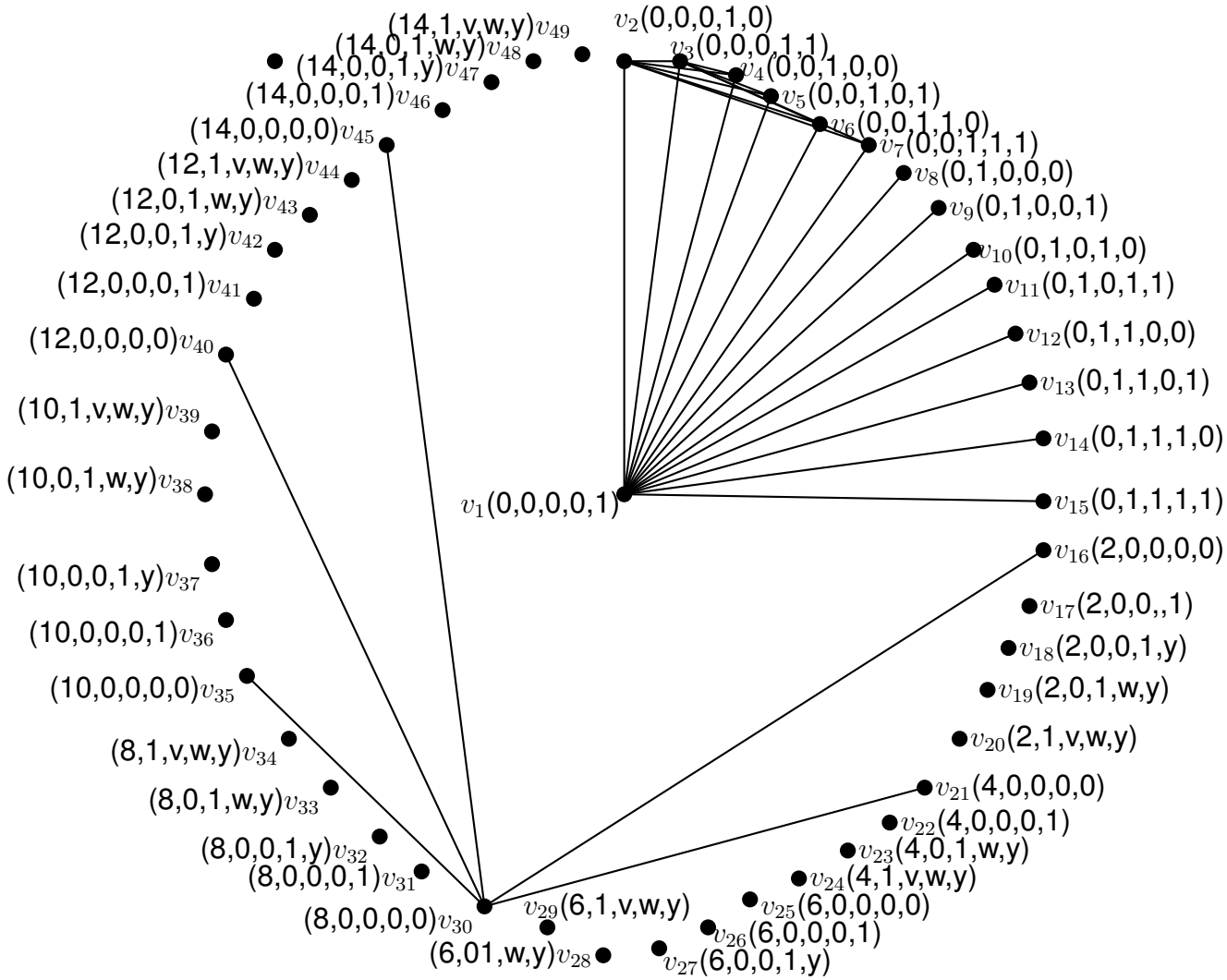
When $charR = p^3$ the graph will appear as below;



Example 4.4. *Let* $Z(R) = 2\mathbb{Z}_{16} \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus (Z)_2$. *In this case* $p = 2$, $r = 1$,$e = 1$ $f = 1$,$g = 1$, $g = 1$ *and* $h = 1$. *Then*

$\Gamma(R)$ is $32-$ partite with $diam(\Gamma(R)) = 2$, $gr(\Gamma(R)) = 3$ and $b(\Gamma(R)) = \frac{31}{96}$.

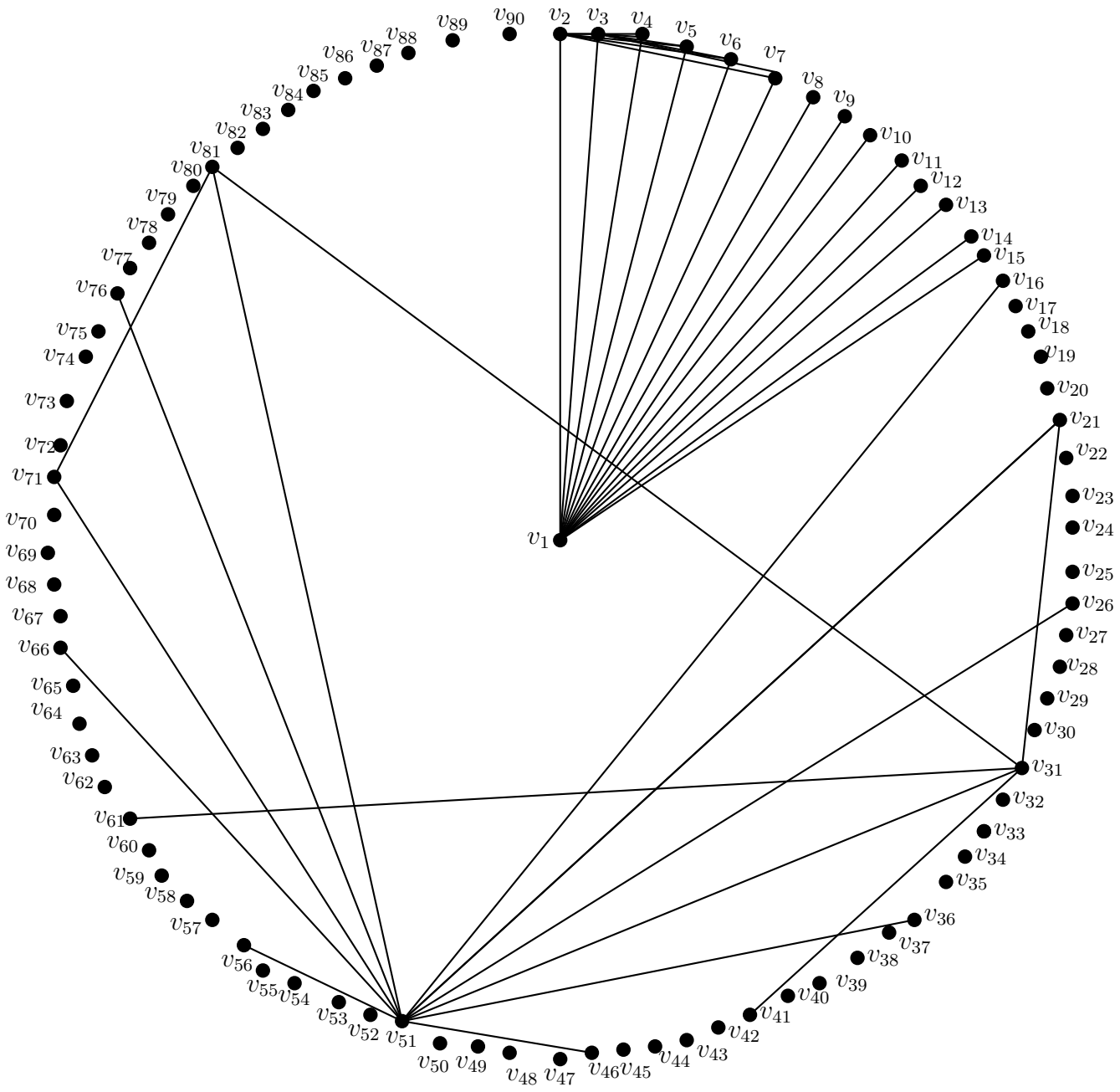
When $charR = p^4$ the graph is as follows;



Example 4.5. *Let* $Z(R) = 2\mathbb{Z}_{16} \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus (Z)_2$. *In this case* $p = 2$, $r = 1$,$e = 1$ $f = 1$,$g = 1$, $g = 1$ *and* $h = 1$. *Then*

$\Gamma(R)$ is $64-$ partite with $diam(\Gamma(R)) = 2$, $gr(\Gamma(R)) = 3$ and $b(\Gamma(R)) = \frac{21}{64}$.

When $charR = p^5$ the graph is as follows;



Remark 4.1. *The graphs provided in the above examples represent complex structures akin to cyclic structures and their metric dimensions and local fractional metric dimensions obey the bounds provided in the previous section of this paper.*

5 Graph Numbers

5.1 The Wiener Index of $\Gamma(R)$

The Wiener index denoted as W and also known as the path number or the Wiener number is a graph index defined on a graph by n nodes and defined as

$$W = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(d)_{ij}$$

where $(d)_{ij}$ is the graph distance matrix. The Wiener index $W(\Gamma(R))$ of the graph G with vertex count $\mid V(\Gamma(R)) \mid$ has a relationship with the average disorder number of the zero divisor graph $A(\Gamma(R)) = \frac{2W(\Gamma(R))}{|V(\Gamma(R))|}$ and the average distance $\mu(\Gamma(R))$ between the vertices of $\Gamma(R)$ which is given by

$$\mu(\Gamma(R)) = \frac{W(\Gamma(R))}{\begin{pmatrix} \mid V(\Gamma(R)) \mid \\ 2 \end{pmatrix}}.$$

In Topological Graph Theory, computations for Wiener indices for cyclic carbon-chained organic compounds and its applications is fundamental. The index is useful in determination of the boiling points and polarity number of alkanes and their branched isomers. Further, the most and common natural field in the application of the Wiener index is the quantitative structure relationships especially in the estimation of emission spectra of the ultra violet radiations of α and β-unsaturated ketone. We therefore present the following results on the Wiener index of $\Gamma(R)$ and other results describing average disorder number and the average distance indices of $\Gamma(R)$ due to their close interdependence with the Wiener index.

Proposition 5.1. *Let $\Gamma(R)$ be the zero divisor graph of the classes of 5-index zero finite rings. Then for any prime integer $p, r \in \mathbb{Z}^+$ and s fixed, the Wiener index, $W(\Gamma(R))$*

$$= \begin{cases} \frac{1}{2}(2p^{(\frac{2(s^2+3s)}{2}-1)r} + p^{2(\frac{(s^2+3s)}{2}-1)r} - p^{(\frac{(s^2+3s)}{2})r} - 7p^{(\frac{(s^2+3s)}{2}-1)r} + 2), & \text{if } char(R) = p; \\ \frac{1}{2}(2p^{(\frac{2(s^2+3s)}{2}+2)r} + p^{2(\frac{(s^2+3s)}{2})r} - p^{(\frac{(s^2+3s+2)}{2})r} - 7p^{(\frac{(s^2+3s)}{2})r} + 2), & char(R) = p^2; \\ \frac{1}{2}(2p^{(\frac{(2s^2+8s+4)}{2})r} + p^{2(\frac{(s^2+3s+2)}{2})r} - p^{(\frac{(s^2+5s+2)}{2})r} - 7p^{(\frac{(s^2+3s+2)}{2})r} + 2), & char(R) = p^3; \\ \frac{1}{2}(2p^{(\frac{(2s^2+10s+6)}{2})r} + p^{2(\frac{(2s^2+10s+4)}{2})r} - p^{2(\frac{(s^2+5s+4)}{2})r} - 7p^{(\frac{(s^2+5s+2)}{2})r} + 2), & char(R) = p^4,\ p^5. \end{cases}$$

Proposition 5.2. *Let $\Gamma(R)$ be the zero divisor graph of the classes of 5-index zero finite rings and $W(\Gamma(R))$ be its Wiener index. Then for any prime integer p, positive integer r and s fixed, the average*

distance of $\Gamma(R)$

$$\mu(\Gamma(R)) = \begin{cases} \frac{\frac{1}{2}(2p^{(\frac{(2(s^2+3s)}{2}-1)r}+p^{2(\frac{(s^2+3s)}{2}-1)r}-p^{(\frac{(s^2+3s)}{2})r}-7p^{(\frac{(s^2+3s)}{2}-1)r}+2)}{(p^{(\frac{(s^2+3s)}{2})r}-1)(p^{(\frac{(s^2+3s)}{2})r}-2)}, & \text{for } char(R)=p; \\ \frac{\frac{1}{2}(2p^{(\frac{2(s^2+3s)}{2}+2)r}+p^{(\frac{2(s^2+3s)}{2})r}-p^{(\frac{(s^2+3s+2)}{2})r}-7p^{(\frac{(s^2+3s)}{2})r}+2)}{(p^{(\frac{(s^2+3s+2)}{2})r}-1)(p^{(\frac{(s^2+3s+2)}{2})r}-2)}, & char(R)=p^2; \\ \frac{\frac{1}{2}(2p^{(\frac{(2s^2+8s+4)}{2})r}+p^{2(\frac{(s^2+3s+2)}{2})r}-p^{(\frac{(s^2+5s+2)}{2})r}-7p^{(\frac{(s^2+3s+2)}{2})r}+2)}{(p^{(\frac{(s^2+5s+2)}{2})r}-1)(p^{(\frac{(s^2+5s+2)}{2})r}-2)}, & char(R)=p^3; \\ \frac{\frac{1}{2}(2p^{(\frac{(2s^2+10s+6)}{2})r}+p^{2(\frac{(2s^2+10s+4)}{2})r}-p^{2(\frac{(s^2+5s+4)}{2})r}-7p^{(\frac{(s^2+5s+2)}{2})r}+2)}{(p^{(\frac{(s^2+5s+4)}{2})r}-1)(p^{(\frac{(s^2+5s+4)}{2})r}-2)}, & char(R)=p^4,p^5. \end{cases}$$

Proposition 5.3. *Let* $\Gamma(R)$ *be the zero divisor graph of the classes of 5-index zero finite rings. Then for any prime integer* p*, positive integer* r *and* s *fixed, the average disorder number of the zero divisor graph*

$$A(\Gamma(R)) = \begin{cases} \frac{2p^{(\frac{(2(s^2+3s)}{2}-1)r}+p^{2(\frac{(s^2+3s)}{2}-1)r}-p^{(\frac{(s^2+3s)}{2})r}-7p^{(\frac{(s^2+3s)}{2}-1)r}+2}{(p^{(\frac{((s^2+3s)}{2})r}-1)}, & \text{for } char(R)=p; \\ \frac{3p^{(\frac{2(s^2+3s)}{2}+2)r}+p^{(\frac{2(s^2+3s)}{2})r}-p^{(\frac{(s^2+3s+2)}{2})r}-7p^{(\frac{(s^2+3s)}{2})r}+2}{(p^{(\frac{((s^2+3s+2)}{2})r}-1)}, & char(R)=p^2; \\ \frac{3p^{(\frac{(2s^2+8s+4)}{2})r}+p^{2(\frac{(s^2+3s+2)}{2})r}-p^{(\frac{(s^2+5s+2)}{2})r}-7p^{(\frac{(s^2+3s+2)}{2})r}+2}{(p^{(\frac{((s^2+5s+2)}{2})r}-1)}, & char(R)=p^3; \\ \frac{3p^{(\frac{(2s^2+10s+6)}{2})r}+p^{2(\frac{(2s^2+10s+4)}{2})r}-p^{2(\frac{(s^2+5s+4)}{2})r}-7p^{(\frac{(s^2+5s+2)}{2})r}+2}{(p^{(\frac{((s^2+5s+4)}{2})r}-1)}, & char(R)=p^4,p^5. \end{cases}$$

5.2 The Zagreb Indices of $\Gamma(R)$

Let $G = \Gamma(R)$ be a simple graph such that $G = (V, E)$ whose vertex set $V(G)$ consist of elements $\{v_1, \cdots, v_n\}$ such that $\mid V(G) \mid = n$ and the set of edges $E(G)$ of order m. Given that the minimum degree of G is denoted by $\delta(G)$ and $\Delta(G)$ the maximum degree. Let $d_i = deg(v_i)_{\Gamma(R)}, i = 1, 2, \cdots, n$ be the vertex degrees of $v_i \in \Gamma(R)$ so that $d_i \geq d_2 \geq \cdots \geq d_n$. The first Zagreb index is the sum of the squares of degrees of the vertices and the second Zagreb index is the sum of the products of the degrees of the pairs of adjacent vertices. We denote the first and second Zagreb indices of $\Gamma(R)$ by $Z_1(\Gamma(R))$ and $Z_2(\Gamma(R))$ respectively. Therefore,

$$Z_1(\Gamma(R)) = \sum_{v_i \in V(\Gamma(R))} d_i^2,$$

$$Z_2(\Gamma(R)) = \sum_{v_i - v_j \in E(\Gamma(R))} d_i d_j.$$

5.2.1 The First Zagreb Index, $Z_1(\Gamma(R))$

Proposition 5.4. *Let $\Gamma(R)$ be the zero divisor graph of the classes of 5-index zero finite rings and $\Gamma(R)$ be a zero divisor graph of order $p^{(e+f+g+h)r}-1$ with m edges. If $\Delta(\Gamma(R))$ and $\delta(\Gamma(R))$ are the maximum and minimum degrees of $\Gamma(R)$ then for any prime integer $p, r \in \mathbb{Z}^+$,*

(i) $Z_1(\Gamma(R)) \geq \frac{((\Delta(\Gamma(R)))^2+(2m-\Delta(\Gamma(R))))^2}{(p^{(e+f+g+h)r}-2)} + \frac{2(p^{(e+f+g+h)r}-3)}{(p^{(e+f+g+h)r}-2)^2}.(\Delta_2(\Gamma(R)) - \delta(\Gamma(R)))^2$, *where $\Delta_2(\Gamma(R))$ is the second maximum degree of $\Gamma(R)$.*

(ii) $Z_1(\Gamma(R)) \leq 4m^2+2((\Delta(\Gamma(R)))^2-4m((\Delta(\Gamma(R)))-((p^{(h+(k-1)r)}-2)((p^{(h+(k-1)r)}-3))[\frac{T(\Gamma(R))}{(p^{(e+f+g+h)r}-2)\Delta(\Gamma(R))}(I(\Gamma(R)))-\frac{1}{\Delta(\Gamma(R))}]\frac{2}{p^{(e+f+g+h)r}-3}$.

Proposition 5.5. *Let R be the classes of rings with h as the dimension of the modules in R'. Let $\Gamma(R)$ be a zero divisor graph such that $\mid \Gamma(R) \mid = p^{(h+(k-1))r}-1$ with m edges. If $\Delta(\Gamma(R))$ is the maximum degree of each $v_i \in \Gamma(R)$ then for any prime integer $p, r \in \mathbb{Z}^+$,*

$$Z_1(\Gamma(R)) \leq (p^{(e+f+g+h)r}-1)m - \Delta(\Gamma(R))((p^{(h+(k-1))r}-1) - \Delta(\Gamma(R))) + \frac{2(m-\Delta(\Gamma(R)))}{p^{(e+f+g+h)r}-3}$$

5.2.2 The second Zagreb Index, $Z_2(\Gamma(R))$

Proposition 5.6. *Let $\Gamma(R)$ be the zero divisor graph of the classes of rings such that $\mid V(\Gamma(R)) \mid = p^{(h+(k-1))r}-1$ and $\Delta(\Gamma(R))), \delta(\Gamma(R))$ its maximum and minimum degrees respectively. If m is the number of edges of $\Delta(\Gamma(R))$ then for any prime integer p, positive integers r, k,*

(i) $Z_2(\Gamma(R)) \geq 2m^2 - m(p^{(h+(k-1))r}-2)\Delta(\Gamma(R)) + \frac{1}{2}(\Delta(\Gamma(R))-2)[(\Delta(\Gamma(R)))^2 + \frac{(2m-\Delta(\Gamma(R)))^2}{p^{(h+(k-1))r}-2} + \frac{2(p^{(e+f+g+h)r}-1)}{(p^{(e+f+g+h)r}-2)^2}(\Delta(\Gamma(R)) - \delta(\Gamma(R)))^2]$.

(ii) $Z_2(\Gamma(R)) \geq 2m^2 - m(p^{(e+f+g+h)r)\delta(\Gamma(R))+\frac{1}{2}(\delta(\Gamma(R))-1)[m(p^{(e+f+g+h)r})-\Delta(\Gamma(R))(p^{(e+f+g+h)r}} - \Delta(\Gamma(R)) + \frac{2(m-\Delta(\Gamma(R)))^2}{p^{(e+f+g+h)r}-3}]$.

6 Conclusion

In conclusion, the study succeeds in presenting some findings related to the metric dimensions, Weiner index, and Zagreb index of $\Gamma(R)$ of interest.

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Unethical Practices in Tanzania Public Universities: Does Workload Matter?

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ABSTRACT

Globally, the operations of universities rely on workload distribution. The extent to which the workload shouldered by employees is excessive or insufficient has implications for ethical behaviour and well-being in academic careers. This paper examined the effect of workload on unethical practices in Tanzanian public universities. Karasek's Job Demand-Control Model served as the foundation for the study. The quantitative research approach was utilized, and questionnaires were used for data gathering. The study adopted a correlational research design. The study gathered data from three public universities, aiming to reach 245 respondents who were sampled using stratified sampling and administered questionnaires. However, only a total of 209 respondents completed the questionnaire. The collected data were analyzed using both descriptive and inferential statistics. STATA MP version 17 was used for analysis to conduct the multivariate regression model. The finding revealed workload has a significant positive effect on unethical practices, indicating that both excessive and insufficient workloads contribute to unethical behaviour among academic staff. The study concluded that although workload in higher learning institutions is a lifeblood of curriculum implementation, it is not fairly distributed and thus affects both individual and organizational operations. Therefore, the study recommends that the universities should play a crucial role in developing and implementing internal workload policies that link with academic integrity to foster high-quality services in university operations.

Keywords: Ethics, Job Demand Control Model, Unethical Practices, Workload, Work Overload, Work Underload

I. INTRODUCTION

Workload in universities plays a crucial role in shaping the behavior and ethical standards of both academic staff and students. In that regard, it is considered to be an effective tool to ensure the work of the institutions is performed based on resources directed to each activity. This tool is used by both government and private institutions to smooth operations and foster accountability of individuals for the assigned task. The literature points out that workload covers almost every sector as well as a profession where education is among them (Gonzales et al., 2022). It is without a shadow of a doubt that a well-balanced workload is essential to maintaining high ethical standards in universities. This goes to show that if it is properly allocated, it may ensure that faculty or school and students are engaged, motivated, and capable of meeting their responsibilities without resorting to unethical practices (Mwita et al., 2023). Evidence shows the proper distribution of workload promotes goal achievement, effective utilization, and the well-being of academic staff (Jerrim & Sims, 2020; Mwita et al., 2024).

Anecdote evidence shows that both excessive and insufficient workload can be tantamount to unethical behaviour. According to Park (2017), when academic staff or students are overwhelmed with excessive workload, the pressure can lead to various forms of unethical behaviours. First, numerous lecturers may resort to cutting corners in their teaching or research. For instance, they might recycle old lecture materials without updating them or plagiarize research content to meet publication deadlines. Students, on the other hand, might resort to cheating, copying assignments, or using unauthorized aids during exams due to their inability to cope with the heavy academic demands. In line with that, Buchanan & Hvizdak (2009) are of the view that academic staff with too many responsibilities may neglect certain duties, such as providing timely feedback, adequately preparing for lectures, or mentoring students. This neglect can lead to a deterioration in the quality of education and, in extreme cases, to unethical practices such as falsifying grades to manage workload.

Contrariwise, when the workload is too low, it can also lead to unethical behaviour due to complacency or a lack of engagement (Voss & Gruber, 2006). Members of the academia's low workload might not be motivated to stay up-to-date with new developments in their field, leading to outdated or irrelevant teaching. This can result in a lower quality of education and a disservice to students. Academic staff with insufficient workload may misuse university resources or time for personal projects that do not benefit the institution or students. For example, a lecturer might focus on consulting work rather than their teaching or research duties, which can be seen as an unethical use of university resources.

An excessive or ineffective distribution of it can lead to dissatisfaction. It has been found that most teachers in the teaching industries, including primary and secondary education, are dissatisfied with their workload (Cooper, 2018; Jerrim & Sims, 2020). This is due to the unrealistic number of activities assigned with the pressures for deadlines to accomplish such activities and insufficient resource allocations to support activities and other work-stress-related matters.

In the case of universities, workload streamlines the way in which academic members perform in teaching, research, and consultancies (Mwita et al., 2023; Ujir et al., 2020). However, Danner et al. (2018) highlighted that the allocation of fair and equitable teaching-related duties to academic staff is a problem for worldwide universities because individual faculty members frequently believe that their workloads are more than normal while considering the assignment process in their department is unjust. This can directly affect the employee's morale and how they act to meet the demands of the workload. For this reason, adherence to ethics while executing their assigned duties may also be a challenge due to unethical practices among academic staff. Supportively, the challenge in workload distribution at all educational levels and unfair work distribution among academics may jeopardize the quality of teaching and research at the university level (Ballet & Kelchtermans, 2009; Danner et al., 2018; Kanwal et al., 2023).

1.1 Statement of the Problem

Workload is now a key component of decentralization and supports worker equity on a global scale. Thus, the degree of responsibility entrusted to an individual determines the efficacy of their work. For Tanzania universities, workload assignment to academic staff needs to comply with standards and guidelines for university education in Tanzania. The Commission reorganizes that academic staff workload in universities is quite enormous as the staff members are involved in various tasks (Tanzania Commission for Universities [TCU], 2019). While literature shows excessive and heavier workloads make academic staff experience burnout, stress, emotional exhaustion, depersonalization, reduced personal accomplishment, anxiety/depression, and fatigue (Fernet et al., 2012; Jerrim & Sims, 2020; Mwita & Mrema, 2023; Smith, 2019). Furthermore, Fournier et al. (2011) and Hewett (2022) found heavy workload influences employees to always diverge from an ethical point of view to create conducive atmospheres that make them feel comfortable and out of stress. Consequently, the workload may result in malpractices that question the ethical behaviour of academic staff while executing their assigned work.

1.2 Research Objective

(i) To examine the effect of work overload (excessive) on unethical practices
(ii) To examine the effect of work underload practice (insufficient) on unethical practices

1.3 Research Hypotheses

H_1: Work overload significantly influences unethical practices
H_2: Work underload significantly influences unethical practices

II. LITERATURE REVIEW

2.1 Theoretical Review

2.1.1 The Job Demand-Control Model

The Job Demand-Control (JDC) model, developed by Robert Karasek in 1979, is a widely recognized framework used to understand how job demands and control over work result in academic staff or employee well-being, stress levels, and productivity. In this context, the model underpins this piece of work as it describes how job characteristics affect the well-being of employees and is designed to predict negative outcomes in the workplace (Kain & Jex, 2010). The model displays job demands such as heavy workloads, tight deadlines, higher targets, role ambiguity, and work pleasure that, if not controlled, may affect employees' well-being. The model delves into two aspects: job demand and job control. The former (activities/job roles assigned to individuals) stress at work is caused by the high demand for a job. In a nutshell, job demand covers the physical, psychological, and emotional requirements of a job, such as workload, time pressures, and the complexity of tasks. On the other hand, decision latitude or job control (how

individuals control their work and take action over them), where stress increases for individuals when they fail to have control over their activities. This implies that academic staff reduces stress from work, anxiety, depression, and all pressure from work caused by the heavy workload such as teaching (preparing lectures, marking assignments, tests, and examinations), research, and publication (submission of book chapters, manuscripts), consultancy, and other administrative functions. They might find a simple path (control mechanism) including manipulation of normal rules and procedures established. Hence, they violate the integrity and ethical principles to ensure the intended results are met and to have control over the high-demand job at that moment. Kain and Jex (2010) claim when the assigned work is in high demand and employees have little or no control, it attracts high job stain, while if there is high job demand that matches with decision latitude, the results are active, which means low stress and high job satisfaction.

2.2 Empirical Review

2.2.1 Work Overload and Unethical Practices

The literature points out that there are mixed feelings when it comes to the work assigned to employees. The idea of the way an employee feels comfortable when they are assigned a huge amount of work may not be perfect for all employees. Researchers have shown high levels of workload among employees attract them to be creative at the same time to find any way possible to attain the intended results, at this point adherence to the professional code of ethics among employees remains moderate (Dadkhah-Tehrani & Adib-Hajbaghery, 2022). On the other hand, Abbasi, (2015); and Yulihardi et al. (2024) showed there is a link between work stress and heavy workload among employees which also affects employees' satisfaction with healthcare work. Similarly, Calais, (1985); Zahednezhad *et al.,* (2021) and Mwilongo *et al.,* (2024) point heavy workload has adversely affected the mental health of staff and patients/customers as the service provider may intend to reduce stress to receivers to find relief from workload burden, hence they may commit unethical practices. This shed a view that a higher workload may compromise ethical standards and influence inappropriate action. This view is supported by Cleaton-Jones, (2012) when claiming the increased workload is a serious challenge and has to be tackled at the workplace.

H_1: Work overload significantly influences unethical practices

2.2.2 Work Underload and Unethical Practices

Employees who are assigned heavy tasks consider employees with low workloads to be lazy. However, the literature points such kind of employees are in a position to perform better compared to those with heavier workloads with deadlines (Utari, *et al.,* 2022). Despite the lower pressure, they can effectively utilize their workload. However, this doesn't guarantee them to act more ethically when executing duties. Westgate, (2020) highlighted that due to the fact they experience boredom when they accomplish the assigned task, may easily engage in unofficial/unacceptable practices such as malpractices. Aldino and Franksiska, (2021) show people who don't have a heavy workload often feel undervalued and disregarded at work, which can lead to job stress. The reason behind work underload in an academic setting may be influenced by academic rank, which tends to differentiate workload distribution among academics. Furthermore, the ability of academic members determines the workload to be assigned. Furthermore, Schunk, (2023) added people who have a high ability to perform are always assigned a wide range of activities to perform while people who are unable to perform are assigned few responsibilities.

H_2: Work underload significantly influences unethical practices

III. METHODOLOGY

3.1 Research Design

The study embedded a quantitative research approach while involving a correlational research design to assess the relationship between workload level (excessive or insufficient) assigned to academic staff and the likelihood of the occurrence of unethical practices in Tanzania public universities. Correlation research design is effective for identifying patterns of relationships/associations between variables as it involves collecting data from the target population at a single point in time to ensure the robustness of the results (Mwita, 2022b).

3.2 Study Population

The participants for this study were selected from academic staff at various public universities in Tanzania. The target population included academic members with different academic ranks and education qualifications who were directly involved in teaching and research activities. To be eligible for inclusion in the study, participants had to meet the following criteria: must be currently employed as academic staff at a Tanzanian public university, must be actively involved in teaching and research responsibilities, and must have at least one year of experience in their current role to

ensure familiarity with the workload and academic expectations. Meanwhile, participants were excluded from the study if they; were on any kind of leave during the data collection period, and did not have direct teaching or research responsibilities, such as purely administrative staff.

3.3 Sample Size and Sampling Techniques

A stratified random sampling method was employed to ensure representation from different academic ranks and faculties. This approach aimed to capture a diverse range of experiences and perspectives related to workload and unethical practices. The sample size was determined using power analysis to ensure sufficient statistical power to detect meaningful relationships between the variables (Mwita, 2022a). The final sample included 209 participants, which is deemed adequate based on the calculated power analysis.

3.4 Data Collection Tools and Analysis

Data for the study were gathered using a well-structured questionnaire by strictly ensuring the confidentiality of the respondents during the process of filling them. Only 209, out of 245 questionnaires that were administered to respondents using the drop-and-pick approach were completed and taken into account for analysis. The data gathered was then analyzed using the multivariate linear regression model to quantify the effect of the workload level of academic staff on the incidence of unethical practices. After analysis results were presented using descriptive and inferential statistics.

IV. FINDINGS & DISCUSSIONS

4.1 Demographic Characteristics of Respondents

The study involved a total of 209 respondents with consideration of their unique demographic characteristics. Out of 209 respondents, 124(59.33) were males and 85(40.67) were females. On the other hand, 14(6.7) respondents had ages ranging from 20-24, while 62(29.67) respondents had ages ranging from 25-29, the ranging age from 30-34 had 44(21.05) respondents, the age from 35-39 had 30(14.35) respondents, and 58(27.75) respondents were above 39 while 1(0.48) is a missing value. On the other hand, the Education level of respondents was considered whereby 46(22.01) respondents hold a bachelor's degree, while 92(44.02) respondents hold a master's degree, 59(28.23) hold a doctoral degree, and 12(5.74) respondents hold a postdoctoral degree. Additionally, the marital status of respondents demonstrates that 81(38.21) of respondents were single, while 124(59.33) respondents were married. Lastly, the year of experience shows 30(14.35) respondents had 0-4 years of experience, 69(33.01) respondents ranged from 5-9 years, 61(29.19) ranged from 10-14 years, and 46(22.01) were above 14 years while 3(1.44) is a missing value. Table 1 below provides a summary of demographic characteristics.

Table 1

Demographic Characteristics

Characteristics	Category	Frequency (%)
Gender	Male	124 (59.33)
	Female	85 (40.67)
Age-group	20-24	14 (6.7)
	25-29	62 (29.67)
	30-34	44 (21.05)
	35-39	30 (14.35)
	Above 39	58 (27.75)
	Missing value	1 (0.48)
Education level	Bachelor’s degree	46 (22.01)
	Master’s degree	92 (44.02)
	Doctoral degree	59 (28.23)
	Postdoctoral degree	12 (5.74)
Marital status	Single	81 (38.21)
	Married	124 (59.33)
	Missing value	4 (1.91)
Years of working experience	0-4 years	30 (14.35)
	5-9 years	69 (33.01)
	10-14 years	61 (29.19)
	15 and above	46 (22.01)
	Missing value	3 (1.44)
Total		**209**

4.2 Descriptive Results

Descriptive statistics were used to calculate the mean and standard deviation of each variable involved in the study. The results are presented below.

4.2.1 Work Overload

The results from Table 2 below provide a weighted mean and standard deviation score on work overload statements. It shows the weighted mean score was 3.89 which is close to 4 in the 5-point Likert scale, implies on average respondents agreed with the constructed statement on work overload. This is to say most respondents agree that work overload is a serious issue that impacts their workplace behavior and ethics. The weighted standard deviation was 1.170 which implies there is a moderate level of variation in responses. This is to say while others agree with the constructed statements on work overload and their effects on unethical behavior, others might be neutral or disagree with it.

Table 2

Descriptive Results for Work Overload

Statements	Mean	S.D
WOL1: I believe that the pressure from overwork can contribute to academic staff engaging in unethical practices	3.94	1.156
WOL2: Overwork and unrealistic workload expectations negatively impact my ability to maintain ethical standards	3.85	1.298
WOL3: I sometimes feel forced to compromise my ethical principles due to the demands of my workload	3.92	1.228
WOL4: Overwork leads to exhaustion, which increases the likelihood of engaging in unethical practices	4.03	1.199
WOL5: I often experience high levels of stress and burnout due to my workload, and I find it difficult to adhere to or maintain high ethical standards in teaching, research, and consultancy	3.73	.968
Weighted mean & standard deviation	3.89	1.170

4.2.2 Work Underload

The results in Table 3 indicate the weighted score for work underload in mean score was 2.91 which is close to 3 which represents neutral on the 5-point Likert scale. This implies on average respondents neither strongly disagree nor strongly agree with the constructed statement related to work underload. On the other hand, the weighted standard deviation was 1.303 which implies there is a moderate to high level of response variation. This is to say while other respondents strongly agree/disagree, others are more neutral about the effects of work underload towards unethical practices. This might be due to work pace demand, and differences in ranks and job roles between each member (Mwilongo et al., 2024).

Table 3

Descriptive Results for Work Underload

Statements	Mean	S.D
WUL1: I believe that underwork can contribute to academic staff engaging in unethical practices out of boredom or frustration	3.08	1.257
WUL2: Underwork can lead to a lack of fulfillment, which may increase the likelihood of academic staff engaging in unethical practices	3.06	1.256
WUL3: I sometimes feel compelled to engage in unethical practices due to a lack of challenging or meaningful work	2.63	1.356
WUL4: I sometimes engage in unethical practices to fill the time when faced with underwork	2.47	1.319
WUL5: I lack sufficient time and resources to meet my job requirements, which puts me in a position where I may compromise my ethical standards.	2.86	1.314
WUL6: The workload in my academic position is overwhelming, making it challenging for me to meet my responsibilities effectively	2.92	1.287
WUL7: Academic staff at my institution lack access to resources and support to handle their workload effectively	3.18	1.231
WUL8: The workload distribution among academic staff is unfair and inequitable, to the extent that it can adversely affect ethical observation in discharging duties.	3.05	1.324
WUL9: Workload pressures sometimes tempt one to compromise ethical standards to meet job demands	2.98	1.385
Weighted mean & standard deviation	2.91	1.303

4.3 Inferential Statistics

This section presents the findings of the study, focusing on the relationships between workload (overload and underload) and unethical practices (plagiarism, fabrication, and falsification) among academic staff in Tanzanian public universities. The analysis includes Cronbach's alpha values for internal consistency, multivariate analysis of variance

(MANOVA) results, and multivariate linear regression models. The findings provide a comprehensive overview of how varying levels of workload impact the prevalence of unethical behaviors in an academic setting.

4.3.1 Internal Consistency Test

Cronbach's alpha is a widely used statistic to measure the internal consistency or reliability of a set of scale or test items. Higher values indicate greater internal consistency of the items in the scale. A commonly accepted threshold is that a Cronbach's alpha of 0.70 or above indicates acceptable reliability (Tavakol & Dennick, 2011).

Table 4

Cronbach Alpha Values for the Variables

Variable	Cronbach's Alpha	No. of items
Work overload	.822	5
Work underload	.932	9
Unethical Practices	.795	3

A Cronbach's alpha value of 0.822 for the work overload variable indicates good internal consistency among the five items used to measure this construct. This suggests that the items are well-correlated and reliably measure the concept of over workload. Values between 0.80 and 0.89 are typically considered good which supports the reliability of this scale (Boß *et al.,* 2016).

The work underload variable has a Cronbach's alpha value of 0.932, which signifies excellent internal consistency. This high alpha value indicates that the nine items used to measure work underload are highly correlated and provide a reliable measurement of the construct. For this case, values above 0.90 are considered excellent (George & Mallery, 2016).

For the unethical practice variable, Cronbach's alpha value is 0.795, indicating acceptable internal consistency among the three items used to measure unethical practices (fabrication, falsification, and plagiarism). This value falls within the range of acceptable reliability, suggesting that the items are reasonably correlated and effectively measure the intended construct (Mackenzie *et al.,* 2011).

4.3.2 Multivariate Linear Regression Model

The Multivariate Analysis of Variance (MANOVA) tests whether there are any statistically significant differences between the means of multiple dependent variables (D1, D2, D3) across levels of the independent variables (Work Over Load [WOL] and Work Under Load [WUL]). In this analysis, the dependent variables are plagiarism (D1), fabrication (D2), and falsification (D3), and the independent variables are WOL) and WUL. Wilks' lambda, Lawley-Hotelling trace, Pillai's trace, and Roy's largest root tests provide a slightly different way of testing the multivariate effect, but all are generally consistent in construal. Table 2 shows the Multivariate Analysis of Variance (MANOVA) generated from the STATA 17 whereby the statistical tests consider two workload conditions including overload (excessive) and underload (insufficient).

Table 5

Multivariate Analysis of Variance

Source	Statistic	Df	F(df1, df2) =		F	Prob>F
Model	W 0.8431	2	6.0	408.0	6.06	0.0000 e
	P 0.1615		6.0	410.0	6.00	0.0000 a
	L 0.1806		6.0	406.0	6.11	0.0000 a
	R 0.1419		3.0	205.0	9.70	0.0000 u
Residual	206					
WOL	W 0.9464	1	3.0	204.0	3.85	0.0104 e
	P 0.0536		3.0	204.0	3.85	0.0104 e
	L 0.0566		3.0	204.0	3.85	0.0104 e
	R 0.0566		3.0	204.0	3.85	0.0104 e
WUL	W 0.8876	1	3.0	204.0	8.61	0.0000 e
	P 0.1124		3.0	204.0	8.61	0.0000 e
	L 0.1266		3.0	204.0	8.61	0.0000 e
	R 0.1266		3.0	204.0	8.61	0.0000 e
Residual	206					
Total	208					

e = exact, a = approximate, u = upper bound on F

The overall MANOVA test shows significant multivariate effects for the model:

Results for Wilks' Lambda (W: 0.8431, F(6, 408) = 6.06, p < 0.0001), Pillai's Trace (P: 0.1615, F(6, 410) = 6.00, p < 0.0001, Lawley-Hotelling Trace (L: 0.1806, F(6, 406) = 6.11, p < 0.0001) and Roy's Largest Root (R: 0.1419, F(3, 205) = 9.70, p < 0.0001) indicate that there are significant differences in the combined dependent variables (D1, D2, D3) based on the levels of the independent variables (WOL and WUL). Since the p-values are less than 0.05, we reject the null hypothesis, which states that the population means of the dependent variables are equal across the independent variable levels.

Besides, the results for work overload indicate that this variable significantly affects the combined dependent variables. The p-values for all statistics are less than 0.05, leading us to reject the null hypothesis for work overload. This suggests that work overload significantly impacts the levels of fabrication, falsification, and plagiarism. Meanwhile, the results for work underload also indicate a significant effect on the dependent variables. With p-values less than 0.001 for all test statistics, we reject the null hypothesis for work underload. This suggests that work underload significantly affects fabrication, falsification, and plagiarism.

After performing a (MANOVA) to assess the overall effect of workload on unethical practices, it is essential to understand the specific relationships between the independent variables (work overload and work underload) and each of the dependent variables (fabrication, falsification, and plagiarism). To achieve this, researchers fit a multivariate linear regression model for each dependent variable. The multivariate linear regression model provides detailed insights into how changes in workload variables predict each unethical practice. The coefficients of the model will indicate the direction and magnitude of the relationship between workload factors and unethical behaviors.

Table 6 below presents the coefficients from the multivariate linear regression analysis, showing the relationship between the independent variables (work overload and work underload) and each of the dependent variables (fabrication, falsification, and plagiarism).

Table 6

Multivariate Linear Regression Model

Equation	**Obs**	**Parms**	**RMSE**	**F**	**P**	
Plagiarism	209	3	0.1079	12.45887	0.0000	
Fabrication	209	3	0.1106	12.81439	0.0000	
Falsification	209	3	0.0598	6.550774	0.0017	
UP	**Coef.**	**Std. Err.**	t	**P>|t|**	**[95% Conf. Interval]**	
Plagiarism						
WOL	-.0977754	.1134937	-0.86	0.030	-.3215336	.1259828
WUL	.3961752	.0801322	4.94	0.000	.2381908	.5541597
_cons	2.833428	.4996942	5.67	0.000	1.848257	3.818598
Fabrication						
WOL	-.2404175	.1156874	-2.08	0.039	-.4685006	-.0123345
WUL	.3827755	.081681	4.69	0.000	.2217376	.5438135
_cons	3.518957	.5093524	6.91	0.000	2.514745	4.523169
Falsification						
WOL	-.54803	.2189568	-2.50	0.013	-.9797136	-.1163465
WUL	.4175298	.1545944	2.70	0.007	.1127398	.7223199
_cons	4.907567	.9640306	5.09	0.000	3.006936	6.808199

Source: Research data generated by STATA 17

4.3.3 Model Fit

The results of the multivariate linear regression models indicate that both WOL and WUL significantly influence the dependent variables of plagiarism, fabrication, and falsification.

The plagiarism model shows that the overall model is statistically significant with an F-value of 12.45887 (df = 3, 205, p < 0.0001). This indicates that the independent variables (WOL and WUL) significantly predict the dependent variable (plagiarism). The Root Mean Square Error (RMSE) for this model is 0.1079, which represents the standard deviation of the residuals or prediction errors. This value suggests that the model's predictions are relatively close to the actual observed values, indicating a good fit (Black & Babin, 2019).

The fabrication model also shows statistical significance, with an F-value of 12.81439 (df = 3, 205, p < 0.0001). This result suggests that the independent variables significantly predict fabrication. The RMSE for this model is 0.1106, indicating the standard deviation of the residuals or prediction errors. This value reflects the model's accuracy in predicting fabrication, with lower RMSE values generally indicating a better fit (Tabachnick & Fidell, 2021).

For the falsification model, the F-value is 6.550774 (df = 3, 205, p = 0.0017), which demonstrates that the overall model significantly predicts falsification. The RMSE for this model is 0.0598, reflecting the standard deviation of the residuals or prediction errors. This relatively low RMSE indicates that the model provides a reasonably accurate prediction of falsification, confirming the model's effectiveness in capturing the relationship between the independent and dependent variables (Field, 2024).

4.3.4 Regression Coefficients

The regression coefficients provide insights into the specific impact of WOL and WUL on plagiarism, fabrication, and falsification.

The coefficient for WOL is -0.0978 with a p-value of 0.030, indicating that higher levels of overwork are associated with a slight decrease in plagiarism. Although statistically significant, this effect is relatively weak. In contrast, the coefficient for work WUL is 0.3962 with a p-value of 0.000, indicating a significant positive effect. This suggests that higher levels of work underload are strongly associated with increased plagiarism. The intercept (_cons) is 2.8334, indicating the baseline level of plagiarism when both WOL and WUL are zero (Kish-Gephart et al., 2010).

For fabrication, the coefficient for WOL is -0.2404 with a p-value of 0.039. This indicates that higher work overload is significantly associated with a decrease in fabrication. On the other hand, the coefficient for WUL is 0.3828 with a p-value of 0.000, showing a significant positive effect. Higher work underload levels are associated with increased fabrication. The intercept (_cons) is 3.5190, indicating the baseline level of fabrication when both WOL and WUL are zero (Kish-Gephart et al., 2010).

The coefficient for WOL in the falsification model is -0.5480 with a p-value of 0.013, indicating that higher work overload significantly reduces falsification. The coefficient for WUL is 0.4175 with a p-value of 0.007, showing a significant positive effect. This means that higher work underload is associated with increased falsification. The intercept (_cons) is 4.9076, indicating the baseline level of falsification when both WOL and WUL are zero (Osborne, 2017).

4.4 Discussions

The study results indicate that work overload has a significant negative effect on fabrication and falsification and a weak negative effect on plagiarism. Specifically, higher levels of overwork are associated with decreased instances of fabrication and falsification. This may be because overburdened staff are too occupied with their responsibilities to engage in these activities, or they may perceive the risks and consequences as too high (De Clercq et al., 2019). Literature showed that work overload could also lead to burnout, which is characterized by emotional exhaustion, depersonalization, and reduced personal accomplishment. Burnout might reduce the inclination to engage in time-consuming unethical practices such as fabrication and falsification (Brady et al., 2020; Rumschlag, 2017). However, the weak effect on plagiarism might be due to the relatively lower effort required to commit plagiarism compared to fabrication or falsification. In disparity, work underload shows a significant positive effect on all three unethical practices: plagiarism, fabrication, and falsification. This depicts higher levels of underwork are strongly associated with increased instances of these behaviors. This finding aligns with the literature suggesting that insufficient workload may lead to boredom and a lack of engagement, which can foster unethical behavior as individuals seek ways to fill their time or achieve undeserved recognition (Clemons, 2020; Schaufeli & Salanova, 2013; Schaufeli & Salanova, 2013; and Westgate & Wilson, 2018). The job demand-control model reflects that when there is low job demand then people may experience "boredom proneness" which suggests that individuals with a low workload may experience higher levels of boredom, which is associated with increased risk-taking and unethical behaviors when demonstrating an inability to have total behavior control (Dhal et al., 2022; Verwaeren & Nijstad, 2022). Moreover, work underload can lead to decreased job satisfaction and organizational commitment, further increasing the propensity for unethical conduct as a form of retaliation or as a means to seek excitement (Kish-Gephart et al., 2010). On the other hand, this is in line with the broader literature on the relationship between workload and unethical behavior. For instance, studies in organizational psychology have shown that both excessive and insufficient workload can lead to negative outcomes, although through different mechanisms (Grobelna, 2021). Work overload can lead to stress and burnout, reducing the likelihood of engaging in complex unethical behaviors due to lack of energy and motivation. Conversely, work underload can lead to boredom and a lack of engagement, increasing the likelihood of unethical behaviors as individuals seek to alleviate their boredom or artificially inflate their performance metrics.

V. CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

To guarantee that universities are providing high-quality education and high-quality research that observes ethical values, the workload distribution should be considered a major priority in ensuring that the activities these institutions perform fulfill national and international standards. Ensuring a balanced, equitable, and just distribution of workloads is crucial in mitigating misconduct that may result from excessive or insufficient workload, which could jeopardize university operations. Universities should ensure adequate training and support for academic staff to help mitigate the effects of workload imbalances. Training programs focused on time management, stress reduction, and ethical standards so that staff can possess the skills needed to handle their responsibilities effectively and ethically.

5.2 Recommendations

There is a need to develop and implement internal workload policies that link with academic integrity. These policies should include mechanisms for detecting and addressing unethical practices, as well as support systems for staff who may be struggling with workload-related issues.

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Examining Microfinance Loan Repayment Challenges and Procedures by Small Business Owners: The Case of Vikenge Village, Morogoro, Tanzania

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ABSTRACT

The research surveys the factors affecting the ability of small business owners in Vikenge village to repay microloans. This study used institutional theory framework to investigates the constraints encountered by these firms, their means for handling these constraints, and offer hints for improvement. Applying a qualitative research design, semi-structured interviews were performed with 13 small business proprietors. Participants were purposively selected due to their experience in microfinance lending and efficient reimbursement. Through thematic analysis, substantial issues concerning repayment challenges, techniques, and recommendations for enhancement were proven. The survey reveals a variety of obstacles that influence the repayment of loans, for instance; unexpected events, personal and familial matters, fluctuations in the economy, seasonal fluctuations in business, and complications with cash flow. Methodologies for repayment include strategies like saving money daily, options for flexible repayment, clear communication, and seeking outside financial assistance. Surveyed respondents also offered supportive recommendations, such as using loans for their intended purposes, decreasing interest rates, allowing more flexibility in repayment schedules, providing enhanced customer education and support, ensuring loan amounts are appropriate, and improving monitoring and collection procedures. This research survey enhances value to the current body of knowledge by delivering detailed analysis on the repayment of microfinance loans in small businesses in Vikenge village. The results proposed practical recommendations for microfinance institutions to acquire tailored repayment strategies that meet the varied requirements of borrowers, eventually leading to improved financial accessibility and long-term viability.

Keywords: Repayment Procedures, Repayment Challenges, Rates of Interest, Borrower Perspectives, Collection Processes

I. INTRODUCTION

It has been documented that microfinance is a potent tool in poverty reduction and global economic development in countries where access to traditional banking services is limited. In these developing economies, microfinance institutions (MFIs) provide small enterprises with access to credit. However, despite the clear impacts of microfinance, the problem of ensuring the repayment of loans has been a huge issue. Many such studies have been made in this regard, which shed light on the interest rates, loan terms, borrower profiles, and economic conditions that propel trends in loan repayment (Kama, 2021; Islam, 2022; Masood et al. 2023).

An extensive survey of microfinance loans has been conducted on a worldwide scale, which has highlighted issues with the repayment of microfinance loans (Masood et al. 2023; Kurniawan et al. 2022; Islam 2022). A more recent study by Cruz et al. (2023) found that higher interest rates and shorter loan terms impact repayment rates most severely. High interest rates mean that borrowing is more expensive, which, in turn, has the potential to restrict borrowers from earning the required income to repay their loans. Moreover, since the duration of these loans is short, there is a demand to repay in quick time, posing obstacles for businesses which experience different cash flow. Similarly, Endris (2022) reveals the impacts of borrowers' discipline and acumen in their financial matters on timely loans repayment. Those who are in need of financial knowledge and managerial skills are more likely to abuse the loan funds and have problems in making timely payments. Financial training and education efforts have resulted in successful efforts to enhance borrowers' ability to manage their finances and repay their loans on time.

To this end, literature also identifies the effect of external influencing factors like economic recessions and changes in the business environment to borrowers upon the effect of repayments. For example, economic instability affords borrowers a more difficult situation in accumulating enough income to meet their obligation. For example, if there is a downturn, this would result in a fall in consumer demand which would lead to declines in sales and revenue for small businesses. Moreover, seasonal variation in the size of business operations (such as a fluctuation in sales

because of the seasonal nature) may also prevent a borrower from paying back loans on a regular basis (Alperovych et al. 2022; Mahadevan et al. 2022).

According to Islam (2022), the users of microfinance services are usually vulnerable, because they are unable to escape from their social economic situations. A dearth of resources, the limited structural prevalence of sufficient social support systems and susceptibility to many other risks including health concerns and environmental catastrophes among others can aggravate payment issues. As a result, borrowers sometimes choose to cater to their short term survival needs ahead of loan repayments; hence defaults.

MFIs are very significant in determining repayment behavior. According to Yusuf et al. (2021) research, that shows that the policies and processes of the MFIs, including loan monitoring, flexibility in repayment terms and supportive services, contribute to the sustainability of loan repayments. In proposing flexible repayment options and grace periods, Cruz et al. (2023) allow borrowers the ability to manage their finances to avoid default.

Academic discourse has extensively discussed group lending models as a strategy to increase repayment rates (Protter and Quintos, 2020). Microfinance institutions extend loans to groups rather than individuals and can reap on peer pressure and mutual assistance among group members to inspire promotion of prompt repayment. However, this approach has revealed great success in societies where social ties are strong and people want to aid each other in periods of economic hardship (López-Sánchez et al., 2022).

Research at the global level helps paint the bigger picture, but it also needs to reflect specifically the conditions in each country and region. In Tanzania, small businesses experience a number of operational and financial challenges — including varied levels of business and cash flow, lack of financial literacy and access to support services. Tanzanian small business proprietors who operate in the informal sector, and who experience income curtailing varies, find it difficult to adhere to commercial banks' stringency repayment terms (Mwankemwa, 2022).

1.1 Statement of the Problem

In recent years, a number of individuals in Tanzania, including small business owners and private citizens, have found themselves trapped in cycles of debt after borrowing from financial institutions that impose stringent terms and high interest rates. These loans exhaust their saved funds, commonly known as "mikopo umiza au kausha damu" (loans that hit hard or drain blood). Many of those affected are individuals who obtain loans from smaller financial service providers rather than larger institutions like commercial banks with more extensive service offerings. Additionally, there are individuals who secure these loans through agreements with unregistered private lenders (Kihwelo, 2023; Banks et al., 2019).

To address this issue, the government, through the Bank of Tanzania (BOT), enacted the Microfinance Act, 2018 and established the Microfinance Regulatory Authority (MFRA) to oversee the microfinance sector and ensure that MFIs and Community-Based Organizations (CBOs) operate in compliance with regulations. This law prohibits conducting financial business and charging interest without a license. It also sets a maximum monthly interest rate of 3.5%, equivalent to 42% per year. Despite the presence of this legislation, many borrowers have continued to borrow from informal and unauthorized lenders. Consequently, they continue to struggle with hefty debt repayment, leading to the seizure of their assets, such as houses, land, furniture, and other properties. What many are unaware of is that some of these hardships could have been avoided if they had been more cautious before entering into agreements with these loan providers.

Despite an important role played by MFIs, many small business owners continue to encounter significant obstacles in loan repayment, threatening borrowers' financial stability and survival of MFIs.

1.2 Research Objectives

(i) To study challenges in the repaying microfinance loans by small business owners
(ii) To examine repayment processes in microfinance institutions
(iii) To gather opinion from business owners concerning ways of improving microfinance loan repayment processes

II. LITERATURE REVIEW

2.1 Theoretical Review

Utilizing the institutional theory, this study investigates microfinance loan repayment challenges and processes of small business owners. Further, the theory explains factors that shape, facilitate and channel the influence of institutional environment on social decisions (Bamfo et al. 2023; Atiase et al. 2023). Loan repayment challenges are in part due to several factors, e.g., institutional procedures, external shocks, borrower features. Bamfo et al. (2023) explains that repayment behavior is a distinctive outcome affected by policies as well as practices instituted by institutions. For instance, inflexible repayment schedules might lead to higher default rates (Sangwan, 2023). In the

case of an MFI, the repayment process is central to the success. Institutional theory stresses the importance for the MFIs to have matched their processes with institutional norms and practices for their success (Millan et al., 2023). In theory, by improving institutional processes, the sustainability of MFIs is given. As an institutional strategy, provision of financial literacy programs are some of the ways which can help improve repayment outcomes that address structural and behavioral challenges faced by borrowers (Sangwan, 2023; Bamfo et al., 2023).

2.2 Empirical Review

A number of studies have been carried out on loan repayment in microfinance (López et al. 2022; Rahim et al. 2024; Ahlin and Debrah, 2023). However the existing research on the topic is replete with evidence of higher default rates without paying attention to the challenges that borrowers face in serving the loans. These reasons are of prime importance as they have immediate impact on both borrowers' financial stability and MFI viability.

The literature offers considerable empirical evidence that the borrower's profile plays a role in explaining repayment difficulty. Management skills and financial literacy, according to Rahim et al. (2024) even have significant impacts on the performance in the loan repayment. The researchers discovered that borrowers with substandard business management skills and little financial awareness are more prone to having trouble getting their loans repaid. But poor financial literacy makes it difficult to budget out loan repayments.

Mwambuli et al. (2023) show that institutional factors could lead to poor loan management practices that cause loan repayment behaviors to be affected. The authors contend that these poor loan management practices are due to social economic factors such as regulatory frameworks. In agreement with Scott (1995), he says that in most developing economies, employing actors with weak regulatory frameworks and attitudes can be healthy when compared to developed markets. Also, slow institutional development of e.g. banks and other financial institutions could explain loan repayment practices in developing countries. For example, based on the work by Surmanidze et. al., (2023), banking sector has a crucial role in financing growth of small business as well as in financial awareness. This is crucial, the study indicated, because the people with a reliable financial support system tend to effectively manage their repayment obligation whether in good times or bad times.

Looking from the lenders side, Msomi (2024) points out that there are several factors within MFIs that influence loan repayment. The study found that effective borrower support programs, including regular monitoring and financial education are essential for successful loan repayment. Similarly, Dorfleitner et al. (2022) analyzed the impact of institutional policies on loan repayment. They found that policies that promote timely repayments demonstrate empathy towards borrowers' repayment difficulties. Other institutional factors frequently reported in the literature to influence loan repayment rates include are high collection frequency, the extent of control tightness, incentives for loan officer, financing costs, and the timing of credit dispensing (Doering & Wry, 2022). Repayment processes within microfinance play a critical role in ensuring the long-term viability of MFIs and the financial well-being of borrowers. Efficient repayment processes are crucial for achieving high repayment rates, while poorly structured processes can lead to loan defaults and financial instability for both MFIs and borrowers.

The influence of unforeseen events, such as public health crises, environmental disasters, and economic downturns, on borrower's ability to repay a loan has been widely documented in the literature. Collier et al. (2024) used data from the U.S. Census Bureau and Business Credit reports that firms that applied for a government-provided recovery loan following a natural disaster. The study found that for borrowers with limited social safety nets, such events can negatively affect household finances, therefore causing borrowers to focus on immediate survival needs and thus delay in loan repayment (see also Lesego, 2020).

Group lending models designed to expand access to microcredit programs and mitigate risk through shared responsibility, may also encounter repayment challenges due to group dynamics (Kodongo & Kendi, 2013; Muchnick & Kollamparambil, 2015). Ahlin and Debrah (2023) conducted a field experiment with group lending models in Ghana. The findings suggested that peer pressure and social collateral within group lending arrangements can influence repayment patterns both positively and negatively. A cohesive group can encourage timely repayments, but a default by one member can erode trust within the group lending leading to increased defaults. More specifically, Muchnick and Kollamparambil (2015) studied the impact of group homogeneity on repayment performance. Their findings indicated that large group sizes and mixed-gender groups (as opposed to groups with only females) were more likely have a negative impact, while more homogeneous groups showed better repayment performance.

Another strand of the literature has examined how behavioural and psychological traits of borrowers predict the credit default. Goel and Rastogi (2023) confirmed that certain non-financial factors can be assessed when granting a loan to a borrower. These factors can be used to develop a subjective credit scoring model that can quantify and verify the soft information, such as character and reliability of debtors. Friedetzky et. al., (2024) found that aligning repayment schedules with borrowers' unpredictable income patterns can significantly improve repayment rates.

Similarly, Sangwan (2023) noted that flexible repayment terms are beneficial as they accommodate income fluctuations, leading to better repayment outcomes, while rigid repayment schedules increase the risk of default.

The advancement of technology has also revolutionized the way MFIs disburse and manage loan repayment performance. Millan et. al., (2023) for example, explored the implications of mobile banking technologies on the loan performance of DT SACCOS in Mombasa. The study found that the incorporation of digital payment systems and mobile banking significantly improves repayment efficiency by lowering transaction costs and enabling borrowers to experience convenient payment plans. However, Rozzani et. al. (2015) who examined the implementation of technology in the disbursement and repayment of MFIs from the clients' perspectives, reported that while clients were fairly satisfied with the disbursement process through a mobile solution, the same conclusion could not be drawn for the repayment process. It can therefore be safely concluded that while technology is acknowledged to have a positive impact, its benefits in loan management may vary between different aspects of the loan process.

III. METHODOLOGY

3.1 Research Design

Research utilised a qualitative approach to ensure effective understanding of successful microloan repayments, challenges, and processes. Creswell (2009) said that this design enables the researcher to go further into the responder's experiences, point of view and ideas concerning the factors affecting loan repayment mechanisms. As for obtaining more detailed insights about the processes and issues related to stable micro-loan repayment, this survey employed a qualitative methodology as the research method.

3.2 Population and Sample Size

The study was carried out at Vikenge village in Mvomero District, Morogoro region, Tanzania. For the purpose of the study, purposefully selected thirteen (13) small business owners each having microfinance knowledge and (2) effective practices of loan repayment so as to have their thoughts on the distinct factors that make loan reimbursement a reality. This selection match with suggestion by Streubert and Carpenter (2003) that advocated that the researchers select cases that allow broader information for the in-depth study.

3.3 Data Collection Methods

Interviews were conducted with the use of a semi-structured interviews guide. This was to understand the loan repayment behaviour and the challenges that borrowers face. Open ended questions were used to allow researchers to gain a deep understanding of the elements contributing to borrowers' capacity to fulfill their financial obligations, as suggested by Creswell (2009).

Informed agreement before the interviews was mandatory in the participants. Interviews were conducted either in person or remotely (remote option being considered based on logistical considerations). To ensure accurate analysis and transcription, they were audio recorded. Interview sessions with participants took between 30 to 45 minutes, during which participants shared their perception and viewpoint of loan repayment practice. This methodology refers to Creswell (2009) in the design of interviews; hence participants can freely express their opinions which can be used for data collection.

3.4 Data Analysis

Every bit of information gathered was recorded to give exact data. As part of the assessment I read through the recordings severally in order to identify trends and insights. Open coding was used to categorise important data sections, while axial coding was used to create links among these sections. These categories were streamlined by selective coding to fundamental subjects that addressed research analyses. Evaluation, recognition and explanation of these topics were supported by thematic analysis, enabling the identification of meaningful patterns in the data. For in depth grasp of the result we interpreted them in light of the research questions as well as the other existing literature (Braun and Clarke, 2019). The method used to confirm data saturation, which ensures the collected data adequately answered the research inquiries and matters, was the methodology defined by Braun and Clarke (2019) for thematic analysis.

IV. FINDINGS & DISCUSSIONS

4.1 Challenges in the Repaying Microfinance Loans by Small Business Owners

As discussed above, the main aim of this research was to conduct studies about people's views on debt repayment problems. Results indicated that unpredictable circumstances, personal and family problems, market fluctuations, economic turbulence, market downturns, and financial restrictions were identified as key factors of disputes concerning loan repayment. Below we discuss the details for these aspects.

Unexpected events have a huge impact on borrowers' ability to meet repayment commitments. In a personal example, respondent R8 replied that suffering from illness put him in a situation where he couldn't meet his loan repayment plan, "I once suffer to pay my loan on time." I used the money I had saved, but I used it also to aid me paying for medical bills when I fell sick for the whole month. To repay the loan I had to borrow from a relative…." These results are in line with research which shows borrowers are susceptible to unforeseen events (Collier et al., 2024).

Facing problems with a repayment of a loan is directly related to the problems of individual and family nature. Respondent R2 echoed this concern of repaying money at times 'due to family issues." That is why the money that I saved to pay off the loan is what I used to solver family matters." "I also encountered difficulty in making my first payment due to a personal problem (health problem) thus requesting an extension to repay my loan from the microfinance institution", was also mentioned. These challenges in turn result in delays in payments, and associated penalties. This finding, in a way, confirms Islam (2022), which attributes loan repayment to personal factors. The research emphasizes that individuals availing of microfinance services are usually constrained by their socio economic situations.

In addition, the periodic nature of commercial activities constitutes a major obstacle. According to participant R13, agricultural companies are seasonal in nature since "Agricultural enterprises follow seasonal patterns, profits are realised after harvest, loan repayments are monthly." In addition, R3 said: "Payments in construction projects come with elongated payment horizons thus leading to delays of payment of loans." This is consistent with studies which identified problems in other sectors where borrowers are seasonal. According to the results, adding adaptability to the repayment conditions would ease the financial pressure bearing on borrowers and, in turn, improve the repayment performance (Sangwan, 2023).

Small business owners often struggle with cash flow problems. "I do have some challenges in paying back my loan on time; because some of the shop owners whom I do sell my products to, don't pay me on time," said respondent R4, highlighting cash flow-related difficulties. This is consistent with research by Sangwan (2023), who found that small business owners frequently deal with considerable cash flow fluctuation. Another respondent, R5, said, "The majority of the issues were brought on by the quickest payment schedule. There are weeks when sales are really less, which makes it challenging to have the money to repay the installment." This result is consistent with research by Friedetzky et al. (2024), who found that shorter repayment periods can raise the burden on borrowers, mainly in weeks of low sales. These results emphasize on the variety of loan repayment challenges that small business owners have, from personal and health difficulties to business and economic constraints.

4.2 Repayment Processes in Microfinance Institutions

This section analyzes the approaches used by small business owners in Vikenge village to repay microfinance loans and how these approaches impact their capability to fulfill their financial obligations. Research indicates that the loan repayment tactic involves a variety of challenges and approaches, like; daily savings, notifications adaptable payment options, and efficient communication, as well as the influence of external financial assistance. The findings for approaches used in the repayment processes are presented below:

Maintaining regular savings is crucial in making sure loans are repaid on schedule. "I have to save some money, for example 10,000 to 20,000 Tanzanian shillings (TZS), when I collect money from shop owners," responded Respondent R1. "I do this almost every time I get the money." They said, "It might come from my own selling. I can make TZS 25,000, for instance, on Sundays. I save 20,000 and use the remaining TZS 5,000 for other requirements." Respondent R4 also said, "I am saving TZS 10,000 every day because I am worried about defaulting." This habit of saving a percentage of daily income is consistent with the findings of Rahim et al. (2024), who found that disciplined saving is essential to effective loan repayment.

Another important consideration is the repayment process's flexibility. Respondent R7 revealed, "microfinance is flexible; if I want to, I can pay with any amount at any time." Two weeks before to the due date, I usually begin saving the repayment. The microfinance provides me extra time to pay if I am running late." Respondent R12 further stated, "The payment was quite flexible, you are allowed to pay the installment using mobile money, cash deposit, or bank deposit." This flexibility makes it easier for borrowers to better manage their repayments. These

findings align with the findings of Sikira et al. (2024) who found that rigid repayment plans and a lack of flexibility could increase the likelihood of default, especially for borrowers with inconsistent income streams.

Microfinance organizations have to provide reminders and proper communication. Respondent R2 was also recorded to have stated "… microfinance call me one day before the due date and I usually pay one day before the due date." "If I don't have enough cash to pay off at the time die want, I will lose credibility from my microfinance," they continued. The result coincides with the findings of Msomi's (2024) study, which highlighted that a consistent reminder increase borrower payback rate significantly.

Respondent R6 emphasized the significance of keeping daily revenues separate for various uses: "And I ensure I top up the phone (Tigo Pesa) everyday…a certain amount." "We've also listed my profit here… How much I've spent on foods and goods. I detach, totally!" This shows that doing money management in disciplined manner, as advised by Msomi (2024), when financiers engage the borrowers in financial education programs this will provide the borrowers with knowledge and skills to prioritise loan repayment, handle their money and understand loan conditions. This can be saving, budgeting or instruction on sound financial planning.

External funding is also used to pay loans. For instance, respondent R10 said, 'If I don't have enough to pay back at the right time I normally borrow from family and friends to repay loans every month.' I am an employee, R6 added, stating that I wield the power and are compelled to pay my monthly loan installments by deducting it from my monthly salary. This result is consistent with findings from Surmanidze et al. (2023) that external financial aid significantly contributes to loan repayment performance.

Higher interest rates and administrative problems were also mentioned. Respondent R3 said 'I have found even in my area (Morogoro) there are different microfinance institutions that dictate different and relatively high interest rates depending on factors such as capital investment, insurance coverage and turnover without that uniformity that is there in the traditional banks'. They also had problems during the application process, and say: 'I once had difficulties when applying for one of the MFIs because they asked for premises information (which I did not have), and business details I provided later enabled the loan to be granted to somebody, without a proper review of the customer profile.' The observation is in accordance with research done on the micro finance sector highlighting problems of high interest rates and administrative obstacles. For one, Cruz et al. (2023) discovered that interest rates and short repayment periods have an important relationship with repayment rates. Hence, increased interest rates make borrowing more expensive, which may stop the borrowers from earning enough income to pay the loan. Islam (2022) pointed out that loan repayment processes might be made more convenient for the savings of transaction costs as well.

Flexible repayment terms can work to ease borrowers' financial strain and allow for the ability to make payments on time, the survey's outcomes indicate. For example, pressure of not having sufficient money to pay back at the right time, helped by respondent R5 who said, "If I do not have enough money to pay back at the right time, I will lose my trust with the microfinance". As mentioned above, Sangwan's (2023) also has a similar finding that by providing flexible repayment alternatives, default risk is reduced, as well as maintaining trust between the borrowers and microfinance institutions.

4.3 Suggestions for Improving the Repayment Process

This section aims to present and critically discuss suggestions made by small business owners in Vikenge village on how to remedy microlending loan repayment problems. It recommended loans be used for its purpose, reducing interest rates, making repayments more flexible, customer education and support, making sure proper loan amounts are given and improving loan monitoring and collection processes.

The most common of these advices is to ensure that loans are used for their intended purpose. According to Participant R2, "The paramount concern is how those funds will actually be used (i.e. for business expenses)." "Individuals may misuse the loan for non-loan reasons resulting in their eventual failures." This brings to attention the critical requirement for borrowers to be financially prudent. This result is in line with Endris's (2022) research that shows that people with no financial literacy or managerial skills stand a high probability of misusing loan monies and failing to pay back loan arrears on time.

A big problem for people who need to borrow is when a high interest rate is involved. For our kind of small business R8, the interest rate is too high and ought to be lowered especially given our present financial performance, Respondent R8 added. Respondent R3 also offered: 'The interest should be reduced.' Microfinance, allowing repaying early, is great, but the interest rate is still a bit high." According to Cruz et al. (2023), imposition of high interest rates could be difficult for small businesses, as lowering of interest rates could enhance repayment rates and sustain long term business viability.

Respondents also suggested enhancing customer support and education. Respondent R13 advised, "Microfinance does not allow customers to pay back prematurely and should monitor its customers from time to time rather than depending on the call reminder. Educate customers on the importance of opening a bank account."

Education and financial literacy are crucial for effective loan management, supporting the findings of Msomi (2024) and Loreño and Teves (2023) who advocate for comprehensive borrower education and personalized support from loan officers can significantly enhanced repayment results.

Establishing backup accounts was another proposal. Respondent R12 suggested, "I recommend... that when a customer visits, the microfinance institution should establish a backup account for her... When you obtain a loan, starting at fifty thousand, designate ten thousand as a deposit to be held with the microfinance institution as your savings. If you borrow one hundred thousand, deposit twenty thousand, ... these are simply illustrations, to ensure that in case of default, the balance in your reserve account can provide assistance." This relates with the idea of compulsory savings explored by Rahim et al. (2024), which can serve as a safety mitigation for borrowers.

One of the sampled respondents, R3, states that there is an acknowledgment of ensuring loan amounts are suitable compared to the borrowers' income levels into an important factor. 'MFIs should make loans aligned to their borrower's income levels to mitigate default and non-performing loans,' they said. In addition, they emphasised the significance of rigorous follow up procedures which would 'involve MFI targeting rigorous follow up procedures including assessment of credit worthiness and regular visits to business premises.' As noted, proper loan sizing and monitoring is critical to the overall health of loan portfolios. The above finding is consistent with research conducted by Msomi (2024) which suggests that borrowers will be exposed to better borrowing services and as a result their understanding of loan terms, budgeting and late payments will be enhanced. These initiatives, by driving borrowers to make better financial choices, can help bring down default rates.

According to Respondent R5, taking into account the business prospects compared to collateral would bring dramatic impact since MFIs "should not only think of the customers with collateral but also think of their business prospects and plans." The recommendation conveys a view in line with Ahlin and Debrah (2023) that, if a company's potential is fairly assessed, non-collateral lending is possible.

Another recommendation was to extend the loan repayment period. Respondent R10 said, "the microfinance should increase the loan repayment duration for loans from TZS 500,000 to at least six months." They further added, "MFIs should also be more understanding when a customer is challenged with repayment; they shouldn't harass their customers." Borrowers may not feel as pressured by longer repayment terms. This result is consistent with that of Cruz et al. (2023), who found that short loan terms have a major influence on payback rates.

Respondent R12 appreciated the current repayment procedures while emphasizing the significance of installment payments. They noted, "The current repayment process is suitable as it permits repayment on an installment basis, facilitating on-time loan reimbursement." Installment-based repayments have been identified to enhance repayment responsibility and borrower ease, as acknowledged by Sikira (2024).

Respondent R1 explained the group lending approach, pointing out that "microfinance lent money to their members based on groups. The group members are the ones who follow up to ensure that the other member pays if one of them forgets to make their installment." Similarly, Ahlin and Debrah (2023) found that group lending can enhance payback rates through peer pressure and mutual support. A strong group dynamic can encourage prompt repayments, but when one member defaults, trust in the group is damaged, which can encourage other members to default as well.

Finally, Respondent R9 recommended the MFI to emulate bank-style top-up procedures and stressing the value of borrower education and collateral verification: "The MFI ought to think about employing a Top-Up method that banks utilize, as it facilitates repayment. When granting a loan, the MFI must take into account the need for collateral verification. The borrower's traits should be equally evaluated, according to MFI. Micro and medium-sized businesses in particular should receive financial management training from MFI." These recommendations are consistent with Msomi's (2024) finding, which emphasizes the significance of thorough evaluation and training for the implementation of effective microlending initiatives.

The suggestions for enhancing the repayment process for the small-scale businesses in Vikenge village comprise of the following: Checking the validity of the requirements of loans, reducing the interest rates, flexibility in the repayment terms, enhancing the customer awareness, creating emergency funds, ensuring that suitable loan amounts, tightening up the internal controls, and enhancing the collection procedure.

V. CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

When investigating the obstacles inhibiting small business owners of Vikenge village from repaying loans, it was clear that the pillars of these hurdles are very complex. Personal and family troubles, unexpected occurrences, economic downturns, cash flow issues, or fluctuations in business operations can all present serious challenges to preventing borrowers from completing repurchases on their schedule. Obstacles are presented that expose small

business owners to disruption from both internal and external origins that threaten their financial strength and ability to make loan payments.

Micro business owners rely on repayment schedules to stress the importance of tactics such as daily consistency in savings, the offer of flexible payment plans, and effective communication with MFIs. Finally, outside financial assistance furthers the connections of financial networks within communities. Today, challenges such as excessive interest rates and inefficient administrative processes still persist – stressing the need for MFIs to implement more borrower-friendly processes and guidelines.

The results emphasize the importance of understanding the overall landscape of repayment for small business owners. MFIs should recognize and monitor to the unique problems they face in order to sustain microfinance programs and yield better repayment results from the customers. Addressing these issues is necessary to improve the financial inclusion and economic resilience of small business owners.

5.2 Recommendations

Finally, MFIs should be encouraged to make available flexible repayment arrangements (such as roll over clauses, installment plans, moratorium, capitalisation, etc.) tailored to the cash flow patterns of small enterprises. They might, for instance, design seasonal loan repayment schedules for agricultural businesses, or forbear repayment when economic conditions tighten. To sustain repayments, frame accordingly, especially in periods of low economic activity, MFIs are suggested to consider reducing interest rates or adopt a tiered interest rate system based on the extent of risks of borrowers.

Enhanced financial education initiatives are essential. Thus, microfinance institutions must also offer clients comprehensive financial literacy and business administration courses that encompass issues like budgeting, creating and executing a saving plan, as well as efficient cash flow management. An emergency fund or a loan savings account proves to be a financial cushion for borrowers who face uncertain situations. Optional minor accumulations from borrowers may accrue into this fund and can be used to cover borrowings when the need arises.

By strengthening their communication techniques, MFIs can send reminders that can be easily understood and offer personalised support to borrowers. Regular monitoring and advisory services help borrowers handle their loan repayments effectively and resolve any financial challenges swiftly. The flexibility in utilizing funds availed through the loan can enable the borrowers to deal with urgent personal or familial needs without declaring themselves in default.

Strong, efficient monitoring and evaluation mechanism must be formed in order to adequately check a venture's performance and the behavior of borrowers in repaying loans. Regular assessments can help catch potential repayment problems early on, and the possibility of early intervention and support for borrowers.

Borrowers should make use of the loans for its intended purpose. This may be for company ventures rather than unplanned spending. It means that borrowed loan can earn profits or revenue which can in turn be utilized to repay loan. Policy makers must come up with policies that promote MFIs to offer flexible, loan repayment options that are tailored towards industries like agriculture that are characterized by income variability. Putting in place regulations which restrict the interest rates can protect small entrepreneurs from exorbitant borrowing expenses.

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Determinants and Extent of Tax Compliance among SMEs in Arusha City Council, Tanzania

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ABSTRACT

Taxes have been a nourishing commodity in generating revenues liable for financing governments' budgets all over the world. This being the fact, this study examined the determinants and extent of tax compliance in SMEs in Arusha City Council which was the specific objective of this study. This was a cross-sectional study which employed a double Hurdle regression model to analyze primary data collected from a sample of 100 SMEs' owners obtained by simple random sampling to represent the entire population of SMEs owners via questionnaire and analyzed with STATA software version 17. Using the economic deterrence theory to guide the analysis of this study, the analysis found that Male entrepreneurs dominate the SME sector, SME owners understand tax regulations and undergo frequent tax audits. Further, the findings revealed that privately owned SMEs, primarily ran by persons in their mid-30s are flexible with an average workforce of nine employees. Furthermore, SMEs were found to generate feasible profits and fulfill tax compulsions by paying an average tax rate. Moreover, the double hurdle regression model shown that age and profits have negative effect on tax payments, and higher household size of SMEs owners are associated with higher tax payments. Additionally, publicly owned enterprises positively comply with tax. Therefore, given these findings, this study recommends to policymakers to streamline tax procedures and develop a platform to discuss tax rates with tax payers before implementing the planned tax obligations.

Keywords: Double Hurdle Model, SMEs, Tax Compliance, Tax Payments, Tanzania

I. INTRODUCTION

Tax revenues have emerged as a critical component of tax policy in developing countries, enabling governments to bolster their financial reserves (Dudine & Jalles, 2018). Particularly in times of global economic sluggishness, such domestic resources are essential to promote sustainable development, diminish unemployment and poverty disparities, provide superior public commodities and services, and invest in capital infrastructure to expedite growth (Gangl & Torgler, 2020). By relying on tax revenues, developing countries can consolidate their fiscal positions and enhance their capacity to meet the evolving needs of their citizens (Matarirano et al., 2019).

In the world context development of the states depend on responsible and the successful strong tax collection systems (Giesi & Bishagazi, 2022). In developed economies such as that of Europe and America suggests that, the good plans between the government and the local business firms provides good force for the development of the economies which lead to attainment of the goals of financial as well as fulfilling residential requirements (Giesi & Bishagazi, 2022). Globally the small and medium businesses are the ones which drives economies which account about the 90 % of the business entities as well as employs more than 60% of the people (Gamage et al., 2020).

In Africa, taxation has been the important tool in rising revenue to support the growing public sectors development (Mashauri, 2021). Local business firms is one among the biggest contributors to the economies of developing countries through tax payments (Matarirano et al., 2019). Yet, the tax system in the majority of developing nations, like Tanzania, are unable to generate enough revenue to finance their budgets, which in turn causes incurring debt and run deficits as a result of borrowing (Jalles, 2017).

Tanzania's primarily source of revenues comes from taxes, which generate a steady stream of income to fund projects like infrastructure improvements, health care, water services and education (Omary & Pastory, 2022). The Tanzanian government imposes taxes with the goal of maintaining the social welfare of its residents through the country's economic development, as well as to effect consumption, production, and distribution (Sifuni, 2017). In fulfilling the revenue function, a well-designed tax system should be efficient in minimizing the distortionary impact on resource allocation to different groups in the society (Mashauri, 2021).

Tanzania Revenue Authority (TRA) is responsible for collecting the taxes from various sources including local businesses (Kimata & Churk, 2020). According to URT (2022), business firms with annual turnover above 40

million TZS are registered for Value Added Tax (VAT) purpose while those with annual turnover of bellow 40 million are also subjected to taxation (presumptive approach of taxation) system. The TRA is responsible for setting out tax systems and policies that ensure that local businesses comply to tax payments (Sifuni, 2017). Small and medium business firms in Tanzania evade taxes heavily especially the business done through cash transactions (Dudine & Jalles, 2018). This is the result of corrupt systems, poor management, social and economic reasons (Marandu et al., 2015). Introduction of the advanced technologies by TRA to capture the local business transactions has yet failed to ensure all revenues are collected (Aladejebi, 2018). Customers fail to demand for the receipt after transactions as well as the business owners fail to provide them. This led to low tax collection and problems in financing budgets (Gangl & Torgler, 2020).

1.1 Statement of the Problem

It is undeniable that taxes are the primary source of income for developing nations (Jalles, 2017). Developing countries like Tanzania faces a major problem of low tax revenues from business firms. Keraro (2017)claims that the main factor contributing to tax authorities' inability to collect enough revenue from businesses in developing nations is failure to comply with tax laws. Wadesango et al. (2020) pinpointed that difficult filling procedure, high rate of taxes, lack of awareness and multiple taxation are some of the reasons for local firms to fail to comply with taxes.

Various initiatives have been put forward to ensure that small and medium business firms pay their taxes. One of the methods was introduction of the Electronic Fiscal Devices (EFD) (Giesi & Bishagazi, 2022). This shows the advancement in tax collection but yet local businesses fail to comply with the tax rules and also the local authorities fail to track non-tax payers (Gangl & Torgler, 2020). Also, other firms do not use those machines at all while other firms created a strong opposition on the use of tax machines (Kyule & Wang, 2024).

Most studies have examined the factors to why local firms fail to comply with tax. Some of these studies include but not limited to Giesi and Bishagazi (2022); Kimata and Churk (2020); Keraro (2017); Matarirano et al. (2019) and Okpeyo et al. (2019)). However, the scholarly eye has been so narrow to further examine the extent of tax payments after tax compliance. This signifies that there is a gap in literature concerning the factors affecting the extent of tax payments after tax compliance. Therefore, this study examined the determinants and extent of SMEs in Arusha to Pay taxes to local authorities which is significant to responsible authorities to restructure tax system and payment procedures.

1.2 Specific Objective

This study aimed to specifically study the determinants and extent of tax compliance among SMEs in Arusha City Council, Tanzania.

1.3 Research Hypotheses

Ho: SMEs in Arusha City Council are not complying with tax
H_1: SMEs in Arusha City Council are complying with tax.

II. LITERATURE REVIEW

2.1 Theoretical Foundations

This study is based on the Economic Deterrence Theory, developed by Allingham and Sandmo in 1972 (Allingham & Sandmo, 1972). The theory was developed specifically to give an understanding on the tax payers' compliance behavior. The theory assumes that the tax payer is economically rational (Kimata & Churk, 2020) whereby the choice to pay taxes is influenced by several factors. First and foremost, the decision of complying with the tax rules involves some levels of risk, therefore the higher the risk the more the chances of not complying with the tax rules. The tax payer has to analyze the costs and benefits of compliance action. Secondly, tax payers need to be well equipped with knowledge concerning the taxation regime and system (Hasseldine & Bebbington, 1991).

Rate of taxation as well as the possibility of being detected after failure to comply with the tax rules is among the aspects that influence tax compliance (Omary & Pastory, 2022). The theory implies that, tax evasion tends to increase when audit chances are minimal and fines are severe, but it tends to decrease if audit possibilities are high and punishments are severe (Kimata & Churk, 2020). Therefore, this means that the presence of strictly laws, penalties and high rate of auditing reduce the chances of people to fail to comply with tax rules (tax evasion).

The theory also in other point of view includes the individual factors of the tax payer. This means that the process of making rational decision involves thinking and making choices based on the tax payers characteristics (Omary & Pastory, 2022). The theory of economic deterrence is applicable to this study because it assumes the individual tax payer as a rational human being who can make choices of whether to comply with tax rules or not basing on the assessment of the cost and benefits of such act (Hasseldine & Bebbington, 1991). This study is guided

by the objective of examining the key main factors influencing the tax compliance among small and medium businesses therefore from that point of view and based on the literature applying this theory to this study is suitable as it provides the socioeconomic factors that affect tax payers' compliance. The factors are tax rates, enough knowledge of the tax system, chances of audits and compliance costs.

2.2 Empirical Review

Nguyen et al. (2020) conducted a study to examine the key factors that influence firms in complying with tax rules and regulations. Both qualitative and quantitative methods were used. Total of 200 respondents who were business owners provided the data for the analysis through interview. The study's findings indicate that the three variables which are audit likelihood, company image, and firm ownership, directly influence the willingness to pay taxes. The likelihood of an audit and the intensity of the penalties possess the greatest effects on tax compliance. As a result, the responsible authorities are recommended to increase frequency of tax inspection as well as appropriate handling of the tax matters.

Inasius (2019) conducted a study to investigate the factors that influence small and medium business to comply with tax rules. The study used the survey study design and data were collected through well-structured questionnaires from a total of 328 small businesses eligible to pay taxes. The study applied multiple regression model in making analysis and the following results were revealed that tax knowledge, referral group, probability of auditing and perception had the high influence of the tax compliance. Furthermore, the study suggests that policy makers and the responsible authorities should focus on improving the tax systems.

The study done by Marandu et al. (2015), on the factors influencing the tax compliance revealed that subjective, normative, and attitudinal control variables were generally effective determinants of tax compliance. The study used the data from 18 empirical published studies across the world. The study suggested that a moderate approach to tax enforcement that will also encourage voluntary compliance through changes in attitudes and norms is advocated for tax policy makers instead of solely using the traditional coercive means typically employed to force tax compliance.

According to the study conducted by Inasius (2015), on the tax compliance of small and medium businesses, a total of four variables of tax compliance were investigated which were rate of taxation, likelihood of auditing, referral group as well as education on the tax procedures. The results of 319 individual business firms that responded to the survey reveal that tax compliance is favorably connected with tax knowledge, referral networks, and audit likelihood while negatively associated with income rate of taxation. The influence of tax knowledge on individual tax compliance is greater, however.

Aladejebi (2018) investigated how well small and medium business owners in Nigeria adhered to tax laws. The Nigerian economy is largely dependent on small businesses. 250 people were included in the sample size for the study, and 223 Small business owners answered to the questionnaire that was sent out. The data gathered were examined using a quantitative approach. Results indicated that female small business owners are much more complying with tax laws than their compared to men. SME owners need to be educated on taxes. There are several taxes. Tax amnesty may boost tax compliance levels.

Mwangi (2014) aimed to pinpoint the factors determining tax compliance in Kenya. A descriptive survey was used in the study. A sample size of 150 was chosen. Self-administered questionnaires and an interviewing guide were used to gather the data. The study's findings showed that the majority of respondents said that Kenya's high tax rates and variety of tax heads hindered small and medium businesses' degree of compliance with laws and regulations. A significant portion of non-compliance was caused by the lack of easily accessible information regarding tax matters. Another instance of non-compliance, the taxpayers' failure to accurately compute the taxes due was a result of the information gap. The SMEs believe that more information would substantially encourage them to be compliant if it were made available to them in the form of tax seminars and books.

In Tanzania, and more especially the Ilala municipality, Omary and Pastory (2022) did a study on the factors influencing small and medium enterprises' tax compliance. The current study employed a descriptive cross-sectional survey design and a quantitative research strategy. Total of 5324 Small and Medium Businesses from the Ilala Municipality made up the study's sample population, from which 98 respondents were chosen at random. Data was gathered via structured questionnaires, and the Multiple Linear Regression was utilized to evaluate it. The regression analysis shows that the willingness of taxpayers to comply with tax laws is influenced by taxpayer, economic, and institutional factors. As a result of the study Tax morality can aid in boosting the nation's rate of voluntarily filed taxes by ensuring that the general public has a favorable opinion of government expenditure. Taxpayers' bound to comply can be encouraged by sustaining equitable taxation, enforcing reasonable and reasonable levels of punishment, and minimizing tax rates. Fairness in the application of tax laws should be institutionalized, and strict controls should be put in place to prevent revenue fraud.

Kimata and Churk (2020) conducted a study to investigate the factors that influence wholesalers' compliance to tax laws in Dodoma. The study used cross-sectional study design and data were collected from 95 business owners through a well-structured questionnaire. Tax rates and actual income have statistical significance and influence on the tax compliance. Furthermore, the policy should also be implemented and modified to be more advantageous to the present economic situation in order to assure tax compliance. Also, the business owners should engage in giving out suggestions and ways so as to avoid failure to comply.

Elly (2015) investigated main factors for small and medium business tax compliance. The study used survey methodology and 158 respondents were involved for giving out data. Economic factors, such as audit frequency, compliance and non-compliance costs, punishments for non-compliance/severity of penalty, difficulty of the tax structure, possibility of being discovered, and tax rate levels, were found to have an impact on small and medium tax compliance.

III. METHODOLOGY

3.1 Research Design

In this study, a cross-sectional research design was applied. This was because the study focused on small and medium business owners at one point in time and data was collected once because of limited resources such as time. In addition, compared to other study designs like longitudinal research design, cross sectional research design was excellent for identifying the relationship between the variables and collecting data for a number of variables at once (Utouh, 2024).

3.2 The Area of Study

The study was conducted at Arusha City, specifically Sakina ward. After the division of Elelai and Ngarenaro wards, the new Sakina ward was formed, and it now has a population of 21,988 (11,572 women and 10,416 men). Arusha DC forms the northern boundary, Sombetini ward and Ostabay streets form the southern boundary, Ngarenaro ward forms the eastern boundary, and Eleraikwa ward forms the western boundary. The district covers an area of 267 km2 (103 square meters) and has an average elevation of 1,331 m (4,367 feet). The latitude of Arusha, Tanzania is -3.386925, and the longitude is 36.682995. Arusha, Tanzania. Arusha shares borders with Kajiado and Narok counties to the north, Kilimanjaro region to the east, Manyara and Singida regions to the south, and Mara and Simiyu regions to the west.

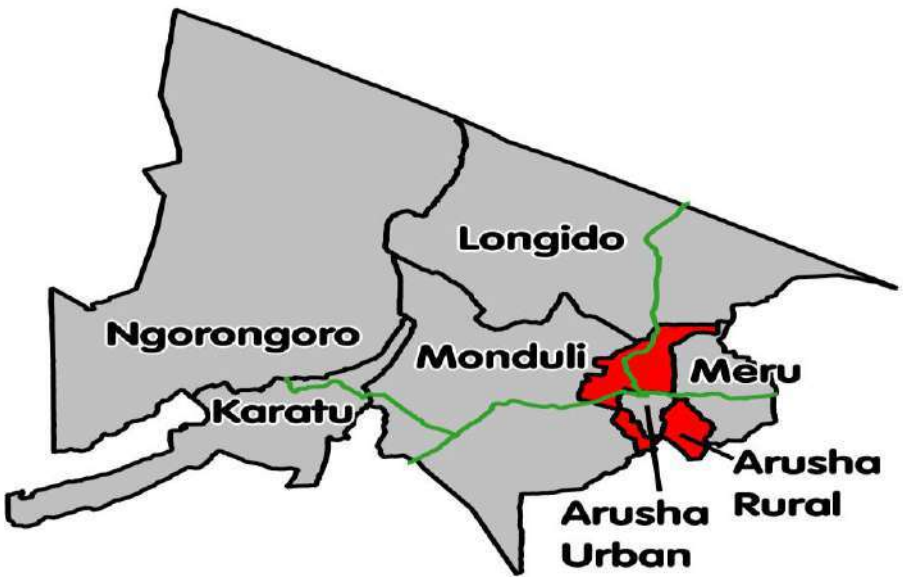

Figure 1
Map of Arusha Region
Source: URT (2023)

This study area was chosen because it has a significant number of SMEs accounting to 21,988 as reported by National Bureau of Statistics (NBS, 2023). Also, the location was easier to get to, closer to the researcher. Access to the majority of respondents was made possible by this, which provided the necessary data to meet the goals and objectives of the research.

3.3 Sample Size

The formula created by Yamane in 1967, which determines the sample size from the target population (Mazhar, 2021), was used to compute sample size for this investigation.

$$n = \frac{N}{1+N(e)^2}.$$

Whereby:

n = sample size
N = Targeted population is 21,988
e = sampling error percentage assumed at 10%

$$n = \frac{21{,}988}{1 + 21{,}988\ (0.1^2)}$$

$$n = \frac{21{,}988}{220.88}$$

$$n = 99.5944726548 \approx 100$$

Therefore, the data analysis for this study was based on information from a sample of 100 respondents.

3.4 Data Collection and Analysis Method

This study used questionnaire as the method of obtaining data from the small and medium firms concerning tax activities. This data collection strategy was chosen because it is the most effective in getting data for quantitative studies, particularly cross-sectional studies (Creswell, 2014). This study was quantitative in nature therefore descriptive analysis and econometrics analysis was used.

3.5 Empirical Model

3.5.1 The Double-Hurdle Model Regression Model

First Hurdle (SMEs Tax Compliance)

The probability that SMEs comply to tax was assumed to be determined by an underlying response variable that explains the water user's demographic, institutional and socio-economic characteristics (Tibamanya et al., 2022), thus can be illustrated as:

$$D_i^* = x_i'\beta + \varepsilon_i \quad \text{..... (3.1)}$$

Where D_i^* is a latent variable that shows whether SMEs comply to tax or not, β denotes the vector of unobserved served parameters to be estimated, x_i'denote the vector of observed independent covariates explaining the event, lastly ε_i denotes unobserved error term capturing other factors and was assumed to be independent and normally distributed (Tibamanya et al., 2022). That is μ_i

N~ (0, 1), and $D_i = 1\ if\ D_i^* > 0$

$$D_i = 1\ if\ D_i^* \leq 0$$

The variable D_i present the value of 1 if the SMEs comply to tax and the marginal utility over compliance is greater than non-compliance and zero (0) otherwise (Tibamanya et al., 2022). The binary variable of SMEs compliance or non-compliance D_i was assumed to be a probit model and is specified as:

$$Pr(D_i = 1/x_i)' = \phi(x\beta) + \varepsilon_i \quad \text{..... (3.2)}$$

Where Pr presents the probability of SME to comply: D_i is the binary variable of SMEs to comply: ϕ denotes the cumulative normal distribution: x is the vector of a SMEs demographic, socio-economic and institutional characteristics denote the coefficient to be estimated and ε_i denote the random error term distributed normally with zero mean and constant variance (Andrew et al., 2022).

The Second Hurdle (The Extent of Tax Compliance)

The extent to which SMEs comply to tax D^* is assumed to be truncated normal distribution with parameters to be different from the Probit model that can be estimated as follows (Tibamanya et al., 2022):

$$D^* = x_i'\alpha + \mu_i \quad \text{..... (3.3)}$$

Where D^* is the observed extent of tax compliance, x_i indicate the vector of covariates that explain the extent, α is a vector of unobserved parameters to be estimated and $\varepsilon_i$is a random variable that denotes all other factors apart from X. Since the assumption of independence of the two error terms, later, it was suggested that the hurdles can be estimated by the maximum likelihood method of Probit and truncated regressions (Tibamanya et al., 2022). The analysis of the model was performed by STATA software version 17.0 by considering the assumption that the two error terms are normally distributed and uncorrelated.

In this Model the estimation is as follows:

$$P(y = {}^{1}/_{x}) = \beta_0 + \beta_1 Ag + \beta_2 Gender + \beta_3 Mstt + \beta_4 SMEsize + \beta_5 Txrate + \beta_6 Knowledge + \beta_7 Audits + \beta_8 Ownership + \beta_9 Workers + \beta_{10} Profits + \beta_{11} Corruption + \beta_{12} Penalties + \beta_{13} Taxsystem + \beta_{14} Trust + \beta_{15} Hsize + \beta_{16} Educ + \mu \quad \text{..... (3.4)}$$

3.6 Variable Measurements and Descriptions

Table 1
Variable Measurement and Description

Variable	Measurement	Category	Expected Sign
Dependent variable			
Tax amount paid by SME	TZS	Continuous	
Independent variables			
Age of SME owner	Years of living SME owner	Continuous	+/-ve
Gender of SMEs owner	1 if a male 0 otherwise	Dummy	+/-ve
Marital status of SME owner	1 if married 0 otherwise	Dummy	+/-ve
Household size	Number of people in a household	Continuous	-ve
Education level of SME owner	1 if SME owner has primary education 0 otherwise	Dummy	+/- ve
SME capital	Amount of money invested in business	continuous	+/ve
Business ownership	1 if private owned 0 otherwise	Dummy	+/-ve
Possibility of audits	1 if there are chances of being audited 0 otherwise	Dummy	-ve
Tax rates	Amount of tax to business firm per month	Continuous	+/- ve
Number of workers	Number of workers	Continuous	+/-ve
SMEs profits	Amount of money in TZS of monthly profits earned by SME	Continuous	+ve
Corruption	1 if presence 0 otherwise	Dummy	-ve
Penalties	1if penalties are present 0 otherwise	Dummy	+ve
Government trust	1 if SMEs trust the government 0 otherwise	Dummy	+ve

Table 1 shows various factors influencing the tax amounts paid by SMEs. The dependent variable is the tax amount paid by SMEs, measured in Tanzanian Shillings (TZS) and categorized as continuous. Independent variables are the age of the SME owner (years of living, continuous, expected sign +/-ve), gender (1 if male, 0 otherwise, dummy, +/-ve), marital status (1 if married, 0 otherwise, dummy, +/-ve), household size (number of people, continuous, -ve), education level (1 if primary education, 0 otherwise, dummy, +/-ve), SME capital (amount invested, continuous, +ve), business ownership (1 if privately owned, 0 otherwise, dummy, +/-ve), possibility of audits (1 if chances of being audited, 0 otherwise, dummy, -ve), tax rates (amount per month, continuous, +/-ve), number of workers (continuous, -ve), SMEs profits (amount of monthly profits in TZS, continuous, +ve), corruption (1 if present, 0 otherwise, dummy, -ve), penalties (1 if present, 0 otherwise, dummy, +ve), and government trust (1 if SMEs trust the government, 0 otherwise, dummy, +ve). Each variable is categorized as either continuous or dummy, with expected signs indicating the hypothesized relationships.

IV. FINDINGS & DISCUSSIONS

4.1 Findings

Table 2

Descriptive Analysis for Discrete Variables

Variable (n = 100)	Frequency (percentage)
Sex	Male = 73(73%) Female = 27(27%)
Marital status	Married = 14 (14%) Widowed = 10 (10%) Separated = 20(20%) Single = 56 (56%)
Tax education	Knowledgeable = 68 (68%) Not knowledgeable = 32(32%)
Tax audits	No = 26(26%) Yes = 74(74%)
Business ownership	Private = 68(68%) Public = 32(32%)

The descriptive analysis of the sample (n=100) presented in Table 2 reveals that 73% of SMEs owners are male, and 27% are female. Regarding marital status, 14% are married, 10% are widowed, 20% are separated, and 56% are single. Concerning the receiving of tax education, 68% are knowledgeable meaning that they have acquired tax education, while 32% have not acquired tax education. When it comes to tax audits, 74% have undergone tax audits in the previous year, whereas 26% have not. Additionally, 68% of the SMEs are privately owned business while 32% are public owned enterprises. Descriptive analysis for continuous variables is shown in table 3

Table 3

Descriptive Analysis for Continuous Variables

Variable	Observation	Mean	Standard deviation	Minimum value	Maximum value
Age (years)	100	32.80189	12.46207	19	66
Number of workers	100	9	2.658337	1	17
SMEs profits (TZS)	100	512000	221540.1	100000	9,000,000
Tax rate (TZS)	100	50,000.41	6000.142	20,000	1,500,000

Table 3 presents a descriptive analysis of continuous variables from 100 SMEs, revealing that the average age of SMEs owners is approximately 32.8 years, with a standard deviation of 12.46 years, whose minimum and maximum years are ranging from 19 to 66 years. Businesses have an average of 9 workers, with a standard deviation of 2.66, whose minimum and maximum values are ranging from 1 to 17 workers. The average profit for SMEs is TZS 512,000, with a standard deviation of TZS 221,540.1, and profits range from TZS 100,000 to TZS 9,000,000 showing minimum and maximum profits respectively. The average tax rate is TZS 50,000.41, with a standard deviation of TZS 6,000.142, and tax rates vary between TZS 20,000 and TZS 1,500,000.

Table 4

Double Hurdle Results on the Determinants and Extent of Tax Compliance

TAX COMPLIENCE				TAX PAID			
Variable	**Coefficient**	**St. Error**	**p-value**	**Coefficient**	**St. Error**	**p-values**	**Marginal Effect**
Age	0.02271	0.014864	0.127	-0.049752	0.02732	0.069*	0.02088388
Sex	0.149643	0.357512	0.676	-0.2004895	0.780335	0.797	0.2478574
Household size	2.76E-02	9.55E-02	0.773	0.4651407	0.230071	0.043**	0.4285954
Marital	-0.22897	0.201859	0.257	4.88E-01	4.53E-01	0.282	-0.2205357
Edu_level	0.081661	0.168336	0.628	-0.0369065	0.40769	0.928	0.1839072
Income	8.99E-07	7.27E-07	0.216	4.02E-08	1.74E-06	0.982	2.37E-06
Profits	-0.24031	0.369951	0.516	-1.645067	0.777428	0.034**	-1.908007
Government trust	0.057672	0.319137	0.857	0.6878952	0.721645	0.34	0.6784657
Penalties	0.558526	0.391718	0.154	0.5220551	0.835888	0.532	1.852204

Corruption	-0.30304	0.200159	0.130	0.0580259	0.446855	0.897	-0.7430541
Tax rate	-1.00604	0.542444	0.064*	0.5736294	0.492982	0.245	-1.9203
Business ownership	0.717829	0.359448	0.046**	0.9196113	0.849255	0.279	2.653333
Workers	-0.01725	0.025958	0.506	-0.0369381	0.069231	0.594	-0.0731904

****p< 0.01, ** p<0.05, *p<0.1*

4.1.1 Age of SMES Owner

Age was negatively associated with tax paid, the relationship was statistically significant at 0.1 significance level (p = 0.069). The coefficient of -0.0497529 suggested that on average younger individuals on average have lower tax payments. However, in tier one age was found to do not have a statistically significant relationship with the tax compliance (p = 0.127). The coefficient of 0.0227099 indicates a positive association between age and the likelihood of tax compliance, although the effect size is small. The marginal effects of 0.02088388 indicated that older SMEs owners comply with tax by 0.02088388 more compared to younger SMEs owners. However, since the coefficient is not statistically significant, it suggests that age alone may not be a significant factor in determining the likelihood of compliance.

4.1.2 Household Size

In tier one, the variable Household size was found to be statistically insignificant relationship with tax compliance (p = 0.773). Household size shown a statistically significant positive relationship with tax paid (p = 0.043). The coefficient of 0.4651407 suggested that larger household sizes were associated with higher tax payments. Marginal effects indicated that more household size increases amount of tax paid.

4.1.3 SME Profits

Profits had no statistically significant relationship with the tax compliance. However, Profits had a statistically significant negative relationship with tax paid (p = 0.034). The coefficient of -1.645067 whose marginal effects of -1.908007 suggested that higher profits are associated with lower tax payments. This indicated that individuals or businesses with higher profits tend to have lower tax liabilities.

4.1.4 Tax Rates

Tax rate had a statistically significant negative relationship with the tax rates (p = 0.064). The coefficient of -1.00604 suggests that a higher tax rate was associated with a lower likelihood of paying tax. This implied that individuals or entities subjected to higher tax rates are less likely to pay tax. The marginal effects of -1.9203 signified that higher tax rates are accompanied with less likelihood to comply with tax.

4.1.5 Business Ownership

Business ownership was positive and statistically significant (p = 0.046) at 5% significance level. The coefficient of 0.7178294 suggests that a public owned SMEs was associated with a higher likelihood of tax compliance. A marginal effect of 2.653333 indicated that on average public owned SMEs comply with tax by 2.653333 higher compared to their counterparts.

4.2 Discussions

This study aimed to investigate the factors and extent of tax compliance among SMEs in Arusha City Council. The study findings revealed that the age of SME owners had a negative relationship with tax payments, which was statistically significant at a 0.1 significance level (p=0.069). Younger SMEs owners tended to have lower tax payments, but in tier one, age did not have a significant nexus with tax compliance (p=0.127). Furthermore, the SMEs owners' household size had no significant relationship with tax compliance in tier one (p=0.773), but it had a positive relationship with tax paid in tier two (p=0.043). SMEs owners with larger households had higher tax payments. SME profits did not have a statistically significant relationship with tax compliance, but they had a negative relationship with tax paid (p=0.034). Higher profits were associated with lower tax payments, indicating that entities with higher profits had lower tax liabilities. Tax rates had a negative relationship with tax compliance (p=0.064), where higher tax rates meant a lower likelihood of paying taxes. Lastly, business ownership had a positive and statistically significant relationship with tax compliance (p=0.046). Public-owned SMEs had a higher likelihood of tax compliance than their counterparts.

The findings of this study are in alignment with the findings of earlier studies conducted by Nguyen et al. (2020) in Vietnam and Mwangi (2014) in Kenya, both of which found that firm ownership and tax rates as pivotal determinants influencing tax compliance among SMEs. Specifically, these studies suggest that the public owned

SMEs and the level of tax rates to be paid play a significant part in determining whether SMEs adhere to tax regulations.

However, the current study's results go antagonistically from the findings of several other studies. For instance, Inasius (2019) found that gender, specifically if an owner is a female, plays a significant role in tax compliance among SMEs. Inasius (2019) added that this is due to the several social and economic gravities faced by female business owners, who rank compliance higher to avoid lawful concerns that could unreasonably impact them. Aladejebi (2018) emphasized the position of taxpayer ethics, suggesting that individual ethical standards and the alleged lawfulness of the tax system meaningfully impact compliance behavior. Omary and Pastory (2022) highlighted the protagonist of economic circumstances, remarking that SMEs are more likely to comply with tax responsibilities during epochs of economic steadiness when their returns are more foreseeable. Kimata and Churk (2020) explored the effectiveness of tax audits as a deterrent to non-compliance, suggesting that consistent and systematic audits increase the perceived risk of evasion.

Elly (2015) also emphasized the importance of tax education, fair implementation, and institutional factors, arguing that when SME owners are well-informed about tax laws and perceive the implementation process as fair and transparent, they are more likely to comply with tax regulations. Moreover, this study offers an exceptional contribution to the literature by classifying the aspects that impact tax payments among SMEs that are already compliant with tax obligations. The findings indicate that once SMEs have decided to comply, the amount of tax they pay is negatively triggered by the age of SME owner and profitability of the SMEs. This finding proposes that older businesses, possibly with more established financial streams and greater access to tax planning resources, find ways to lawfully decrease their tax burden. Similarly, more profitable SMEs participate in more erudite tax minimization plans, applying available deductions and credits more effectively to lower their overall tax responsibility.

On the other hand, the study finds that the household size of SME owners has a positive influence on tax payments. This result advocates that larger household sizes are allied with a superior sense of financial responsibility.

V. CONCLUSION & RECOMMENDATIONS

5.1 Conclusions

The study examined determinants and extent of tax compliance among SMEs in Arusha City Council using a sample of 100 SMEs. The analysis of small and medium enterprises (SMEs) in Arusha City Council has provided important details into tax compliance and the extent of tax payment. Male entrepreneurs dominate the SME sector. SME owners exhibit a strong understanding of tax regulations and undergo frequent audits associated with tax education, showing their pledge to compliance. Privately owned SMEs, chiefly ran by individuals in their mid-30s, and are flexible with an average workforce of nine employees. Notably, SMEs generate viable profits and fulfill tax obligations by paying an average tax rate. Age negatively links with tax payments, while household size shows a positive relationship. Although SME profits do not significantly impact compliance, higher profits are associated with lower tax payments. Higher tax rates are linked to a decreased likelihood of tax payment, and public ownership is positively correlated with compliance.

5.2. Recommendations

The analysis on SMEs in Arusha City Council reveals the need for policy interventions to provide tax education and support, simplify tax procedures, support record-keeping and financial management, offer targeted assistance to financially constrained SMEs, and foster collaboration between tax authorities and SMEs to improve tax compliance and payments. Policymakers should develop a conducive platform to discuss tax rates between tax payers and tax authorities. These measures aim to create a supportive environment, enhance tax compliance, and facilitate the growth of SMEs in Arusha City Council.

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Students' Subjective Well-Being in Cape Coast Metropolis, Ghana: The Influence of Sex, Parenting Style, and Self-Esteem

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ABSTRACT

This study sought to determine senior high school students' level of self-esteem (SE) and subjective well-being (SWB), establish the influence of their sex and parents' parenting style on their SWB, and determine the effect of their SE on their SWB. The cross-sectional survey research design was used. The study was guided by Life circumstances theory (LCT). Final year senior high school students formed the population. Data was collected from 347 senior high school students drawn from a population of 7,665 in Cape Coast with a questionnaire. The spread of the students and their dispositions led to the use of the proportionate simple random sampling technique to select 347 students from the 10 senior high schools in the metropolis. The instrument used to collect data for this study was a questionnaire. Descriptive (means and standard deviation) and inferential statistics like Independent samples t-test, Analysis of Variance (ANOVA), and PLS-SEM were utilised to analyse the responses. Results indicated students' SE and SWB levels were high. Further, there was no statistically significant difference in students' SWB with regard to their sex. There were statistically significant differences in students' SWB levels with regard to their parents' parenting styles. Results of the study further indicated that senior high school students' SE significantly and positively predicts their SWB. It is concluded that even though SE is not the sole determinant of SWB, senior high school students will enjoy their learning, feel part of and connected to their school, have a high purpose of learning, and feel highly efficacious academically (SWB). It was recommended that senior high school authorities in Ghana put in place policies and strategies that will sustain the high levels of SE and SWB. This is very important for the realisation of SDG goals 3 and 4, which are critical for the total development, and well-being of students.

Keywords: Parenting Style, Sex, Self-Esteem, Subjective Wellbeing, Students

I. INTRODUCTION

Well-being generally pertains to the condition of being content, healthy, or joyful. Wellbeing is seen as the amalgamation of a sense of worth and living up to expectations where people feel emotions like joy and satisfaction. On the other hand, students' well-being denotes the situation of total good feeling of the student (Adams et al., 2000). Research has shown that the exhibition of well-being is contentment, which is seen as the crucial objective of human survival. However, Allin and Hand (2017) posit that well-being is not only about hedonism and the hunt for pleasure worldwide, but it also includes living up to expectations (eudemonism). Joy brings about the decent living of individual contentment, which is a universal drive of the hedonic understanding of well-being (Kahneman et al, 1999; Ryan & Deci, 2001; Waterman, 1993).

Diener et al. (1999), point out that subjective well-being (SWB) has several aspects that mention individuals' well-being, subjectively assessed by their overall fulfilment with their lives, significant life territories, as well as their related passionate conditions. Diener (1984) defines SWB as a combination of two things: the cognitive component (life satisfaction) and the affective component (positive and negative affect). Ryan and Deci (2001) also conceptualised SWB to include hedonic well-being and eudaimonic well-being. Hedonic well-being denotes an individual looking for pleasure and avoiding troubling emotional endeavours. Eudaimonic well-being on the other hand relates to the endeavour of individuals to make sense of life and come to self-actualisation leading to discovery of the potential. Students' SWB has to do with an overall evaluation of how satisfactory students feel about themselves at a particular time within the school context. How well students feel about their studies (joy of learning), how well they feel connected to the school environment (school connectedness), how best they think they are

achieving their educational purpose, and how they feel they are performing academically (academic efficacy) give them that feeling of wellness. These four attributes (joy of learning, school connectedness, educational purpose, and academic efficacy) help to better explain and appreciate the level of students' SWB (Ryan & Deci, 2001).

First, the joy of learning pertains to the positive emotions and satisfaction experienced by individuals in the process of acquiring knowledge and skills (Ryan & Deci, 2000). Research has shown a substantial association between the joy of learning and students' SWB (Smith, 2018). According to Fredrickson (2001) and Pekrun et al. (2009), students who are joyful during the learning process are more likely to participate, feel satisfied, and have a sense of purpose. Second, school connectedness is a crucial component of students' SWB in education. It describes how much a student feels supported, involved, and personally engaged in the educational setting. It entails having a feeling of acceptance, having good relationships with students and teachers, and having a sense of being an essential member of the school community (Resnick et al., 1997; McNeely et al, 2002). Students who feel a strong sense of connection to their school environment are inclined to report higher levels of life fulfilment and overall well-being (Huebner, 1991; Resnick et al., 1997).

Another component of students' SWB is "educational purpose," which refers to the more expansive objectives and goals of the educational process. It includes the improvement of critical thinking and problem-solving skills, the gaining of knowledge and skills, and personal growth. It goes beyond the mere dissemination of knowledge to the development of well-rounded individuals who can make significant contributions to society (Dewey, 1897; Bruner, 1966). Students may experience a feeling of fulfilment when they believe that their education serves a meaningful purpose, which has a favourable effect on their SWB (Wang & Eccles, 2012).

Finally, academic efficacy denotes the confidence that one has in accomplishing academic obligations and reaching learning objectives. According to Bandura (2013) and Pajares (1997), it has to do with a student's self-assurance in their ability to comprehend and retain the content, perform well on tests, and live up to the standards required by the educational system. Students with a high degree of academic efficacy are more likely to experience positive feelings and a sense of achievement, which contributes to their well-being (Bong & Clark, 1999; Lane et al, 2004). These attributes are what constitute students' SWB. However, students' SWB is likely to be influenced by factors such as sex, parenting styles, and self-esteem (SE).

Primarily, the influence of sex on students' SWB has been a subject of increasing scholarly interest. Martínez-Marín and Martínez (2019) conducted a study on SWB and gender-typed qualities in teenagers and their study confirmed the sex gap in the SWB report (Senik & Clark, 2015), which states that at age 18, males exhibit more happiness than females. In their research, girls presented more negative emotions than boys. Lokeshwari and Monika (2021) also concluded that females have better subjective well-being as compared to males. Agormedah et al. (2024) found that there was no significant difference in SWB levels between younger male and female students. This inconclusiveness in the literature is addressed by this study.

In addition to sex, parenting style also influences students' SWB. Baumrind (1991) and Durbin et al. (1993) define parenting style as a set of routine activities, attitudes, and techniques affecting a child's warmth, responsiveness, and control levels. Baumrind (1968) classified parenting styles into authoritative, permissive, and authoritarian styles. Martin and Maccoby (1983) developed a different typology, which led to the development of four PSs by combining high and low demands and warmth. The authoritative parenting style, characterised by warmth, responsiveness, and control, is considered advantageous for positive child outcomes, while permissive, authoritarian, and neglectful parenting styles emphasise discipline and warmth (Martin & Colbert, 1997; Čudina-Obradović & Obradović, 2006). Studies have repeatedly shown that students who experience authoritative parenting have a higher SWB (Ryan & Deci, 2000). In contrast, there is a negative correlation between students' SWB and parenting approaches that prioritise strict discipline, a lack of warmth, or overbearing control, such as those adopted by neglectful parents (Baumrind, 1991; Durbin et al., 1993).

Besides, SE serves a pivotal role in peoples' SWB. Definitions of SE given by scholars show that SE can be seen as a great predictor of students' SWB. Blascovich et al. (1991) explain SE as an individual's sense of his or her value or worth. According to Crocker and Wolfe (2001), SE denotes a worldwide verdict of the worth or value of the self as a whole. Diener (1984) recognises global SE as a vital ingredient of the subjective value of life (positive affect and life satisfaction). Diener and Diener (1996) found that satisfaction in the lives of students is related to their SE. SE was also seen to be related to other aspects of SWB, like positive and negative affect (Robins et al, 2001), understanding life (Steger et al., 2006), and subjective energy (Ryan & Frederick, 1997). Du et al. (2017) found that both Personal Self-esteem (PSE) and Relational Self-esteem (RSE) were positive predictors of SWB but Collective Self-esteem (CSE) was weakly associated with SWB. Zhang (2005) discovered a weak correlation between CSE and SWB among Chinese youth and adults. In contrast, Bettencourt and Dorr (1997) also found a mediating role that CSE plays in the association between communalism and SWB among U.S. college students. Simsek's (2013) found an association between CSE and SWB and a mediating role of PSE in this relationship.

1.1 Statement of the Problem

SWB is a crucial constituent of a person's psychological growth, mostly during teenage years and young adulthood, when students practice significant life changes and formulate their self-identity. It plays a main part in how students steer several facets of life, including academic accomplishment, social affairs, and emotional regulation. The well-being of students in academic achievement has attracted attention in recent years (Huppert, 2009; Adams et al., 2000; Allin & Hand, 2017). In terms of dimensions related to academic performance, some research has focused on teacher characteristics, curriculum factors, home factors, and student characteristics. In the Ghanaian sphere, some researchers (Mensah & Owusu, 2022; Akaboha & Kwofie, 2016; Mpiani, 2012) have focused on student factors that may influence performance. These several studies did not look at the SWB aspect of students' characteristics. We must look at this aspect as reports from various senior high schools and WASSCE chief examiners paint a gloomy picture of the academic performance of students.

Literature again draws attention to SE as an overall subjective sense of personal worth or value, which is key to SWB. Research suggests a positive relationship between SE and SWB. Apart from SE, sex and parenting styles have been found to influence SWB levels in students (Martínez-Marín &Martínez, 2019; Lokeshwari & Monika, 2021; Martin & Colbertg, 1997; Čudina-Obradović & Obradović, 2006). Despite the established effect of SE on SWB and the specific influences of sex and parenting styles on SWB, the senior high school context in Ghana remains unexplored. There is a need for the exploration of how these variables interrelate and affect students' sense of well-being, which is a determinant of academic performance.

Over the years, studies on the influence of sex, PS, and SE on SWB have disclosed opposing findings. While some researchers have found a positive correlation between these variables, others have found a negative or no relationship. Sex differences have been found in SWB (Agormedah et al., 2024; Esteban-Gonzalo et al., 2020; Lokeshwari & Monika, 2021; Martínez-Marín & Martínez 2019; Senik & Clark, 2015) and also SWB on SE (Butt, 2009; Hill, 2015; Padhy et al., 2011). Again, parenting styles influence SWB, with authoritative and permissive parenting styles positively correlated with SWB (Pavićević, 2020; Pavićević & Zivkovic, 2021; Xie et al., 2016), but authoritarian parenting styles negatively correlated with SWB (Xie et al., 2016). Apriliya and Hastuti's (2023) findings contradict those of the others. It has been established that SE predicts SWB among students in universities (Betterncourt & Dorr, 1997; Butt, 2009; Du et al., 2017; Maluka, 2004; Misbach et al., 2023; Padhy et al., 2011; Hill, 2015; Tan et al., 2023; Zhang, 2005).

Most of the studies reviewed were conducted outside Africa, and it seems only one study has been conducted on the influence of SE on SWB in Africa. Agormedah et al. (2024) are the only scholars to have studied the effects of sex on the SWB of senior high school students in Northern Ghana. The current study is conducted in the Cape Coast metropolis. There is no study on the influence of parenting styles on SWB in Africa. In Ghana, it seems no study has been conducted on the influence of either parenting style or SE on SWB. In addition, previous studies did not combine these three (sex, parenting style, and SE) variables on SWB. This means that there is a knowledge gap in Africa and knowledge and geographical gaps in Ghana. These gaps make it relevant to explore these variables in the Ghanaian context to create awareness of how sex, parenting styles, and SE affect SWB. With this understanding, learning in a more inclusive setting, accommodating, and catering to the requirements of each student may be improved, and education can be elevated. These would lead to the attainment of Sustainable Development Goals 3 and 4 and the development of potential strategies for supporting students' psychological development and promoting healthier outcomes for their SWB, which relates to their academic performance. This study aims to address these gaps by these research questions and hypotheses:

1.2 Research Questions

i. What is the level of students' self-esteem?
ii. What is the level of students' subjective well-being?

1.3 Research Hypotheses

$H_{0\,1}$: The level of students' subjective well-being does not differ with respect to their sex
$H_{0\,2}$: The level of students' subjective well-being does not differ with respect to their parents' parenting style
$H_{0\,3}$: There is no statistically significant effect of students' self-esteem on their subjective well-being

II. LITERATURE REVIEW

Well-being is now a concept that drives research, policy, and decision-making in education. The determination of students' SWB has become crucial for students' engagement and accomplishment. Well-being as a psychological construct has emotional and affective dimensions. This theoretical review focuses on the Life circumstances theory and how it relates to this study

2.1 Theoretical Review

The study was anchored on the Life Circumstances Theory (LCT). The theory postulates that a person's living circumstances, both positive and negative have an impact on their subjective well-being (SWB) (Maddux, 2017). The idea offers a framework for comprehending how both internal and external factors affect an individual's SWB. Internal factors include an individual's self-worth, gender, age, etc. The implication is that an individual who rates himself/herself as valuable will have a high SWB and vice versa. As per the theory, an individual's external circumstances, such as income, health, social (family) relationships, employment status, and living conditions, significantly influence their SWB (Salmela-Aro & Tuominen-Soini, 2010). It suggests, for instance, that those who are healthier or wealthier ought to be happier overall. From this perspective, those who were born into favourable situations (stabilized family life, financial security, etc.) and who experience more positive events than negative ones will generally feel more satisfied with their lives than those who are less lucky or advantaged. This means that an individual's pleasure and life satisfaction level is mostly determined by their external, objective surroundings.

The relationship between SWB and LCT emphasises how crucial it is to take into account how a person's circumstances affect his/her overall well-being. Research has indicated that negative life experiences might raise stress and anxiety levels, which in turn lowers subjective well-being (Diener et al., 1999). According to Lyubomirsky et al. (2005), a good living environment can enhance SWB by providing opportunities for growth, stability, and support. In addition, life events like unemployment, poverty, or strife in the family can lead to stress and negatively affect SWB (Agnew, 2006). Social contacts, financial security, and physical health have all been found to be positively correlated with measures of SWB (Diener et al., 1999). The influence of living circumstances on SWB is also moderated by individual factors such as personality traits, coping strategies, and cognitive assessments.

Furthermore, the LCT offers an invaluable structure for comprehending how students' subjective well-being (SSWB) is influenced by a range of external circumstances both within and outside of the educational setting, including the school environment, social connections, family history, and health affect students' overall well-being, happiness, and life satisfaction. The academic environment is one of the external elements that significantly shapes students' subjective well-being, according to the Life Circumstances Theory (Suldo et al., 2011).

Another important aspect of a student's life that affects their SWB is their social ties, with parents (parenting styles), siblings, peers, and teachers. Research has indicated that adolescents who experience excellent peer interactions and solid friendships typically have greater levels of life satisfaction and happiness (Tian et al., 2016). On the other hand, students who endure bullying or social isolation could be less socially comfortable. The Life Circumstances Theory also highlights how the home environment and socioeconomic status affect students' SWB (Reyes et al., 2020). Overall, the most important life event that influences a student's SWB is their academic achievement, which informs their self-esteem. According to the Life Circumstances Theory, students with high self-esteem are more likely to be content and pleased with their lives (Salmela-Aro & Tuominen-Soini, 2010).

2.2 Conceptual Framework

Figure 1 conceptualises the effect of sex, parenting styles, and SE on students' SWB. It also portrays the relatedness of the variables to the problem identified as well as the objectives undergirding the study. Empirical evidence from the literature reviewed indicates an influence of sex and parenting style on the SWB levels of people. In addition, some literature shows an effect of SE on the SWB of individuals. It is worth noting that the literature is not conclusive and homogenous about these influences and effects. It is therefore hypothesised that students' sex and their parents' parenting styles will make a difference in their SWB, as seen with hypotheses 1 and 2. It is also hypothesised that students' level of SE will affect their SWB levels, as seen in Hypothesis 3.

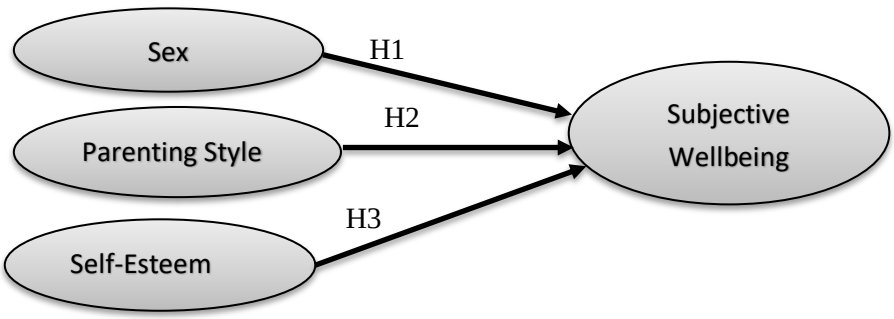

Figure 1

Conceptual Framework Depicting the Effects of Sex, Parenting Style, and Self-Esteem on Subjective Wellbeing

III. METHODOLOGY

3.1 Procedures

The cross-sectional survey research design was utilised to conduct this study. The design allowed the gathering of data from final-year senior high school students from the 10 schools in the Cape Coast metropolis. The final-year students in the 10 senior high schools were 7665. The final-year students were the target population for the study because they have spent almost three years in the school and can share their SWB levels that are essential in determining academic success in both their final certificate examinations (WASSCE) and tertiary (higher) education. They were deemed an ideal target for the study because they are dependent on their parents' directions, which may give an idea of their SE and can have an effect on their SWB.

Table 1

Population and Sample Size Distribution from Schools

School	Population	Sample
University Practice SHS	724	31
Wesley Girls SHS	867	41
Mfanstipim SHS	1008	44
St. Augustine's College	710	32
Academy of Christ the King SHS	188	8
Adisadel College	1,050	48
Oguaa Sec. Tech School	434	20
Ghana National College	804	40
Holy Child College	730	32
Aggrey Memorial School	1,150	51
Total	**7,665**	**347**

The spread of the students and their dispositions led to the use of the proportionate simple random sampling technique to select 347 students from the 10 senior high schools in the metropolis. According to Adams (2020), when dealing with a population close to 8,000, a sample size of 259 is representative if you collect continuous (numerical) data and analyse it at a 95% confidence interval. The sample size was, however, increased to 347. Table 1 shows details of the population of each school as well as the number of students selected to participate in the study. The questionnaires were distributed to all the students, and responses were received from all of them (100% return rate). Table 2 shows the demographic characteristics of the final-year students sampled for the study.

Table 2

Demographic Characteristics of Students

Variable	Sub Scale	No.	%
Sex	Male	196	56.5
	Female	151	43.5
Parenting Style	Permissive	44	12.7
	Authoritative	285	82.1
	Neglectful	9	2.6
	Authoritarian	9	2.6

To ensure there were no ethical breaches, the research protocol was sent to the Institutional Review Board of the University of Cape Coast for ethical clearance. After clearance was granted, an introductory letter was sent to the various schools to seek permission to collect data. Permission from the headmasters and mistresses of the 10 schools gave us access to the final-year students' classes. Informed consent was sought verbally. This was done by explaining the purpose of the study to the students. After students gave their consent verbally, sampling was done in each of the schools to select participants. Before the administration of the questionnaires, instructions about how to answer the questions were given, and respondents were guaranteed anonymity and confidentiality. They participated voluntarily.

3.2 Measures

The instrument used to collect data for this study was a questionnaire. The questionnaire had 28 items in three (3) sections. Section A had two items that sought information on students' sex and the parenting style of their parents. Section B measured students' SE levels with 10 items adopted from the Rosenberg Self-Esteem Scale (RSE) developed by Rosenberg (1979). The SE levels of students were measured using 1–5 as a scale (1= Undecided; 2=Strongly Disagree; 3=Disagree; 4=Agree and 5=Strongly Agree). Section C of the instrument collected information

on students' SWB levels with 16 questions adopted from the Students' Subjective Wellbeing Questionnaire (SSWQ) developed by Renshaw (2015). The items for determining students' SWB are in four aspects but were put together. Students' SWB was measured on a scale of 1–5 (1=Never; 2=Rarely; 3=Sometimes; 4=Often and 5=Always).

3.3 Validity and Reliability

Face and Content Validity

The questionnaire for data collection was tested to ensure it was valid even though items measuring the major variables were adopted from already validated scales. The instrument was scrutinised to guarantee that the questions reflected the objectives of the study before the main data collection. We first did this, and later gave the instrument to colleagues to review.

Reliability

A pilot test was conducted in the Komenda Edina Eguafo Abirim Municipality using 103 final-year senior high school students to ensure that there is internal consistency in relation to the items in the instrument. The Cronbach alpha reliability coefficient for students' SE and SWB were determined as 0.84 and 0.86 respectively. The overall reliability coefficient for the instrument before main data collection was determined as 0.85.

Test for outer model loadings

Measurement biases were measured with the assessment of construct reliability, convergent validity, and discriminant validity (Hair et al., 2014). Table 3 and Figure 2 detail the outcomes. Key: SE= Self-esteem, SWB=Subjective Wellbeing

Table 3

Construct Reliability and Convergent Validity for Self-esteem and Subjective Wellbeing

Variable	Item	Loading	α	rho_a	rho_c	AVE
SE	8	0.505-0.780	0.811	0.833	0.858	0.434
SWB	10	0.526-0.780	0.862	0.871	0.889	0.449

Results in the Table show the factor loadings of SE and SWB variables, which ranged from 0.505 to 0.780 and 0.526 to 0.780 respectively. These values were above 0.50 representing proof of convergent validity. This means that items measured the variables (Hair et al., 2014, 2017; Vinzi et al., 2010). The items were also deemed very reliable because of the Cronbach alpha reliability coefficients that were determined from 0.811 to 0.862. The composite reliabilities (rho_a and rho_c) for SE and SWB were also determined to be 0.833 to 0.858 and 0.871 to 0.889 respectively. These coefficients indicated that the internal consistencies of the variables were high (Hair et al., 2017; Henseler et al., 2009). The average variance extracted (AVE) coefficients from 0.434 and 0.449 recorded were not within the usual threshold of 0.5. Fornell and Larcker (1981) and Hair et al. (2017) indicate that in cases where the average variance extracted is below 0.5, we can still maintain the convergent validity of the variable if the composite reliabilities (rho_a and rho_c) are higher than 0.6. The details of the discriminant validity using the Fornell-Larcker criterion and Heterotrait-Monotrait Ratio (HTMT) is presented in Table 4. The structure model after the PLS-SEM Algorithm is also shown in Figure 2.

Table 4

Discriminant Validity between Self-esteem and Subjective Wellbeing

Construct	SE	SWB
Fornell-Larcker Criterion		
SE	**0.658**	
SWB	0.607	**0.670**
Heterotrait-Monotrait (HTMT) Ratio		
SE		
SWB	**0.686**	

In Table 4, the results of the test for discriminant validity of the model show that the average variances extracted for the main variables were 0.658 for SE and 0.670 for SWB. If the square roots of the average variances extracted for the constructs are greater than the equivalent inter-construct correlations then it is said that discriminant validity is achieved (Fornell & Larcker, 1981; Hair et al., 2014; 2017). Again, discriminant validity was confirmed with the Heterotrait-Monotrait (HTMT) Ratio test since the coefficients were below 850 or .90 (Collier, 2020; Henseler et al., 2015).

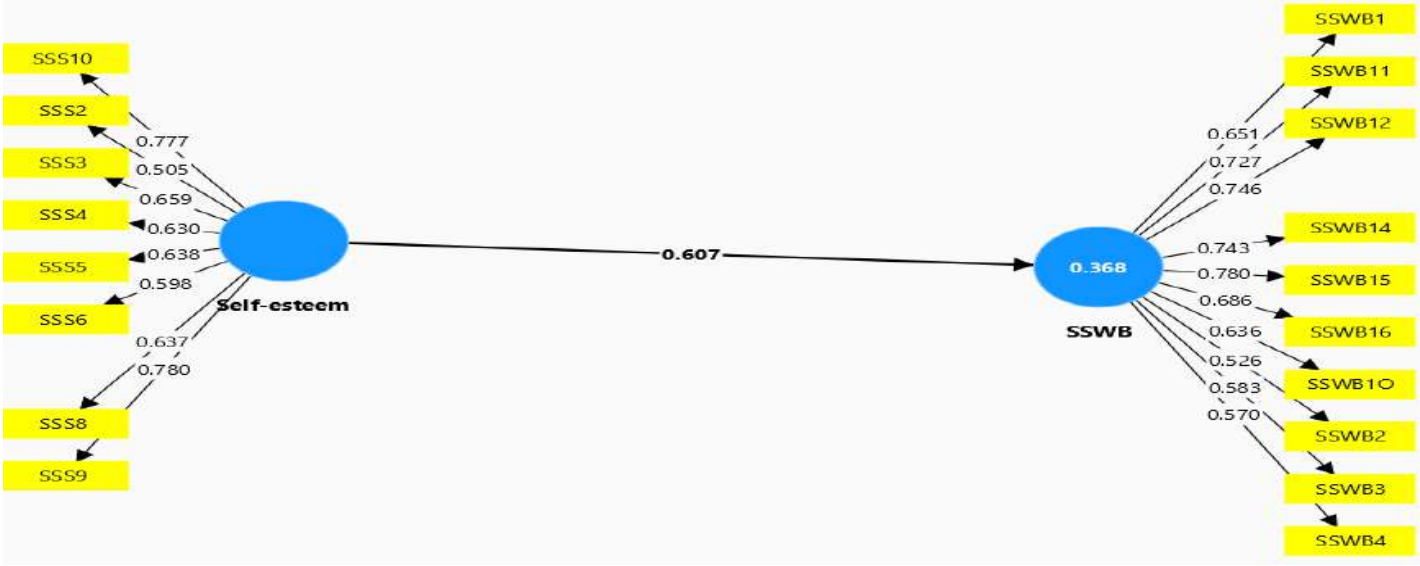

Figure 2
Structure model after PLS-SEM Algorithm

3.4 Data Analysis

Frequency counts and percentages were used to analyse data on students' demographic characteristics, to give a sense of their sex and parents' parenting styles. Frequencies, percentages, means, and standard deviations were used to analyse data related to the descriptive objectives formulated. This helped to determine students' SE and SWB levels. Differences in students' SWB concerning their sex were determined with the use of independent samples t-tests. Analysis of Variance (ANOVA) was utilised to determine if differences exist in students' SWB with respect to their parents' parenting styles. Finally, PLS-SEM analysis was utilised to establish the effect of students' SE on their SWB.

IV. FINDINGS & DISCUSSION

The outcomes of the data analysis of this study are presented in tables and figures. The results of research questions 1, and 2 and hypotheses 1, 2, and 3 are all presented in tables. Some results related to hypothesis 3 are also presented in the figures.

4.1 What is the Level of Students' SE?

One of the objectives of the study was to find students' SE levels. The students were to respond to show their agreement or disagreement with the items to determine their SE levels. The key of the responses were 5=Strongly Agree, 4=Agree, 3=Disagree, 2=Strongly Disagree, and 1=Undecided. The final mean value was interpreted as 1.0-1.6 (Low), 1.7-3.5 (Moderate), and 3.6-5.0 (High). The results of the analysis are in Table 5.

Table 5
Self-esteem Level of Students

Statement	**SA/A**		**DA/SDA**		**U**		**M**	**SD**
	No.	%	No.	%	No.	%		
On the whole, I am satisfied with myself.	263	75.8	55	15.9	29	8.4	4.0	1.2
At times, I think I am damn good.	252	72.7	66	19	29	8.4	3.7	1.2
I feel that I have several good qualities.	285	82.2	39	11.2	23	6.6	4.1	1.1
I am able to do things as well as most other people.	270	77.9	64	18.4	13	3.7	4.0	1.0
I feel I do have much to be proud of.	244	70.3	76	21.9	27	7.8	3.9	1.2
I really feel useful at times.	253	72.9	76	21.9	18	5.2	3.9	1.1
I feel that I am a person of worth, or at least an equal plane with others.	227	65.4	85	24.5	35	10.1	3.8	1.2
I think I have enough respect for myself.	294	84.8	39	11.2	14	4.0	4.2	1.0
All in all, I am inclined to feel that I am not a failure.	309	89.1	20	5.8	18	5.2	4.4	1.0
I take a positive attitude towards myself.	308	88.7	29	8.3	10	2.9	4.4	1.0
Average							**4.0**	**1.0**

Results indicate that most (309, 89.1%, M=4.4) of the students are inclined to feel that they are overall not a failure. Also, most (308, 88.7%) of them indicated agreement (M=4.4) with the fact that they are optimistic about themselves. Besides, the majority (294, 84.8%) of them showed agreement (M=4.2) that they think they have enough respect for themselves. The fact that students feel they have several good qualities is a greater (285, 82.2%, m=4.1) indicator of their SE. Moreover, the statement with the lowest number of agreements was on the thought of students about them being damn good at times. The average mean value obtained shows that the students' SE levels are high. This is shown by the average mean value of 4.0 (SD=1.0)

4.2 What is the Level of Students' SWB?

Another objective of the study was to determine the level of students' SWB. The students were to respond to show their agreement or disagreement with the items to determine their SE levels. The statements measured the four attributes (joy of learning, school connectedness, educational purpose, and academic efficacy) of SWB. The results are displayed in Table 6.

Table 6

Subjective Wellbeing Level of Students

Statement	**A/O**		**S/R**		**N**		**M**	**SD**
	No.	**%**	**No.**	**%**	**No.**	**%**		
I get excited about learning new things in class.	243	70.1	100	28.8	4	1.2	4.2	1.0
I am interested in the things I am doing at school.	214	61.6	121	34.9	12	3.5	4.0	1.1
I enjoy working on class projects and assignments.	212		110	31.7	25	7.2	3.8	1.1
I feel happy when I am working and learning in school.	214	61.6	120	34.6	13	3.7	3.9	1.2
I feel like I belong at this school.	192	55.4	123	35.4	32	9.2	3.7	1.1
I can be myself at this school.	204	58.8	109	31.4	28	8.1	4.7	1.1
I feel like people at this school care about me.	113	32.5	200	57.6	34	9.8	3.1	1.2
I am treated with respect at this school.	181	52.1	147	42.3	19	5.5	3.6	1.1
I feel like the things I do at school are important.	251	72.3	89	25.6	6	1.7	4.3	0.9
I think school matters and should be taken seriously.	290	83.6	54	15.5	3	0.9	4.5	0.9
I feel it is important to do well in my classes.	302	87	41	11.8	2	0.6	4.6	0.8
I believe the things I learn at school will help me in life.	298	85.9	43	12.4	6	1.7	4.6	0.9
I am a successful student.	276	79.6	59	17	10	2.9	4.6	1.0
I do good work at school.	276	79.5	67	19.3	4	1.2	4.3	0.9
I do well on my class assignments.	252	72.7	91	26.3	4	1.2	4.2	1.0
I get good grades in my class.	255	73.4	88	25.4	4	1.2	4.2	1.0
Average							**4.5**	**1.0**

Outcomes of the analysis indicate that most (204, 58.8%, M=4.7) of the students can be themselves at their school. Also, most (302, 87%, m=4.6) of them feel it is important to do well in their classes. Most (298, 85.9%, M=4.6) of them in return believe that the things they learn at school will help them in life. The majority (276, 79.6%) indicated agreement (M=4.6) that they are successful students. Again, the thought that school matters should be taken seriously is a determinant of the students' SWB and the majority (290, 83.6%) of them indicated agreement (M=4.5). The majority (276, 79.5%) of the students indicated agreement that they do good work at school (M= 4.3). More than half (252, 72.7%) showed agreement that they do well on their class assignments (M= 4.2) and 255 (73.4%) indicated agreement that they get good grades in their class (M= 4.2). Lastly, the majority (113, 32.5) indicated agreement that they feel like people at their school care about them (M= 3.1). Put together, it is realized that the students have a high level of SWB as evidenced by an overall mean of 4.5 out of 5 (SD=1).

$H_{0\,1}$: The Level of Students' SWB does not differ with Respect to Their Sex

This hypothesis sought to determine if the sex of students causes differences in their SWB levels. Students' SWB levels were the dependent variable and sex (male and female) was the independent variable. To analyse data, the independent samples t-test was used. Table 7 details the results.

Table 7

Independent Sample T-Test on Differences in Students' Subjective Well-Being about their Sex

	Groups	n	Mean	SD	t-cal	df	P value	Decision
Subjective Well-being of Students	Male	196	4.22	.94	1.523	345	0.129	Accept
	Female	151	4.08	.71				

Results from Table 7 show that male students had a higher mean value (4.22, SD=0.94) as compared to females (4.08, SD=0.71). The mean difference was recorded as 0.14. The results show that the level of students' SWB does not differ with respect to their sex. There is no statistically significant difference in students' SWB concerning their sex (t=1.523, df=345, p=0.129). This implies that students' SWB levels are not influenced by their sex.

H_0 2: The Level of Students' SWB does not differ with Respect to Their Parents' Parenting Style

Further, the study purposed to determine the influence of the parenting styles of students' parents on students' SWB levels. The One-way Analysis of Variance (ANOVA) was employed to establish the influence of parenting styles (independent variable) on students' SWB (dependent variable). Tables 8 and 9 detail the outcomes.

Table 8

Descriptive of the Influence of Parenting Styles on Students' Subjective Wellbeing

Parenting styles	N	Mean	SD
Permissive	44	3.76	0.98
Authoritative	285	4.26	0.77
Neglectful	9	3.17	0.68
Authoritarian	9	3.85	1.40
Total	**347**	**4.16**	**0.85**

Table 9

One-Way ANOVA of Subjective Well-Being and Parenting Styles

Source	Sum of Squares	df	Mean Square	F	Sig.	Decision
Between groups	19.293	3	6.431	9.667	0.000	Reject
Within groups	228.178	343	0.665			
Total	247.472	346				

The results of the analysis showed a statistically significant difference in students' SWB levels with respect to their parents' parenting styles, $F(3, 343) = 9.667$, $p<0.05$. The SWB levels of students are influenced by the parenting styles adopted by their parents. This means that the SWB levels of students are not the same concerning their parents' parenting styles. A Post hoc analysis (Bonferroni) showed statistically significant mean variances between permissive and authoritative parenting styles as well as authoritative and neglectful parenting styles on students' SWB levels (see Table 10).

Table 10

Multiple Comparisons of Parenting Styles and Subjective Well-Being Test Using Bonferroni

(I) Parental style	(J) Parental style	Mean Diff. (I-J)	Std. Error	Sig.	95% C I	
					Lower Bound	Upper Bound
Permissive	Authoritative	-.49555*	.13211	.001	-.8461	-.1450
	Neglectful	.58917	.29839	.295	-.2026	1.3810
	Authoritarian	-.09138	.29839	1.00	-.8832	.7004
Authoritative	Permissive	.49555*	.13211	.001	.1450	.8461
	Neglectful	1.08472*	.27613	.001	.3520	1.8175
	Authoritarian	.40417	.27613	.865	-.3286	1.1369

* Significant at the 0.05 level

H_0 3: There is no Statistically Significant Effect of Students' SE on SWB

The study finally set out to determine if students' SE had any predictive effect on their SWB. PLS-SEM statistics were used to determine the effect of SE on the SWB of students.

Results in Table 11 indicated that students' SE significantly and positively predicted their SWB (β = .607, t = 12.167, p=.000). Students' SE moderately (Chin, 1998), explains 37% of the variances in the SWB of students (R^2 = .368, $R^2_{Adjusted}$ = .336,) with a substantial effect size (f^2= .586) showing the extent of the effect of SE on SWB (Hair et al., 2013). The positive standardized beta value (β) is an indication that a decrease in the SE levels of students will lead to a decrease in their SWB levels and vice versa. Figure 3 shows a pictorial representation of structural model assessment after bootstrapping (Hair et al., 2014; Kock, 2015).

Table 11

Effect of Students' Self-esteem on Subjective Wellbeing

Construct	*B*	*SD*	t-value	p-value	f^2	R^2	*adj* R^2	Bootstrap 95% CI	
								Lower	Upper
SE -> SWB	.607	.050	12.167	.000*	.583	.368	.336	.488	.691

*significant @ .05

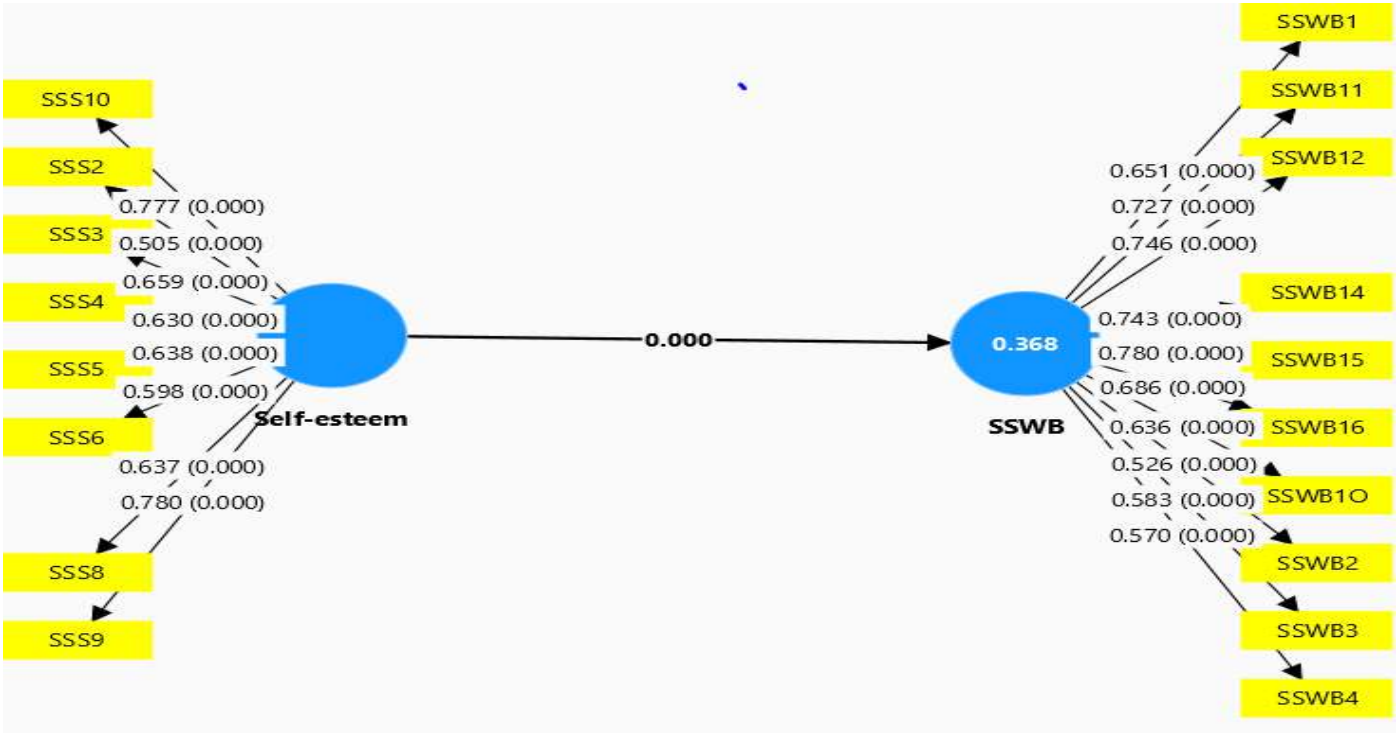

Figure 3

Structure Model After Bootstrapping

4.3 Discussion

SWB of students has attracted a lot of research in recent years. Students' SWB, which encompasses the joy of learning, school connectedness, educational purpose, and academic efficacy, is seen to be crucial in determining the students' total development and attainment of curricula goals. SE of students has been discovered as a pivotal determinant of students' SWB. This study and its findings relate to the United Nations' Sustainable Development Goals 3 and 4. Goal 3 talks about ensuring good health and well-being. This relates to the SWB of the students in this study. Goal 4 talks about quality education for all students irrespective of their state or disabilities (emotional or psychological). The findings provide a snapshot of the SWB of senior high school students in Ghana. This helps in policy formulation about providing quality senior high school education in Ghana. The purpose of this study was to determine the effect of students' sex, parents' parenting style, and SE on their SWB.

Outcomes of the study indicate that senior high school students have a high level of SE (M= 4.0 out of 5, SD=1.0). The high level of SE indicates that they have a high sense of value or worth of themselves (Blascovich &Tomaka, 1991) and a high level of global judgment worth of themselves (Crocker & Wolfe, 2001), which are an essential part of the subjective quality of life (Diener, 1984). This finding corroborates previous studies that sought to measure students' SE. Arshad et al. (2015) found that university students had high levels of SE. Kariuki et al. (2019) studied the impact of students' SE on their academic performance and found high levels of SE among the students but did not influence their performance. Acosta-Gonzaga (2023) and Zhao, et al. (2021) both found high levels of SE

among university students and early adolescents in China and India respectively. All these previous studies affirm that SE relates to the academic performance of students.

The study also revealed that students' SWB level is high (4.5 out of 5, SD=1). Their high level of SWB indicates that they have high joy in learning, which pertains to the positive emotions and satisfaction they experience in the process of acquiring knowledge and skills (Ryan & Deci, 2000), a high feeling of acceptance, good relationships with other students and teachers, and a sense of being an essential member of the school community (Resnick et al., 1997; McNeely et al., 2002), high educational purpose including the development of critical thinking and problem-solving skills, the acquisition of knowledge and skills, and development of well-rounded individuals who can make significant contributions to society (Dewey, 1897; Bruner, 1966) and high academic efficacy which denotes the confidence that one has in accomplishing academic obligations and reaching learning objectives. This finding is in line with the findings of Donald and Jackson (2022), Ratelle et al. (2013), and Özdoğan (2021) who found high levels of SWB among University students. Other studies (Opoku et al., 2021; Sharma & Yukhymenko-Lescroart, 2023) have found moderate SWB levels of students in Colleges of Education in Ghana United States of America. The findings of this study and previous research have shown that the manifestation of students' SWB is considered pivotal in determining the academic engagement and success of students.

Furthermore, the results of the study revealed that there was no statistically significant difference between the SWB levels of male and female students. Therefore, irrespective of sex, students' level of joy in learning, school connectedness, educational purpose, and academic efficacy are the same. Martínez-Marín and Martínez (2019) confirmed that there is a sex gap in SWB of students. Lokeshwari and Monika (2021) also concluded that females have better subjective well-being than males. Buhner et al. (2022) and Clark et al. (2014) found significant effects of sex on their respondents' SWB. Contradicting the current finding, Joshi (2010) and Mahasneh (2022) in their respective studies found that sex is not a determinant of students' SWB. These findings (current and previous) draw attention to the inconclusiveness when it comes to the differences in SWB based on sex.

The study further revealed that there was a statistically significant difference in students' SWB levels concerning their parents' parenting styles**.** SWB levels of students are not the same across their parents' parenting styles. This finding corroborates the previous findings (Manuel & Asuquo, 2021; Huang et al., 2022; Mishra & Sethi, 2023; Kang, 2023) from different locations that found that parenting styles significantly predicted or influenced SWB levels of students and individuals. These studies found that the authoritative parenting style, characterized by warmth, responsiveness, and control, was advantageous for positive child outcomes, while permissive, authoritarian, and neglectful parenting styles emphasize discipline and warmth (Martin & Colbertg, 1997; Čudina-Obradović & Obradović, 2006). Studies have repeatedly shown that students who experience authoritative parenting have a higher SWB (Ryan & Deci, 2000). In contrast, there is a negative correlation between students' SWB and parenting approaches that prioritize strict discipline, lack of warmth, or overbearing control, such as those adopted by neglectful parents (Baumrind, 1991; Durbin et al.1993). This finding backs the debate by showing the Ghanaian perspective on the influence of parenting styles on the SWB of senior high school students.

Finally, the study revealed that students' SE moderately explained 37% of the variation in their SWB. Students' SWB will increase if their SE is increased. Butt (2009) revealed that SE correlated positively with SWB and clarified a substantial aggregate of variances (18%) in SWB and life gratification. Luo (2023) also found in his study that SWB was positively and significantly associated with SE and that SE played a suppressing role in SWB. In addition, Liao et al. (2023) also found that SE mediates a positive indirect effect on SWB. The findings of this study also confirm recent discoveries by Katsantonis et al. (2022) and Yang et al. (2019) who found from their respective studies that the SE of students predicts and correlates with their SWB. Du et al. (2017) found that both PSE and RSE were positive predictors of SWB but CSE was weakly related to SWB. The finding of this study, juxtaposed with previous findings, furthers and contributes to knowledge on the effect of SE on SWB from the Ghanaian perspective, drawing attention to the essentialities of these variables to the total well-being of the student.

V. CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

Students' high level of SE is likely to result in them having a high sense of self-worth and value. This high SE is likely to result in general self-development and an improvement in the academic performance of students. In addition, the high level of senior high school students' SWB is likely to lead them to enjoy what they are learning in school, feel connected and accepted in the school, have a purpose for learning, and feel efficacious academically. The conclusion can also be drawn that both male and female students have the same levels of joy in learning, school connectedness, educational purpose, and academic self-efficacy, which are the constituents of SWB. Therefore, students' SWB levels are not determined by their sex. Furthermore, students whose parents used different styles of parenting to bring them up would not have the same level of SWB. This means parenting style influences senior high

school students' SWB levels. Authoritative parenting style leads to high levels of SWB as compared to the others. Finally, it is conclusive that students will enjoy their learning, feel part of and connected to their school, have a high purpose for learning, and feel highly efficacious academically (SWB) when they have a high sense of self-value and worth (SE). It is, however, worth noting that their SE is not the sole determinant of their SWB.

5.2 Recommendations

We recommend that senior high school authorities in Ghana should put in place policies and strategies that will sustain the high levels of SE and SWB revealed. This is very important for the realisation of SDG goals 3 and 4. In doing this, it is also imperative for authorities not to play the sex card since sex does not influence the SWB levels of students. The findings and conclusions of this study also draw attention to the need for parents to be educated and sensitized on the effect of their parenting styles on the SWB levels of their children. This is because authoritarian parenting results in high levels of students' SWB as compared to the others. Lastly, students' SE explains 37% of their SWB. This draws attention to the fact that other variables affect students' SWB levels aside from SE. It is recommended that further studies should be conducted to find out these other predictors of students' SWB. Further studies can also be conducted on basic school and university students' SWB to expand knowledge in the Ghanaian context of the issue.

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Domestic Violence in Nyeri County, Kenya: What and When?

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ABSTRACT

Domestic violence is a worldwide problem affecting many households. Nyeri County, one of the 47 counties in Kenya has, according to reports, experienced a steep increase. Nyeri County has experienced tremendous development, and therefore, changes in terms of education, occupation, and economic activities, making it one of the richest counties in Kenya. With these changes in levels of education, strong cultural adjustments, and increased economic endowment, the expectation is that there should be harmony in the homes. Contrary to that, there is a lot of domestic violence as captured by scholars and the media. This study sought to investigate three things namely; when the violence takes place, the types of violence, and why the violence tales place. The study was underpinned by the frustration-aggression theory and the social learning theory, where descriptive and evaluative research designs were used. The following respondents were sampled to participate in the study: 384 household heads, 12 assistant chiefs, 15 police officers, 30 religious leaders, 12 NGOs/CBOs, 3 Sub-county administrators, and 20 village elders. Primary methods of data collection were questionnaires, interviews, focus group discussions, and observation. Quantitative data was analyzed using SPSS version 26 to give frequency distribution, means and percentages, and findings were presented using tables, graphs, and charts. Qualitative data was analyzed using thematic narrations, with findings presented using narrations and descriptions. The study found out that the cases increase when the farmers receive the coffee bonuses, when learners are going back to school and fees is needed, during the festive seasons and traditional festivities, as well as during the drought season. The study concludes that domestic violence is evident throughout the year, but there are times when the cases increase as a result of conflict in the gender roles. In this case, therefore, the study recommends that there should be a social talk on the gender roles in the study area so as to ease the adamant rise and spread of domestic conflicts related to the various sets of culturally defined and allocated sex roles. Further, there is need to empower both men and women in Nyeri county so that no gender feels threatened by the other. This will go a long way in ensuring that there is peace and stability in families and social relationships.

Key words: Domestic Violence, Gender, Peace, Social Cultural Dynamics

I. INTRODUCTION

According to Njenga and Njoroge (2021), domestic violence occurs at all societal levels and within all societal classes. Thus, it can occur to infants, children, adolescents, young adults, adults, and the elderly. It can occur at home, learning institutions, and on the streets (Njenga, & Njoroge, 2021). The Inter-Agency Standing Committee (2014) adds that it takes several forms that encompass varying actions which violate human rights including verbal abuse, sexual molestation, physical aggression, threats, intimidation, economic deprivation, and emotional abuse among others (Inter-Agency Standing Committee, 2014).

On the other hand, the term domestic violence which is also referred to as intimate partner violence (IPV), domestic violence, battering, spousal abuse, dating abuse, or family abuse is a behavioral pattern that involves violence perpetrated by one partner against his/her intimate partner, within a family, a cohabitation, marriage, or a dating relationship/courtship (Mongare et al., 2018). Traditionally, domestic violence has been occurring against women across all cultural and socio-economic backgrounds. The society forces women to accept, rationalize, and tolerate violence against them, and remain silent in the name of submission. They should not talk about their experiences even if they are emotionally, physically, and/or psychologically tortured.

Domestic violence has remained a persistent problem in the United States of America (USA) for quite a long time, making it an endemic situation in most American societies. Recent statistics have shown that nearly 10 million adults experience domestic violence annually with women and girls being the most affected group (National Coalition Against Domestic Violence [NCADV], 2020). Domestic violence has greatly impacted both genders since the abused or victims stem from either gender. Regarding this, numerous factors have been found culpable of exacerbating the already dire situation and they include but are not limited to education level, stereotype, cultural practices as well as the history of marital violence of either the victim or the abuser's family genesis. To unravel the sociocultural dynamics that fuel domestic violence, Pan et al., (2006) conducted a study that targeted immigrants from the city of San Diego in the State of California. Three communities namely; Somali, Latino and Vietnamese were sampled through a needs assessment strategy. The study established that domestic violence was a grave concern across the

three communities with many participants acknowledging its frequent occurrences. Similarly, the sampled communities perceived domestic violence as intergenerational and mainly focused on the physical aspect of violence. For instance, in the Somali culture, domestic violence is limited to physical violence and implicates all family members. The changing gender roles were found as the greatest contributor to persistent domestic violence across Somali families, a factor that was attributed to the alteration of the power dynamics upon securing refuge in the United States (Pan et al., 2006). The US immigration laws tend to be favorable to women than men which in turn makes men feel helpless and useless owing to the fear of losing their stature as breadwinners, making most of them resort to violence to reclaim their lost control over the family. For the Vietnamese, domestic violence is seen as a confidential family matter that is indispensable. The study established that the persistent domestic violence across Vietnamese families was a result of economic stressors that widened the economic inequality of both genders. On the other hand, Latinos reported gradually changing gender roles as the greatest contributor to domestic violence. In a nutshell, the study linked the persistent domestic violence, especially among the migrants to some sociocultural dynamics such as strict gender roles, unemployment, cultural identity, and spirituality among others (Pan et al., 2006). A similar picture is painted on the typical households of the Americans with most of these dynamics playing a huge role in perpetuating the menace of domestic violence.

The African continent has not been insulated from the rippling devastations of domestic violence, especially on women. Notably, all the African regions have had a fair share of such violence that ever since has impeded the socio-economic development of a swath of the continent. Quite a similar picture is painted in the Middle East as highlighted in the correlational study by Kisa et al. (2021). The study noted that domestic violence, especially violence against women remains a common phenomenon in both Northern Africa and Middle Eastern countries and serves as an impediment to women's equality and security vis-à-vis their legitimate right to enjoy fundamental freedoms. The study revealed that the regions consist of a multi-ethnic society that is mostly patriarchal, noting that women are disadvantaged by the masculine nature of their societies, rendering them vulnerable to domestic violence. For instance, countries like the Islamic Republic of Iran and the Kingdom of Saudi Arabia embed religious rules in their national laws which have huge impacts on women's perception of domestic violence. Additionally, high illiteracy, pervasive poverty, and low family labor force among others have contributed to the high prevalence of domestic violence in both regions. This is further worsened by the conservative and widespread masculine gender attitudes which significantly impact the status of women. In Northern Africa, domestic violence is inherently linked to the structure of the family. Precisely, the region is largely patriarchal with family serving as the realm of male domination, where women's roles are reduced to child education and rearing. However, societal changes that empowered women through education and job accessibility culminated in a new wave of conflict within the family which was accompanied by new gender negotiations within the family set-up (Kisa et al. (2021). The situation was worsened by the combination of economic stagnation, unemployment and superficial form of religiosity all of which ushered in a period of masculinity crisis in the region which in turn led to intense violence, especially domestic violence (Gichango, 2020). In West Africa, especially Nigeria, domestic violence has been on the rise. Traditionally, like many African countries, most of the domestic societies consider wife and children beating as a form of instilling discipline. This is largely necessitated by the over-dependence of women on men for survival. Additionally, the societies are largely patriarchal which has dealt a blow to women who are considered subordinate. Domestic violence is regarded as a means of enforcing conformity to societal norms as further enabled by privatization of the vice across most societies in Nigeria thus shielding such violence from external scrutiny (Mutahi, 2017). Consequently, the persistent culture of silence over domestic violence strengthens stigmatization rather than condemnation of the perpetration of such crimes. Such among other factors have made domestic violence thrive in most of the Nigerian societies, making it an endemic problem.

1.1 Statement of the Problem

Nyeri County is one of the richest counties in Kenya, and this can be attributed to the changes that have occurred in the community including economic empowerment, high literacy rates, and the industrious culture and horticultural farming zeal among the residents. This means that there has always been a good flow of income in that region. With all the resources, it can be deduced that people in the region do not struggle to get basic things such as food, shelter, and education among other critical things for survival. According to Muhindi (2012), Nyeri is one of the counties with the highest educated individuals in Kenya. As such, education in the region has brought about cultural adjustment and economic empowerment. Unlike in other counties in Kenya, women in the region have greatly been in the forefront seeking education for better means of living. With all this in place, the general society has set some expectations which are normally common among the educated and economically stable societies, one of these being the existence of harmony in homes within Nyeri County. Contrary to the expectations, Nyeri has experienced an increase in domestic violence. Bitrus-Ojiambo and King'ori (2016) argue that a high level of education brings about competition between the father and mother in the house as a result of power rivalries especially in decision-making

processes. Such competition causes unnecessary disagreements that in many cases usually escalate to violence in the house. Scholars such as Bitrus-Ojiambo and King'ori (2016) and some media houses in Kenya have clearly indicated that despite the high level of education and economic stability in Nyeri County, domestic violence still exists. Therefore, the prime interest of this study is to assess when domestic violence is heightened in Nyeri County, Kenya.

In Nyeri County, the high prevalence of domestic violence is far from receding. Notably, Nyeri is one of the Kenyan counties that have and continue to post a remarkable stellar economic performance. It has experienced tremendous development, and therefore, changes in terms of education, occupation, and economic activities, making it one of the richest counties in Kenya. With these changes in society, there is a high level of education which has highly contributed to the holistic empowerment of both genders, thus making some of the women secure the most stable jobs that were primarily held by men, cultural adjustments which have seen the decline in male domination, exchange of gender roles among other inevitable changes. Notably, cultural adjustments have imprinted severe changes in the feminine perception which in turn has influenced the dynamics of domestic violence. Again, economic endowment and empowerment have contributed significantly to the independence of both genders since even women have become a reckoning force in various economic sectors that drive the development of the county.

1.2 Research Objective

The main objective of the study was to asses when the cases of domestic violence increase in Nyeri County.

II. LITERATURE REVIEW

2.1 Empirical Literature Review

The first official definition of GBV was done in 1993 when the UN General Assembly defined GBV as: Violence against women is any act of gender-based violence that results in, or is likely to result in, physical, sexual, or psychological harm or suffering to women, including threats of such acts, coercion, or arbitrary deprivations of liberty, whether occurring in public or in private life (United Nations, 1993).

Further, the UN General Assembly added that:

Violence against women (VAW) is a manifestation of historically unequal power relations between men and women, which have led to the domination over and discrimination against women by men and to the prevention of the full advancement of women. VAW is one of the crucial social mechanisms by which women are forced into a subordinate position compared with men (United Nations, 1993).

According to Bhuvanendra and Holmes (2014), the term domestic violence which is also referred to as intimate partner violence (IPV), battering, spousal abuse, dating abuse and family abuse is a behavioral pattern that involves violence perpetrated by one partner against his/her intimate partner, within a family, a cohabitation, marriage, or a dating relationship/courtship.

Gender Violence Recovery Centre (GVRC) (2014) defines gender-based violence as:

Any act that results in physical, sexual, or psychological harm or suffering, including threats of such acts, coercion, or arbitrary deprivations of liberty, whether occurring in public or private life perpetrated against a person based on socially-ascribed (gender) differences between males and females (GVRC, 2014).

Bitangora (1999) adds that gender-based violence mostly refers to the harm inflicted to a person or people in the society because of belonging to a certain gender. In most of the countries, gender-based violence affects females as they are considered defenseless in comparison to the males (Bitangora, 1999). According to UN Economics and Social Council (2017), women in some countries have been restricted by the law and culture from participating in developing the economy. This has been the reason behind the motive to restructure the law and educate women so that they can withstand the different types of violence they have been subjected to.

In most cases, domestic violence is limited to physical violence. However, Mongare et al. (2018) argue that GBV also includes stalking, kidnapping, endangerment, harassment, trespassing and unlawful imprisonment. In fact, in most patriarchal societies such as Kenya, the society accepts violence against women as a “correct” way of disciplining a wife. The patriarchal society socializes the woman to accept and anticipate such discipline (Mutahi, 2017). As a result, such practices are deeply engrained in the society and the woman should take such punishment from the husband positively.

In most societies, women have since time immemorial been properties of men (Meyersfeld, 2010). When a girl is born, she becomes the property of her father and automatically takes her father’s name as the surname. When the woman gets married, the status changes and she becomes the property of the husband (Bhuvanendra & Holmes, 2014). In most cultural affiliations, the woman moves from her father’s home and joins the husband and starts living there as the man’s property. As a result, the woman is entitled to do everything that the man wants her to do without questioning (Mutahi, 2017). In case the woman fails to obey her husband, she is battered or punished for her actions.

Olufunmilayo (2004) adds that the woman is supposed to apologize to the man and try behaving well so that she will not receive punishment (Mucheru, 2014; Olufunmilayo, 2004).

According to Flemke and Allen (2008), domestic violence against men is not something new in history. Nonetheless, the worrying issue is the increase in the rate of domestic violence against men in different societies. Psychological abuse, sexual harassment and physical battering are the most common typologies of abuse that men in the current world are going through (Flemke & Allen, 2008). Apparently, thousands of men irrespective of their age, color, race, and religion live under the threat of domestic violence (Mongare et al., 2018). However, most of the men do not report the cases because they fear stigmatization that comes with such cases. Unfortunately, there are few and in many cases no efforts to stop domestic violence against men (DVAM) in many countries.

In the Middle East Countries, women are not allowed to enjoy some privileges such as driving and going for higher education. King Salman, from Saud Arabia in May 2017 issued a directive which aimed at empowering women in attaining the highest education level, taking jobs, or undergoing surgery to beautify themselves (Wald, 2017). However, this action was not received positively as the society is not yet ready to empower the women. Carr (2007) argues that women are expected to remain in the silent world where they can be oppressed without raising alarm. Consequently, Salman received a lot of political and religious criticisms because of his action. It is unfortunate that even in the 21st century, the world is still struggling to attain equality and zero domestic violence.

Mucheru (2014) argues that in most societies, domestic violence against women is acceptable as part of culture. Women have been oppressed for many ages with the society taking the harassment and oppression as a cultural practice that is acceptable and unquestionable (Mucheru, 2014). However, in the recent years, men have also become victims of domestic violence. The rate of domestic violence perpetrated by women against men is rising, creating alarm to the entire world. The cultural belief that men should be strong and should not be battered makes the men more vulnerable. They go through hard times as they cannot report the cases to the authorities or to the family because they will be depicted as “incomplete men” (Mongare et al., 2018).

Domestic violence is a general term that entails all harmful actions directed to an individual based on their gender. Violence can appear in various forms including forced marriages, sexual violence, domestic violence, femicide, female genital mutilation, and emotional violence (European Institute for Gender Equality, 2019). Domestic violence is the most common form of violence (Kangas et al., 2015). It commonly involves partners in romantic relationships but can also occur between siblings or between a child and the parent. According to the United Nations, Gender-based violence (GBV) contributors include gender inequality, dangerous norms, and power abuse. The available data on GBV reveals that one in every three women, among about 730 million women worldwide, have been exposed to physical or sexual violence or both at least once in their lifetime (UN Women, 2021). These statistics focused on women aged 15 years and above. Domestic violence is a significant threat and causes adverse effects, evident in all countries globally.

According to the Kenya National Bureau of Statistics (KNBS), statistic results in 2010 on the Kenyan population reveal that the victims usually know perpetrators in most domestic GBV cases. Only 6% of the perpetrators are strangers (KNBS, 2014). KNBS also reported that 47% of women aged 15-49 years had been victims of physical or sexual violence. The number of individuals who have prior exposure to domestic violence increases with age. 35% of them are aged between 15 and 19 years, while 54% are between 40 and 49 years (KNBS, 2014). Domestic violence also occurs in individuals of Kenya's lesbian, gay, bisexual, and transgender (LGBT) community. However, the Kenyan Government has not been more open about this type of abuse.

Several cases of domestic violence have been reported in Nyeri County. For instance, Kimani (2007) reports an incident of domestic violence. In 1998, a Kenyan police officer, Felix, went home and demanded meat for his supper. The fact that there was none in the house made him beat up his wife, and she incurred brain damages and became paralyzed. The wife died five months later during her birthday. The media extensively covered that story, which led to mass protests. The Kenyan government then passed on a protection bill against wife-beating and other ways through which domestic violence is perpetrated.

Central region of Kenya has witnessed rampant increase in the rise and spread of domestic violence, which sometimes leads to early and untimely deaths. The violence is associated with consumption of illicit brew, alcohol, drug abuse, and poverty. Unlike other parts of Kenya, Kiambu County and Nyeri County are recording higher incidences where men are being affected by the domestic violence occurring in the regions. Mutahi (2017) depicts that 3.8 women and 1.3 men per 1000 have experienced domestic violence. However, this statistic is changing as the number of men rapidly increases in comparison to the ladies (Mutahi, 2017). In the year 2012, about 500,000 men experienced beating from their wives; hence, there has been need to bring different mechanisms to end such trends despite increase in female superiority. The current study aims at investigating the contribution of sociocultural factors in promoting the prevalence of domestic violence so that the appropriate steps can be suggested.

Most of the scholars as indicated in the above discussion (Bhuvanendra & Holmes, 2014; Bitangora, 1999; Mucheru, 2014; Njenga, & Njoroge, 2021) focus on domestic violence against women. Thus, the studies fail to

investigate the role played by women as perpetrators thereby exhibiting a gap on scholarly work on the same. There are cultural dynamics that have made women to become perpetrators of domestic violence as well. A lot of concentration and resources have been directed to the empowerment of the girl child consequently leaving out the male child in opportunities that would have been considered appropriate for their growth and empowerment as well. In this case, violence at home has become a two-way occurrence. Women perpetrate violence as men do the same. Further, they did not try to investigate the time of the year when the cases of domestic violence increase or reduce. Thus, it is the prime interest of this study to investigate whether the patterns of domestic violence are the same throughout the year.

2.2 Theoretical Framework

The study was underpinned by a conceptual framework borrowing ideas from the frustration-aggression theory and the social learning theory. The two theories complemented each other in meeting the objectives of the study. The elements of each theory were tied together to form a conceptual framework.

2.2.1 Frustration-aggression Theory

Frustration-aggression theory was developed by scholars such as Dollard, Miller, and Doob among others in the early 1930s and later reformulated by Berkowitz in the late 1960s. According to the theory, aggression largely stems from frustrations, especially when individuals fail to attain their goals. In its first hypothesis, the theory postulated that frustration preceded aggression with the latter being the absolute consequence of the former. Barely two years after its formulation, scholars led by Miller and Sears restructured the hypothesis to denote that while frustration necessitated the need to react, some form of aggression is one possible outcome. Dollard and colleagues asserted that frustration is a condition that existed when an objective suffers distortion while aggression is conceived as an act whose intention is to inflict injury or pain (Breuer et al., 2015) Attached and deeply linked to the theory is displaced aggression which describes a response to a frustration that is directed towards individuals who bear no responsibility for the interference in the attainment of one's objective. For instance, a man may be humiliated in his workplace but is incapacitated to respond in the fear of losing a job and may instead redirect his anger and frustration on the family. Naturally, the requirements for frustration to lead to aggression differ between various forms of aggression which may include but are not limited to direct, indirect, or physical aggression.

2.2.2 Social Learning Theory

The profound development of social learning theory is largely attributed to the influential works of Albert Bandura. As averred by Bandura (1986), people tend to observe behaviors either directly or indirectly mainly from social interactions with others. The theory holds that everything that we undertake has been learned. Much of the learning in mankind stems from behavioral observations of other people's actions as well as from imagining the dire consequences of our actions (Wren, 1982). Similarly, it conceives learning as an evolutionary process that is aided by social imitations that are inherently linked to close contacts, coping, and adapting to a superior understanding of concepts coupled with the role of model behavior. It postulates and recognizes that aggressive behaviors may be conceived by an individual during the growth period or can be emulated through practice from bad influencers (Belanger, 2011). Relating to behavior, the theory mainly focuses on both psychological and environmental factors as the greatest influence on a person's behavior. The theory is underpinned by several assumptions; that people learn mainly through observations and imitations; the rewards, especially punishment that may arise from one's actions pose direct effects on both learning and observations; that behaviors are influenced by the processes of meditation and that change is not necessarily brought by learning. Therefore, the theory comprehends violence as an emulated mechanism that is carried through observations or experience. Additionally, the intergenerational passage of violence remains the critical component of the social learning theory.

III. METHODOLOGY

The study was conducted in Nyeri County, Kenya. The study employed descriptive and evaluative research designs. The study was guided by the frustration-aggression theory by Berkowitz 1960s, as well as the social learning theory by Bandura 1986. The study population included 384 household heads, 12 assistant chiefs, 15 police officers, 30 faith-based organizations, 12 COBs/NGOs, 3 sub-county administrators, and 20 village elders. This makes a total of 476 respondents. The respondents were sampled using purposive, simple random and census techniques.

Data was collected using questionnaires, interviews, focus group discussions, and observations. Qualitative data from interviews, FGDs and open-ended questionnaires was analyzed thematically and presented using thematic narrations and verbatims.

The study focused on Nyeri County, Kenya, with an objective of examining the socio-cultural factors influencing domestic violence in the study area. The study covered all the Sub-Counties in Nyeri County. The periodization of the research study dates from 2010 to 2022. The year 2010 is selected because it is the time the media started reporting many cases of domestic violence in Nyeri County. In addition, 2022 was selected because it is the year the study was conducted and cases are continuously being reported. The span of 10 years gave answers to the research questions. Data collection was conducted from the month of March, 2022 to September, 2022.

IV. FINDINGS & DISCUSSION

4.1 Response Rate

In this study, 384 questionnaires were distributed among household heads, and all the 384 were returned. 12 assistant chiefs, 15 police officers, 30 faith-based organizations, 12 COBs/NGOs, 3 sub-county administrators were interviewed. Similarly, 20 village elders participated in focus group discussions. This indicates that there was 100% response rate.

4.2 Time When Domestic Violence Increases

To understand the extent of domestic violence, the study sought to investigate when domestic violence cases are higher. Thus, household heads were treated to an open-ended question requiring them to write when the domestic violence cases are higher than normal times. The results are discussed in this section together with responses from other key respondents.

4.3 During Coffee and Tea Bonuses

When farmers in Nyeri receive their bonuses, domestic violence tends to increase especially between the husband and wife in the house. This is mainly caused by disagreements that tend to occur while budgeting for the money received. As the head of the house, men feel that it is their mandate to manage the money and, women should wait to be given what to use. On the other hand, women feel that men are unfair as they tend to misappropriate their money luxuriously. It is worth noting that most of the time, children end up suffering when the bonuses are misused and at times they are involved in the commotion as well. In other scenarios, the violence might escalate and lead to divorce. In an interview with a church elder it was pointed out that:

> *Most domestic fights in this area occur during tea and coffee bonus time. Bonuses have been a major cause of domestic violence, especially in the area where coffee and tea are mainly planted. Like here in Othaya, we recorded more than twenty-three cases of domestic violence in last week because of the mini bonus. It is interesting to note that mini-bonuses were paid on last week on Monday. Most of those conflicts were based on accusations that men in the house are misusing money with commercial sex workers who have come from other counties to get their share. Women feel that they are the ones who will suffer after their husbands have misappropriated the funds. I found it reasonable because women and their children take part in maintaining their coffee and tea, but it is unfortunate that once the bonuses are paid, they get nothing. However, men defend themselves on the ground that they are the heads of the family, and therefore, their decision is final* (Interview with religious leader, 8th April, 2022).

The area assistant chief agreed with the church elder on matters to do with domestic violence which tend to occur after the payment of bonuses. The assistant chief stated that:

> *It has been hectic for us because we have to deal with ever-escalating cases of domestic violence especially when bonuses are issued. Sometimes we even have to seek the intervention of religious leaders who play a pivotal role in eradicating and solving domestic violence. This week we have a case where a woman received a bonus and used all the money to take her mother to the hospital against the will of her husband. When the husband realized that he became violent which led to a serious injury to the woman who up today is still hospitalized. We are still expecting more such cases because most of the tea factories are still issuing bonuses to their farmers* (Interview with assistant chief, 8th April, 2022).

Bonuses have been identified to be one of the causes of domestic violence not only in Nyeri but also in other counties such as Kiambu, Murang'a, Kirinyaga and Kericho. According to Mwayuli et al., (2019), in Kiambu, domestic violence has been a norm when tea bonuses are shared. Most of such occurrences are caused by bonuses that are issued at a specific time in the area. The whole family takes part in their farm maintenance but when the bonuses are paid, they do not get their share, which leads to disagreement (Mwayuli et al., 2019). The disagreement might lead to domestic violence which is manifested through different dimensions. Some other times, divorces might occur.

3.2 Back to School Season

There have been conflicts in the families when a lot of money is required to take children to school. As providers, men are usually under a lot of pressure, as they have to cough a lot from their pockets. In most families, both the man and the woman contribute something. As a result of the financial strain, couple disagreements tend to occur. The spouses blame each other for the financial strain. In Nyeri County where majority of the residents are farmers, conflicts have been high when children are going back to school.

In addition to that, there is a lot of confusion about the current education system in Kenya. The confusion is caused by the occurrence of the Covid-19 pandemic and the introduction of Competence Based Curriculum (CBC). Covid-19 altered the normal academic year and in effort to cover up time lost during the quarantine period, the ministry of education tended to shorten terms. With the short academic term that means parents had a short duration of time to look for school fees. Lack of school fees led to disagreements in most of the homesteads in Nyeri County. While interviewing the Nyeri sub-county administrator it was revealed that:

School fee is becoming a big challenge in most families due to joblessness caused by Covid-19. We have received a big number of parents who opt to seek our intervention as they want us to support them financially so that they can afford to pay fees. On Monday I received a case where parents were fighting. When we made a follow-up in the process of trying to solve the conflict, we realized that the father has been unable to pay fees for his child. The child has been at home for a whole week and this was so stressful for the mother. The mother felt that the father was not doing enough to provide basic things such as school fees to his children. Domestic violence related to stress that occurs during back to school is increasing (Interview with sub-county administrators, 21st March, 2022).

In an interview with the NGO leader, he echoed the words of the Nyeri sub-county administrator. The NGO leader had the following to say about back to school period:

Back to school period is so stressful for most of the parents because schools expect students to report back and have to pay a certain amount of money. As an NGO that works toward eradicating domestic violence, we have established that during back to school period, domestic violence tends to be rampant. Currently, we have over fifteen cases that are under investigation. We are also monitoring the health condition of the man who came home drunk and had not paid fees to his children. The woman was agitated when her man who came home drunk yet their children had not yet reported back to school. They had a fight, and the man was severely injured (Interview with NGO leader, 1st June 2022).

According to Mwangi (2017), most of the domestic violence in Nyeri is related to the inability of the parents to meet the daily needs of their children. School fees are one of those basic needs that fathers are expected to cater for. However, it is becoming difficult for them due to the high rate of unemployment. On the other hand, Nyambura (2014) notes blame games between the father and mother in the homes exist when it comes to school fees payment with neither agreeing to take responsibility. In case one of the parents is unable to pay the fee as expected the blaming game tends to emerge and later escalates to domestic violence.

4.4 Festive Season

During the festive season like Christmas, most families spend a lot as it is time to come together for celebrations. The expectation in every family is very high as the head of the family must provide for them. In a situation where the father cannot afford to cater for the festive season, many women tend to feel like he has failed as the head of the family. During the festive season, a high number of domestic violence cases is reported due to the disagreement that occurs in families more so in relation to budgeting.

While I was interviewing a CBO leader, he outlined that:

Being the leader of a community-based organization, I can testify that there are high cases of domestic violence during the festive season. As we are talking, I am handling a case of a church elder from Chinga who was assaulted by his wife on Christmas day after he stated that he has no money to facilitate the Christmas festive. It all started with an insult between the elder and his wife and later ended up being a tragic scenario that led to serious injuries. It was reported that the woman poured kerosene on her husband and set him on fire and later disappeared. The woman is still on the run even though the police are still investigating the occurrence (Interview with CBO head, May 19, 2022).

A youth leader agreed with the CBO leader on matters to do with the occurrence of domestic violence during the festive season. The youth leaders had the following to say:

As a youth leader, I have seen the pattern in which domestic violence occurs in this village. Every year, there is a tendency for domestic violence to increase during December which is the main festive season among the Nyeri residents. For example, at the beginning of the last December, I had a case where the parents of a certain boy who was to be circumcised were having a conflict. The man stated that he cannot facilitate financially the circumcision of his son until the mother proves that the son belonged to him biologically. The mother took that as a claim of infidelity on

her part and she lost her control and attacked the man causing very serious injuries. Such cases are common in the region because some men are running away from their responsibilities (Interview with CBO head, May 19, 2022).

Mutahi (2017) states that the December festive seasons are critical as every family has an intention of creating memories out of it. Most important is the passage of rites such as circumcision. Such activities come with a lot of pressure and might be the root cause of domestic violence in case parents were not prepared for it financially (Mutahi, 2017). On the other hand, there are men who totally fail to take their responsibilities on the ground that leaves all responsibilities to the helpless mother. Mwangi (2017) adds that even though the role of circumcising was a docket bestowed on men, most of them in Nyeri have neglected that role as they mostly spend their time in clubs drinking. Mothers are left feeling helpless as they know nothing about circumcision and in that case, they end up expressing their disappointment to their men violently. This becomes a trigger to domestic violence.

4.5 During Traditional Functions

Generally, traditional occasions are one of the pivotal aspects of the African culture. Regardless of the modernity being witnessed in the current society, there are some cultural practices that are still held so dearly in the heart of many residents from Nyeri. Such cultures include activities such as dowry payments. It is worth noting that dowry payment is a role solely left to men in most African cultures. As such, Nyeri county is an epitome of places exhibiting such a cultural expectation. However, due to the high level of poverty and the influence of western culture, most men are no longer taking some traditions such as dowry payment seriously.

During an interview with a police officer, he stated the following.

> *Ulipaji wa mahari umefifia mno katika jamii ya leo. Kwa pande nyingine wazazi wamefanya ulipaji mahari kitega uchumi kwao. Hii inamaanisha ya kwamba kuna mgogoro unazuka baina ya mume anafaa kulipa mahari na wazazi wa msichana. Kwa mara nyingi utapata wazazi wa msichana hutumia mtoto wao kuwaitishia mahari kutoka kwa mumewe. Hili jambo huleta mgogoro baina ya bwana na mke kwenye nyumba. Tafaruku kama hii huweza kusambaratisha jamii na kuleta vita na kisha baadae utengano (Payment of dowry in current society has decreased while on the other hand, the in-laws have commercialized it. This creates a disagreement between men and their in-laws. In some scenarios, parents use their daughters to demand dowry from their husbands thus leading to conflict in the house. Due to this kind of pressure, some men tend to become violent, especially in situations where they cannot afford to pay dowry. Domestic violence might escalate and later end in divorce*.) (Interview with police officer, 2nd April, 2022).

On another occasion where a religious leader was interviewed, he pointed out that:

> *As a church we find it difficult to solve the cases that are related to traditions, more so on dowry payment. We have experienced domestic violence where dowry paid is not shared equally among the parents and this affects even the marriage of the young couples. A good example is a case where a dowry was paid and the father of daughter went to the nearby town and spent all of it with his friends. The mother became furious and stated that her daughter will not be married. The man as the head of the house, on the other hand, claimed that his orders are final and that her daughter must be married as planned. The man became violent and broke the hand of his wife and later chased her away. We intervened and we are still trying to sort that issue* (Interview with religious leader, 18th April, 2022).

Although change is inevitable, dowry payment has become the most constant thing in modern society. Due to benefits related to dowry payment parents have made it mandatory regardless of the economic situation of the spouses (Mwangi, 2017). In a situation where one is unable to pay dowry the in-laws feel like they have been robbed and this might create a grudge not only between the parents of both sides but also between the couples. Such grudges provide a basic platform where hatred and disrespect can thrive. Lack of respect in home especially to the men prompts them to be violent.

4.6 Drought Season

Recently, drought has become a common thing in Kenya whereby it comes with a lot of suffering to families. Some parts of Nyeri County, especially Kieni Sub-county, often experience severe drought which means individuals have to go for long distances to get water or even food. This implies that the effort of every family member is needed during the time of drought for survival. Any slight misunderstanding caused by the effects of drought such as lack of water, pasture for the animals or even food for the family can therefore become a trigger to domestic violence. Just like any other male-dominated society, most of the men in Nyeri opt to assume that it is the role of women to prepare meals and fetch water to be used at home. On the other hand, women feel that they deserve to be assisted since everyone in the family needs water and food and should therefore engage in the efforts put towards getting the same. Such a misunderstanding becomes an epitomical point that leads to domestic violence.

In an FGD with village elders, one of them commented that:

> *During the time of drought, women must walk for a long distance to look for water. Some even have to carry their babies with them because they have no one to help them as men consider such roles as traditionally female-oriented. Other than searching for water, women have taken the role of feeding their animals as well as preparing meals for the family. This entire role is done by women with men offering little or no assistance. As a result, women tend to be mean and as a way of saving themselves from the tedious role of fetching water, they might try to economize it, something that anger most of the men. This creates tension because some men in Nyeri consider orders from women as disrespect* (FGD with village elders, 4 April, 2022).

In another FGD with a village elder he conquered his colleague as he stated the following:

> *We have had a case where domestic violence occurred between a husband and a wife as a result of what they said was the misuse of the water. The wife claimed that she has been walking for a long distance carrying a baby to search for water to be used at home while the man was in the local town passing time with other men. When the man came home, he used the water in watering some vegetables in the kitchen garden. This angered the wife and she decided to confront her husband. Luckily, neighbors came to separate them before any injuries occurred. Such occurrences tend to increase every time when there is a prolonged drought* (FGD with village elder, 8th March, 2022).

Nowadays, the weather pattern has become unpredictable. This means that one cannot tell when the drought will come thus the occurrence catches most of the people unaware. Scrutinized from the family level, this affects mostly women as they are house managers as per the African culture (Muluneh *et al*., 2019). Men become unbothered but at the same time expect women in their homes to sort everything. Women, on the other hand, are trying to pressurize men as a way of ensuring that they offer assistance to curb the impacts of drought. Nyeri men do not take such pressures lightly. Hence, domestic violence becomes an absolute outcome.

V. CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

The study sought to investigate the time when domestic violence is on the rise in Nyeri County. The results indicated that there are increased cases during coffee and tea bonuses, during the back-to-school period, during the festive season, during the traditional festive season, and when the county is experiencing drought. During the coffee bonuses, men misuse the funds that the family worked hard to earn for a whole year, and this brings conflicts. When children are going back to school, there is financial strain, and this increases the probability of disputes at the family level. Traditional festive seasons such as circumcision also bring disagreements, mostly because of the change of gender roles and financial demands during these periods.

5.2 Recommendations

It is recommended that both men and women should be empowered in Nyeri County. Empowering one gender makes the other vulnerable, especially in a community that has been fighting over dominance between men and women. The woman in Nyeri is more empowered by socio-cultural dynamics, thereby making the man feel endangered.

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Income and Transport Cost: Multiple Responses to Tourists Arrivals in Zanzibar

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ABSTRACT

The tourism sector plays an important role in the Zanzibar economy and continues to be a leading sector and the main contributor to the national GDP but this sector is faced unprecedented challenges and an existential threat from the impact of the COVID-19 virus. In this regard, this study aimed to analyse the determinants of international tourism demand from the top ten tourist's origin countries to Zanzibar from 2005 to 2021 and to quantify their influence. In this study Gray's travel motivation theory, Crompton's motivations for pleasure vacation theory and Dann's theory of push and pull factors were used to analyse behaviour of tourists and choice of destination. The study adopted panel data analysis and used long, strong balanced secondary panel data extracted from various sources to estimate the impact of GDP per capita as proxy of income, exchange rates, tourism infrastructure, price of crude oil as proxy of transport cost, consumer price indices adjusted by exchange rates as proxy of relative price and dummy variables to capture effects of Covid-19 and economic recession of 2008 on number of tourist arrivals as proxy for tourism demand in Zanzibar. Regression results of fixed effect model suggested that; at the 0.05 significance level, a one unit increase in GDP per capita results into 1.999 units increase in number of tourist arrivals and one unit increase in transport cost would decrease tourist arrivals by 64.991 units. Coefficients value of dummy variable COVID-19 implies that, on average, the number of tourist arrivals decreased on the year of occurrence of COVID-19 pandemic by 9,539 units. The coefficients of other independent variables exchange rates, tourism infrastructure, relative price and economic recession of 2008 bear the right sign even though they were not statistically significant at 0.05 levels. This study recommends that, the government and other stakeholders to work on marketing Zanzibar tourism product to the high-income countries and to work on reducing transport cost from origin countries to Zanzibar.

Keywords: Fixed Effect Model, Income, Tourist Arrivals, Transport Cost, Zanzibar

I. INTRODUCTION

The gravity model indicates generally that there is a link between the desire for a vacation and personal income. Many researchers agreed on the fact that the tourism sector without any barrier leads to gross domestic product (GDP) growth and the creation of employment (Rasool et al., 2021; Kyara et al., 2021; Wamboye et al., 2020; World Travel & Tourism Council [WTTC], 2020; Manzoor et al., 2019; Fourie & Santana-Gallego, 2011). It is further argued that, to have a successful destination, the secret is to approach the right target market and to distinguish the tourism products and establish local partnerships between the public and private sector (Buhalis, 2000).

According to the United Nation Statistics Division [UNSD], (2010), tourism is defined as "the activity of a traveller taking a trip to a main destination outside his/her usual environment, for less than a year, for any main purpose (business, leisure or other personal purpose) other than to be employed by a resident entity in the country or place visited". Tourism demand can be defined as "the willingness and ability of consumers to buy different amounts of tourism product at different prices during any one period". International tourism demand can be measured in term of tourist arrivals to a destination country, number of nights spent by a tourist in a destination country and total expenditure of a tourist at a destination country (Song et al., 2019).

Gross domestic product is the most important explanatory variable in modelling tourism demand, it is used to measure the amount of money earned per person in a country. Income affects the ability to pay for the overseas travel (Naudé & Saayman, 2005) and play a vital part in destination choice. Despite recent development in air transport in Zanzibar especially construction of new passenger's terminal three building and establishment of chartered flights between Zanzibar and some of its source markets but still the cost of air transport is high which might decrease the possibility of tourists choosing Zanzibar as their destination. Transport cost between origin country and destination country has been found to negatively impact flow of tourists (Garin-Munoz and Montero-Martin, 2007; Archibald et al., 2008; Dudokh, 2008; Vítová et al., 2019).

1.1 Statement of the Problem

Tourism sector is highly integrated and complex in nature, hence it is important to understand behaviour and needs of tourists as well as its structure. Prior to the mid-1980, tourism in Zanzibar barely existed and arrivals of international visitors were in the interest of fellowship and there were few hotels that could offer accommodation of any quality for visitors (Sharpley & Ussi, 2014). Since early 1980s, tourist arrivals in Zanzibar have increased gradually, from roughly average of 26,000 tourists per year in the late 1980's to an average of 400,000 tourists per year in the late 2020's. Even number of hotels and guest houses has increased rapidly, example; in a year 2021, there were 683 hotels and guest houses available compared with 620 available in a year 2020 (Office of Chief Government Statistician [OCGS], 2022).

Despite the availability of studies which provide in-depth information's on impacts of tourism on Zanzibar economy and surrounding societies (World Bank [WB], 2022; Hafidh & Rashid, 2021; United Nation International Children's Emergency Fund [UNICEF], 2018; Sharpley & Ussi, 2014), the study is set to provides more insight on the performance of both economic variables, such as income and exchange rates; and non-economic variables, such as global health crisis; which contributed to tourist arrivals.

1.2 Research Objective

The specific objective of this study is to analyse determinants for the tourist arrivals in Zanzibar for the period 2005 to 2021.

II. LITERATURE REVIEW

2.1 Theoretical Review

Demand for tourism product is much influenced by tourist's income, tourism prices in destination and tourism price in substitute destinations (Wu et al., 2017). Tourist's income is expected to influence tourism demand in positive way; tourism price in destination country is expected to influence tourism demand negatively while tourism price in substitute destinations is expected to influence tourism demand in a mixed way.

2.1.1 Gray's Travel Motivation Theory

Gray (1970) in answering the big question of 'why do people travel?' suggested only two motives; one, wonderlust, which can be described as the desire to go from a known to an unknown place; and two, sunlust, which is to go to a place which can provide the traveller with specific amenities that does not exist in his or her usual place of residence. He added that, motives which determine their travel choices are pleasure, recreation, new experiences, shopping and cultural interest. This theory is relevant in understanding psychology, behaviour and personality of tourists.

2.1.2 Crompton's Motivations for Pleasure Vacation Theory

Crompton (1979) identified nine motives for selection of a destination. Socio-psychological (push motives) factors which include escape from a perceived mundane environment, exploration and evaluation of self, relaxation, prestige, regression, enhancement of kinship relationships, and facilitation of social interaction. Other two factors (pull motives) are novelty and education. He concluded that, tourism industry must pay greater attention to socio-psychological motives in developing product and promotion strategies. The first seven explains desire for travel and the rest explain the actual destination choice.

2.1.3 Dann Theory of Push and Pull Factors

Dann (1981) identified seven fundamentals of tourist's motivations to travel as "a response to what is lacking yet desired, destination pull in response to motivational push, motivation as fantasy, motivation as classified purpose, motivational typologies, motivation and tourist experience, motivation as auto-definition and meaning". These two fundamentals motivate tourists to travel and be attracted to the desired destination, and they try to answer the big question of what are motives for people travel in term of individual and cultural conditioning.

2.2 Empirical Review

Wamboye et al. (2020) applied panel data model to investigate the relevant determinants of international tourism demand for Tanzania's top fifteen tourist sources countries during the 2000- 2016 period and found that income of tourists and infrastructure development in Tanzania are two main determinants of international tourism demand for Tanzania. They also found that increase in transportation cost and inflation has negatives effects on tourism demand. Naudé and Saayman (2005) applied panel data model to identify the determinants of tourism arrivals in forty-three African countries, taking into consideration tourist's country of origin. Results of their study strongly

suggest that "political stability, tourism infrastructure, marketing and information, and the level of development of destination country are key determinants of travel to Africa".

Kusni et al. (2013) investigated the significance influence of both economic and non-economic factors in determining tourism demand for Malaysia by tourists from the Organization for Economic Cooperation and Development (OECD) countries using panel data model. Findings of their study showed that demand of Malaysia tourism product is sensitive to price changes as variable relative price was found to be statistically significant. Permatasari and Esquivias (2020) employed a dynamic panel dataset of fifteen years from 2000 to 2014 to estimate the impact of selected variables on the total expenditure of travellers as a proxy for international tourism demand in Indonesia. Their study found that per capita income of tourists, relative prices and accommodation capacity have a positive impact on tourism expenditure while distance between origin country and Indonesia has negative impact on tourism expenditure.

Considering the changing structure of consumer preferences, Habibi and Abbasinejad (2011) estimated dynamic panel data model to identify and estimate factors for tourism demand in Malaysia from European countries using annual panel data span from 1998 to 2007 and found that income, accommodation capacity and political stability had positive effects on tourism demand in Malaysia. Mavrommati et al.,(2021) used panel data fixed effect model to investigate the determinants of international tourism demand for Greece from twenty-eight European and Non-European countries for the period from 1996 to 2015 and found out that cost of living for tourists in Greece, population and marketing expenses to promote tourism industry are statistically significant at level in explaining tourism demand for Greece.

III. METHODOLOGY

3.1 Model Specification and Data

Panel data model has been applied in studying relationship among various variables, Proenca and Soukiazis (2005) as cited in Permatasari and Esquivias (2020) explained that a combination of time series and cross-sectional data enables higher degrees of freedom in the estimation process, enabling dynamic specification, reducing multicollinearity effects, and providing more data information. Verbeek (2004) explained that the availability of repeated observations on the same elements allows specification and estimation of realistic and more complicated model than pure cross-section or time series models. In this study, the demand for tourism was measured in terms of the number of tourist arrivals. This study adopts regression model presented by Greene (2008).

The model is as follows:

$$Y_{it} = X'_{it}\beta + Z'_i\alpha + \varepsilon_{it} \quad \text{..... (i)}$$

Where;

X'_{it} are group of k regressors, β and α are parameters to be estimated, Z'_i contain a constant term and the observed panel data set;

$\{(y_{it}, x_{it})$: $1 \leq i \leq , 1 \leq t \leq T\}$

Now, the panel data model for this study took the following form;

$$TA_{it} = \alpha + \beta_1 GDPpci_{it} + \beta_2 ERTshs_{it} + \beta_3 TI_t + \beta_4 TransCost_t + \beta_5 RelativePrice_t + \beta_6 Cov19_t + \beta_7 GGR_t + u_{it} \quad \text{..... (ii)}$$

Where; TA_{it} = Number of tourist arrivals from origin country i to Zanzibar at time t; $GDPpci_{it}$ = Gross domestic product per capita of country i at time t; $ERTshs_{it}$ = Annual average exchange rates between origin country i and Tanzania at time t; TI_t = Tourism infrastructure in Zanzibar at time t; $TransCost_t$ = Price of crude oil at world market at time t proxy for transport cost; $RelativePrice_{it}$ = Relative price at time t adjusted by exchange rates between origin country i and Tanzania; $Cov19_t$ = COVID-19 cases at time t, taking value 1 in a year 2020 and value 0 otherwise; GGR_t = Global great recession at time t, taking value 1 in a year 2009 and value 0 otherwise; u_{it} = Error term, which are normally distributed with mean 0 and variance σ_u^2.

Various literature (Mavrommati et al.,2021; Permatasari and Esquivias, 2020; Wamboye et al.,2020; Kusni et al., 2013; Naudé and Saayman, 2005) emphasize the use of panel data in modelling tourism demand because of its capacity to examine individual specific effects and/or time effects in order to deal with heterogeneity that may or may not be observed (Park, 2011). That is, by panel data we may control for unobserved individual-specific or time-specific heterogeneity (Biørn, 2016).

This study used longitudinal strong balance panel data collected for the period of seventeen years from 2005 to 2021. Since the focus of this study was long-term relationships between tourist arrivals and its influencing factors longitudinal annual data was used. Data used in this study were number of international tourists arrived in Zanzibar from a year 2005 to a year 2021 from ten countries, namely; Italy, United States of America, the United Kingdom,

France, Germany, Australia, Kenya, South Africa, Netherlands and Belgium which were obtained from Zanzibar's Office of Chief Government Statistician's *year statistics abstract.*

3.2 Data Collection

Data of gross domestic product per capita of named ten origin countries from the year 2005 to 2021 were obtained from World Bank data portal (World Bank [WB], 2023). Data of exchange rates which are the average annual market rates of local currency against currencies of sampled tourist arrivals origin countries were obtained from United Nation Conference on Trade and Development. Data of tourism infrastructures which were measured in terms of number of guest houses, lodges and hotels were obtained from office of Zanzibar Commission for Tourism. Data of transport cost which was measured in term of price of crude oil at world market was obtained from organization of the petroleum exporting countries (OPEC) official website. Data of consumer price indices of origin countries were extracted from United Nation Conference on Trade and Development and Zanzibar consumer price indices data were obtained from office of chief government statistician (OCGS) of Zanzibar.

3.3 Unit of Measurements

Data measurements used in this study were deduced from theoretical and empirical findings and are summarized in

Table 1

Units of Measurements of Selected Variables

Name of variable	Description	Unit of measurement
Tourist arrivals	Number of tourist arrivals from origin country to Zanzibar	Number of tourists
GDP per capita	Gross domestic product per capita of countries of origin	Constant (2015) international dollars (PPP)
Exchange rates	Exchange rates between Tanzania and tourist original country	Tanzanian shillings
Tourism infrastructure	Number of hotels and guest houses in Zanzibar	Number of hotels and guest houses
Transport cost	Transport cost between original country and Zanzibar	Price of crude oil (in USD) at world market
Relative Price	Consumer price of Zanzibar divided by that of origin countries adjusted by exchanges rates.	CPI points adjusted by exchange rates
COVID-19	Dummy variable	Taking value 1 in a year 2020 and value 0 otherwise
Global great recession	Dummy variable	Taking value 1 in a year 2009 and value 0 otherwise

IV. FINDINGS & DISCUSSION

4.1 Descriptive Statistics

Summary statistics of tourist arrivals in Zanzibar shows that, a total of 4,457,375 tourists arrived in the time span of this study and 2,653,157 (59.52%) of them came from the top ten nations sampled and used for this study. This is presented in Table 2.

Table 2

Number of Tourists Arrived in Zanzibar from 2005 to 2021

Year	Top ten nations	Other nations	Total tourists	Tourists data bar
2005	99,292	26,151	125,443	
2006	108,783	28,328	137,111	
2007	113,605	29,678	143,283	
2008	98,919	29,521	128,440	
2009	104,398	30,521	134,919	
2010	103,970	28,866	132,836	
2011	131,681	43,386	175,067	
2012	122,675	46,548	169,223	
2013	132,840	48,461	181,301	
2014	187,233	124,658	311,891	
2015	180,452	113,791	294,243	
2016	214,672	161,570	376,242	
2017	241,295	192,179	433,474	
2018	270,631	250,178	520,809	
2019	298,395	239,869	538,264	
2020	109,131	151,513	260,644	
2021	135,185	259,000	394,185	
Total	2,653,157	1,804,218	4,457,375	N/A
% Share	59.52%	40.48%	100.00%	N/A

Country level statistics over the study period are presented in Table 3; Italy was a leading country with over 772,053 tourists arrived in Zanzibar from 2005 to 2021. This is almost two times more than the number of tourists from second placed Germany and third placed United Kingdom with 332,751 and 312,356 tourists respectively.

Table 3

Top Ten Tourist's Original Countries

S/No	Country	Total arrivals	S/No	Country	Total arrivals
1	Italy	772,053	6	South Africa	239,715
2	Germany	334,751	7	Netherland	129,581
3	UK	312,356	8	Kenya	124,914
4	USA	294,825	9	Belgium	93,928
5	France	266,262	10	Australia	84,772

4.1.1 Summary Statistics

Summary statistics of selected variables shows that; an average of 15,606 tourists arrived in Zanzibar between 2005 and 2021 from top ten countries. Median of number of tourist arrivals was 10,412 which is less than the mean and it implies that the distribution was right- skewed.

Table 4

Summary Statistics of Selected Variables

Variable	Observations	Mean	Std. Dev.	Median	Min	Max
Number of Tourist Arrivals	170	15,606.81	14,397.13	10,412	1407	61369
GDP per capita	170	35,731.01	17,688.66	40,221	1221.859	61855.52
Exchange Rates (Tshs)	170	1,662.162	909.049	1,841.55	14.942	3160.32
Tourism Infrastructure	160	28.125	33.713	29.5	-47	99
Transport Cost	170	71.816	22.430	69.1	40.76	109.45
Relative Price	170	0.007	0.037	0.0004	0.0002	0.476

4.2 Unit Roots Test

It was paramount to check the presence of unit root using Im-Pesaran-Shin panel unit root test with the null hypothesis that all panels contain a unit root, this hypothesis was rejected for all variables except for tourism infrastructure but the first difference of this variable was stationary.

Table 5

Im-Pesaran-Shin Unit Roots Test Results

Variable	t-bar statistics including time trend
Number of tourist arrivals	-2.145**
GDP per capita	-2.7148***
Exchange rates	-2.876***
Tourism infrastructure I(1)	-5.1908***
Transport cost	-2.3094***
Relative price	-1.40E+02***

*NOTE: *** p < 0.01, ** p < 0.05, * p < 0.1*

4.3 Covariance-Correlation Matrix

In correlation analysis of panel data, there are some of variables which were highly correlated; instance the exchange rates was positive and highly correlated with GDP per capita with coefficient of 0.729 while the correlation between relative price and GDP per capita is negative with coefficient of -0.3042. The principal diagonal of the covariance matrix shows the covariance between the same variable, such as the covariance of transport cost was 504.96.

Table 6

Covariance-Correlation Matrix Table for the Period 2005-2021

	Number of tourist arrivals	GDP per capita	Exchange rates	Tourism infrastructure	Transport Cost	Relative Price
Number of tourist arrivals	2.10E+08					
	1					
GDP per capita	3.50E+06	3.20E+08				
	0.0153	1				
Exchange rates	4.30E+06	1.20E+07	837426			
	0.333	0.729	1			
Tourism infrastructure	78235.7	18535.2	3085.57	1136.59		
	0.162	0.031	0.1	1		
Transport cost	-41103	-8782.58	-2072.81	47.213	504.96	
	-0.096	-0.014	-0.061	0.062	1	
Relative price	-26.997	-117.117	-5.763	-0.003	0.005	0.000086
	-0.114	-0.321	-0.304	-0.008	-0.0703	1

4.4 Panel Data Models Regression Results

The main objective of panel data modelling is to examine fixed effects and/ or random effects. Results of statistical tests; F-test and the Breusch and Pagan Lagrangian multiplier test of panel data model (ii) above shows that both fixed and random effects were statistically significant although results of Hausman specification test recommended that fixed effect model was the preferred model. Post estimation tests of heteroskedasticity and serial correlation provided in-depth consideration on which model was the best for this panel data. Verbeek (2004) explained that presence of heteroskedasticity and serial correlation implies that error terms in the model are no longer independently and identically distributed. Results presented in Table 7 conclude the presence of heteroskedasticity and serial correlation in the estimated fixed effect model.

Table 7
Fixed Effect Model Post Estimation Test Results

Heteroskedasticity test	Chi-square (10) = 2468.22
	Probability > chi-square = 0.000
Serial correlation test	F (1, 9) = 89.486
	Probability > F = 0.000

4.5 Fixed Effect Model with Driscoll and Kraay Standard Errors

The Driscoll and Kraay standard errors are used to calculate robust standard errors in regression models and are calculated by estimating a long-run variance-covariance matrix of the errors, which allows for the presence of autocorrelation and heteroskedasticity. Presences of serial correlation and heteroskedasticity call for the fixed effects regression model with Driscoll and Kraay's standard errors (Mehmood & Mustafa, 2014). According to Anser et al., (2020), Driscoll and Kraay's standard errors are applicable in the presence of heteroskedasticity and serial correlation.

The coefficient of an independent variable gross domestic product per capita bear positive sign and was statistically significant at 95% level which suggest that gross domestic product per capita was one of the main determinants of tourist arrivals in Zanzibar. In term of size of effect; a one unit increase in gross domestic product per capita; number of tourist arrivals was expected to increase by 1.999 units. This result is consistent with findings from Habibi & Abbasinejad, 2011; Permatasari & Esquivias, 2020; Wamboye et al., 2020 which all included gross domestic product per capita as independent variable.

Exchange rate between the origin country currency and Tanzania's currency in favour of origin country automatically decrease cost of living in Zanzibar. The coefficient value of exchange rate suggests that; one unit increase in exchange rate in favour of origin country would increase tourist arrivals by 5.601 units and this result is supported by previous findings of Archibald et al., (2008).

Table 8
Regression Estimated Results

	FE Estimation	FE Estimation with Driscoll and Kraay SE
Variable	Coefficients (SE)	Coefficient (SE)
GDP per capita	1.999** (0.501)	1.999** (0.329)
Exchange rates (Tshs)	5.601** (2.679)	5.601 (4.105)
Tourism infrastructure	17.236 (20.198)	17.236 (21.367)
Transport cost	-64.991** (31.424)	-64.991** (24.627)
Relative price	-74066.13 (502096.5)	-74066.13 (150177.1)
COVID-19	-9539.658** (2719.637)	-9539.658** (2078.668)
Global great recession	-1893.856 (2823.572)	-1893.856 (1164.728)
Diagnostic statistics		
	R Squared = 0.403	R Squared = 0.403
	F(7,143) = 13.81	Wald $\chi2(7) = 8.01$
	p-val> F = 0.000	p-val>$\chi2 = 0.004$

*NOTE: *** p < 0.01, ** p < 0.05, * p < 0.1, Standard errors in parentheses*

Accommodation capacity as measured in term of number hotels and guest houses has a strong impact on attracting number of tourist arrivals. Regression results indicated positive sign as expected though not statistically significant. This result is supported by tourism statistical release of the office of chief government statistician which shows that bed occupancy average rate is 78.1 percent as of July 2022; meaning that hotel facilities in Zanzibar are satisfactory in term of quantity and number of available rooms even in peak season. Findings of similar findings from Habibi & Abbasinejad, 2011; Permatasari & Esquivias, 2020 and Naudé & Saayman, 2005 revealed a positive sign as expected.

Transportation cost was expected to have negative influences on tourist decision on choosing the travel destination as tourist tend to maximize satisfaction at lower cost. Since most of top original countries for Zanzibar tourism products are from North America and Europe, the results of this variable bear the right sign. The regression results of the coefficient associated with transport cost showed that, a one unit increase in price of crude oil in the world market would decrease tourist arrivals by 64.991 units. In order to decrease the negative effects of transport cost on tourism, Ulucak et al., (2020) suggest that policymakers and marketing officers should aim to attract more tourists from neighbouring countries.

As expected, the coefficient sign of relative price was negative though not statistically significant at level which show that it was inversely proportional with number of tourist arrivals. The negative sign of relative price means that the higher the cost of living in Zanzibar relative to the original country, the lower the demand of Zanzibar tourism products. The result is consistent with finding of Dudokh, 2008; Mavrommati et al., 2021 and Permatasari & Esquivias, 2020.

World Health Organization [WHO], 2020 declared COVID-19 outbreak as a global health emergency of international concern on 30 January 2020. The pandemic had highly impacted tourism industry especially airlines and hotel business all over the world (Foo et al., 2021). Total number of confirmed covid-19 cases in Tanzania was 43,078 and number of deaths was 846 (World Health Organization [WHO], 2023) as of 17/08/2023.(World Bank [WB] 2022) quoted that "In Zanzibar, the impact of the COVID-19 crisis on tourist arrivals and subsequent drop in jobs and business activity is likely to have mostly affected informal enterprises, women, and low-skilled workers". Generally; COVID-19 resulted into decline in real GDP growth of Tanzania from 6.9 percent in 2019 to 4.8 percent in 2020 which caused by "regional trade disruptions and contraction in tourism and related sectors" (Henseler et al., 2022). Coefficients sign of individual-invariant dummy variable covid-19 to capture time-specific effect of COVID-19 pandemic were negative as expected and they were statistically significant at 95% level which implies negative correlation between covid-19 and tourists' arrivals.

The coefficients of COVID-19 represent the average change in the number of tourist arrivals associated with occurrence of COVID-19, while holding constant all other determinants. Coefficients value of dummy variable COVID-19 where 1 represented a year with COVID-19 and 0 represented a year with no COVID-19 implies that, on average, the number of tourist arrivals decreased on the year of occurrence of OVID-19 pandemic by 9,539 tourists. Descriptive statistics suggested a significant decline in tourist arrivals in a year 2020 by 51.58 per cent, this could be the effect of a high degree of concentration of tourist arrivals from its top ten markets and these countries were among those most affected by COVID-19 pandemic. Finding of this study is consistency with findings of Kusni et al., (2013) and findings of Jaipuria et al., (2021) who observed decrease of tourists arrived in India by 67.66 per cent in the first three months of 2020.

Results also indicated that global great recession had negative relationship on tourist arrivals; however, it is not statistically significant at level. This result of dummy variable global great recession of 2009 suggests slightly decline in tourist arrivals by 1.54 per cent in 2010, a year after the 2009 global great recession.

V. CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

Factors for tourist arrivals to Zanzibar may be hard to determine for both government and other tourism sector stakeholders. This study examined how income, transport cost, hotel capacity, tourism price and exchange rates affect tourists arriving in Zanzibar. Using panel data and a fixed effect model, the results shows that income of tourist and transport cost are key determinants of tourists arriving in Zanzibar. Also, dummy variables included in this study to capture effects of COVID-19 and global great recession, and the results show a negative relationship between number of tourist arrivals and occurrence of COVID-19 pandemic and this can vividly be seen on number of tourists arrived in Zanzibar in a year 2020 and 2021.

5.2 Recommendations

It is thus recommended the revolutionary Government of Zanzibar and other tourism sector stakeholders to work together in making Zanzibar tourism sector more competitive by marketing its tourism products to high income countries. Transport cost was found to have negative impact on tourists arriving in Zanzibar, so the Zanzibar commission for tourism should work toward convincing airlines companies to introduced chattered flights between top tourist's origin countries to Zanzibar so as to reduce transport cost and flight time which will encourage tourists aged 65 and above to visit Zanzibar. Regarding the cost of transportation, it is further suggested that the Zanzibar commission for tourism to advertise its tourism product to neighbouring countries which most of them they don't have luxurious of enjoying beach tourism.

5.3 Declaration

The authors declare that they have no known contending financial benefits or personal associations that could have resulted to influence the work reported in this study.

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Exploring the Prevalence and Patterns of Media Exposure and Use among Adolescent Generation Z in Secondary Schools in Western Kenya: A Focus on Kakamega, Kisii, and Siaya Counties

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ABSTRACT

The purpose of this study was to explore media exposure patterns among school-going Generation Z adolescents in Western Kenya, specifically in Kakamega, Kisii, and Siaya Counties. By providing crucial insights into the media habits of these adolescents, the study aims to enhance understanding of youth media behavior and its developmental impacts. The findings could inform educational policies, parental guidance, content regulations, and health initiatives to address issues such as screen addiction and cyberbullying. The study applied Erik Erikson's Psychosocial Theory of Development to explore the media habits of adolescents in an exploratory context. Using descriptive and correlational research designs that included questionnaires, interviews, focus group discussions, and content analysis, data was collected from a sample of 384 learners drawn from a population of 190,555 learners using a stratified random sampling method. Additionally, 56 parents and 48 teachers were selected based on the principle of data saturation. A pilot study conducted in Vihiga County with 42 learners yielded a Cronbach's Alpha coefficient of 0.87. Expert faculty members assessed the instruments for face validity. The study adhered to the Kenya Data Protection Act of 2019, ensuring ethical procedures such as informed consent from principals, participant anonymity, and thorough training for research assistants. The data was analyzed using a mixed-methods approach that combined descriptive statistics for the quantitative data and thematic analysis for the qualitative insights. Findings reveal that Generation Z adolescents have widespread access to media, primarily engaging with social media (38%), with at least 35% using media for over 6 hours, notably on social media and mobile phones. A significant portion (35%) of their media consumption occurs without parental supervision, prominently on social media (40.7%) and mobile phones (28.2%). Media serves dual purposes for these adolescents, with radio and television primarily used for education (32% and 31.8%), while mobile phones and social media are predominantly for entertainment (34.6% and 25%). The study concluded that social media is widely used among adolescent Generation Z learners, with substantial daily engagement and minimal parental supervision. This study recommends implementing digital literacy programs, setting screen time limits, and integrating educational content into popular digital platforms, particularly targeting adolescent Generation Z. Schools should collaborate with content creators to enhance learning through engaging, interactive materials.

Keywords: Adolescents, Generation Z, Media Exposure, Parental Oversight, Social Media

I. INTRODUCTION

Generation Z, born between the mid-1990s and early 2010s, represents a shift from traditional to digital media. As digital natives, they have grown up immersed in the rapid evolution of technology, moving beyond traditional platforms like television and radio to embrace a diverse range of digital media. The term "media" refers to electronic or electromechanical technologies used to store and distribute information to users (Delfanti & Arvidsson, 2019). Traditionally, this included television and radio, but today it also encompasses mobile phones, video games, social media platforms, and software that enable data access, retrieval, storage, transmission, and manipulation. These digital natives are not only early adopters but also innovators, using smartphones, social media, and online communities to express themselves and forge connections on a global scale.

Studies indicate a significant increase in electronic media accessibility among youths, both at home and in school, particularly in technologically advanced countries like the United States, where Generation Z exemplifies this trend. For example, according to the latest available data, every American adolescent of Gen Z will have viewed approximately 500,000 TV adverts by the time he/she leaves school (Kaur et al., 2019). Furthermore, Hurwitz et al., (2020) point out that young adults in the US spend double the time watching television than they spend reading books. These observations underscore how the constant availability of electronic media shapes Generation Z's consumption

habits, identity formation, and cultural integration (Serbanescu, 2022). Their ongoing interaction with digital media profoundly influences the way they learn, communicate, and perceive the world (Bourke, 2019).

A growing body of research highlights Generation Z's increasing dependence on digital media and social platforms, a trend that spans across different countries and cultures. A global analysis of 19 European Union countries showed that 80% of school-age Gen Z use smartphones to access the internet daily (Smahel et al., 2020). Moreover, the study revealed that internet use among this generation has increased significantly over the years to a stage where it has almost doubled since 2010 among this group. Donelle et al,(2021) conducted a study that showed that 57% of the Canadian adolescent Gen Z have embraced the use of social media platforms such as YouTube, Snapchat, and Facebook. More specifically, 79%, 67%, and 33% of the respondents use YouTube, Snapchat, and Facebook, respectively. A cross-sectional study of young people in Austria in 2022 found that as many as 77% of young Austrians use TikTok daily as reported by the Youth Internet Monitor (Sackl-Sharif et al., 2022). Similarly, the 2021 Internet Use and Social Media Report in Denmark revealed that almost all (87%) Gen Zs aged 12 to 18 had at least one social media account, half of them were active on two to five platforms, and the rest were on six to eight (O'Neill, 2023).

In Africa, particularly Sub-Saharan Africa, patterns of mobile phone access among Generation Z adolescents reveal a distinct reliance on shared connectivity a opposed to regions like Europe and the United States. For instance, Wang et al (2022) conducted a survey that determined the level of access to smart devices for Gen Z adolescents in Sub-Saharan Africa and found that possession of mobile phones was much lower for this group than for young adults in America or European countries. Specifically, the findings indicated the rates in Tanzania, Ethiopia, Sudan, and Burkina Faso to be 3%, 13%, 36%, and 40% respectively. However, the rate of access to any phone, whether owned or not was 56%, 80%, 95%, and 70% in the same countries respectively. This suggests that in Sub-Saharan Africa, usage of mobile phones does not solely depend on personal ownership. Rather, there is evidence of prevalent patterns of shared access and connectivity facilitated by these devices within the region.

In Kenya, the rapid growth of internet and technology use is evident, with internet penetration reaching 87.2%, including 46.8 million mobile data users and 7 million Facebook users (Cowling, 2024). Popular social media platforms among Kenyans include WhatsApp, X Space, Instagram, TikTok, and Facebook, indicating a high level of digital engagement. However, access to Information and Communication Technologies (ICTs) and other media platforms remains uneven, with rural areas, the Coastal Region, and the North Eastern parts of the country experiencing significantly lower levels of access (Mwita, 2021). Despite these regional disparities, internet use has surged; in January 2020 alone, the number of internet users in Kenya rose by 16%, reaching 22.86 million and achieving a 43% internet penetration rate (Kemp, 2020).

Among Generation Z youth aged 10 to 21, a 2020 survey revealed that 32% had internet accounts despite age restrictions that require users to be at least 18. Furthermore, 16% of these adolescents accessed adult-themed content, and 5% participated in online gambling (Kemp, 2020).

Residents of Kakamega, Kisii, and Siaya counties have significantly greater access to media technology than their neighbouring counties. For instance, the 2019 Population and Housing Census revealed that 45.8% of the population aged 3 and above in Siaya County owned mobile phones, a higher percentage than those in the neighboring counties of Busia (37.8%), Kakamega (41.4%), Vihiga (45.4%) and Homabay (41.5). Furthermore, household access to radio (65.5%), computer (4.2%), television (28.3%) and internet (8.3%) in Siaya county was higher than that of neighboring Busia (54.5%; 3.7%; 27.2%; 7.4% respectively) as reported by Kenya National Beaureau of Statistics (2019). In Kisii, the same census report indicates that among the population aged years and above, ownership of mobile phones (44.6%) and household access to computer (5%), TV (30.7%), and internet (8.8%) was higher than that of neighboring Narok (32.4%; 2.7%; 19.3%; 6.9%) and Migori (36.5%; 4.2%; 28.1%; 3.9%) Counties. The same trend was observed in Kakamega County. Worth noting too is that access to radio in the three counties (Kakamega, Kisii, and Siaya), exceeded the national one.

It is important to note that the statistics pertaining to media use in Kakamega, Kisii, and Siaya Counties are primarily derived from census reports. These reports provide a broad overview of demographics and technology access, suggesting the extent of media usage within households. Access to media is a recognized predictor of media usage patterns (Cabello et al., 2018). However, census data typically encompasses the entire population rather than relying on sample-based methods, which limits insights into the nuanced behaviors, attitudes, and social implications of media use among adolescents in this specific region. This lack of detailed exploration leaves researchers and policymakers without critical insights into how media exposure influences the psychosocial development, cultural identity, educational experiences, and social dynamics of young people in Western Kenya. Furthermore, there is a significant absence of focused studies addressing media access levels specifically among adolescent Generation Z in these areas. Consequently, this study aims to evaluate the prevalence and patterns of media exposure and use among

adolescent Gen Z learners in the Western Region of Kenya, specifically targeting Kakamega, Kisii, and Siaya Counties.

1.1 Statement of the Problem

In today's digital age, adolescents are immersed in a wide range of media sources, including social media, television, streaming platforms, and online content. Generation Z, in particular, has grown up with unprecedented access to digital platforms, which deeply influence their social interactions, learning processes, and worldview (Weru, 2018). While global research increasingly examines the effects of media on adolescents, there remains limited data on the prevalence and patterns of media exposure among adolescents in rural and semi-urban regions of Kenya, particularly in Kakamega, Kisii, and Siaya counties. Existing census reports provide only ecological data on media access in these areas without specifically addressing how Generation Z learners engage with media, the types of media they consume, or the duration of their exposure (Kenya National Bureau of Statistics, 2019).

Although the World Health Organisation (WHO) does not set specific recommendations for the ideal amount of time children should spend using media, it strongly advises that adolescents should not allow media consumption to overshadow essential activities such as getting adequate sleep, maintaining regular physical activity, enjoying family meals, or spending time on non-digital, 'unplugged' leisure pursuits. Understanding the media consumption habits of adolescents in these regions is critical, as it may have far-reaching implications for their academic performance, social behavior, and mental well-being (WHO, 2020).

Numerous studies globally have examined adolescent access to media. For example, in the United States, a literature review by Twenge et al. (2019), covering 40 years of research, identified clear trends in media use among adolescents, noting a shift from traditional formats like radio and television to modern digital platforms. Similarly, . Boer et al. (2020) conducted a survey across 29 developed countries, focusing on adolescents' access to social media. This study highlighted a significant relationship between social media usage and well-being, concluding that higher social media use correlated with poorer mental health outcomes. In Africa, studies conducted in South Africa (Salubi et al., 2019), Tanzania (Pfeiffer et al, 2014), and Nigeria (Akokuwebe et al., 2019) have all centered on adolescent social media use. In Kenya, research has primarily been conducted in urban areas such as Nairobi (Kharono et al., 2022), Kiambu (Ng'ethe, 2014), and the coastal region (British Broadcating Corporation Media Action, 2020).

Despite the above studies having been conducted both globally and within Kenya, there is a significant gap in localized studies specific to Western Kenya. Though census reports show that counties like Kakamega, Kisii, and Siaya have relatively high access to various forms of media, such as television, radio, internet, and mobile phones there is a lack of detailed insights into how adolescents in these regions actually use media. This absence of focused research poses challenges for educators, policymakers, and guardians who need accurate information to develop effective strategies for mitigating the potential negative impacts of media and encouraging positive media engagement.

To address this gap, this study investigates the prevalence and patterns of media exposure among Generation Z learners in Kakamega, Kisii, and Siaya counties. The findings are intended to provide essential data that can inform interventions and contribute to evidence-based decisions aimed at supporting healthy adolescent development in these regions.

1.2 Research Objectives

i. To identify the types of media accessed by adolescent Generation Z learners in Secondary Schools in Kakamega, Kisii and Siaya Counties, Kenya
ii. To evaluate the average daily duration of media consumption among adolescent Generation learners in Secondary Schools in Kakamega, Kisii and Siaya Counties, Kenya.
iii. To examine the frequency of unsupervised media use among adolescent Generation Z learners in Secondary Schools in Kakamega, Kisii and Siaya Counties, Kenya.
iv. To investigate the primary reasons for media usage among adolescent Generation Z learners in Secondary Schools in Kakamega, Kisii and Siaya Counties, Kenya.

1.3 Research Questions

i. What types of media are accessed by adolescent Generation Z learners in secondary schools in Kakamega, Kisii and Siaya Counties, Kenya?
ii. What is the average daily duration of media consumption among adolescent Generation Z learners in secondary schools in Kakamega, Kisii and Siaya Counties, Kenya?
iii. How frequently do adolescent Generation Z learners in secondary schools in Kakamega, Kisii and Siaya Counties use media without parental supervision?

iv. What are the primary reasons for the use of different forms of media among adolescent Generation Z learners in secondary schools in Kakamega, Kisii and Siaya Counties, Kenya?

II. LITERATURE REVIEW

2.1 Theoretical Framework

2.1.1 Erik Erikson's Psychosocial Theory of Development

This study employs Erik Erikson's psychosocial theory of development to provide a framework for understanding the media use patterns among adolescent Generation Z learners. Erikson's theory outlines eight stages of human development, each characterized by a central psychosocial conflict that individuals must navigate to achieve healthy psychological growth (Youvan, 2024). The fifth stage, Identity vs. Role Confusion, is particularly relevant for adolescents, as this period is critical for forming personal identities and establishing social connections.

In the context of this study, Erikson's theory emphasizes the role of social interactions and media in shaping adolescents' identities. During this developmental stage, individuals are exploring their self-concept, often influenced by peer interactions and societal expectations. Media, particularly social media platforms, serves as a prominent avenue for adolescents to experiment with their identities, seek validation, and navigate complex social dynamics (Vannucci & McCauley, 2019).

Furthermore, Erikson highlights the importance of social relationships in identity formation. As adolescents engage with various forms of media, they are not only consumers of content but also participants in a broader social landscape that can significantly impact their self-perception and social roles. Erikson's psychosocial theory therefore offers a framework for examining the patterns of media consumption among Generation Z learners and its relationship to their identity development during adolescence.

2.2 Empirical Review

The first objective of this study was to determine the different types of media accessed by these learners. Prior research highlights that smartphones are a leading medium, with 75% of teenagers aged 14 to 17 in the United States owning them (Adjin-Tettey, 2018). In a similar vein, studies in India indicate that 85% of children aged 11 to 17 have smartphones (Kumar, 2022). While specific data for Kakamega, Kisii, and Siaya are unavailable, global trends suggest that digital devices are likely significant in media access among Kenyan adolescents. Therefore, this study aimed to investigate similar information specifically within these three counties.

Regarding the second objective, which focuses on the average daily duration of media consumption among adolescent Generation Z learners, it is notable that American youth reportedly spend up to 28 hours per week watching television (Livingstone et al., 2017). In a different context, research conducted in Canada indicates that 93% of 12-year-olds have regular access to television, averaging 3 hours of daily viewing (Fitzpatrick et al., 2019). Worth noting is that that kind of data is missing in the three counties, hence the need for this study to investigate the average daily duration of media consumption among adolescents in these three counties.

The literature further indicates that a significant portion of media consumption occurs without oversight; for example, studies show that a large percentage of adolescents engage with social media independently (Weru, 2023).

Furthermore, this study seeks to investigate the primary motivations behind the use of different forms of media among adolescent Generation Z learners. Previous research suggests that traditional media, such as radio and television, are primarily utilized for educational purposes, with 32% and 31.8% of adolescents reporting their use for learning, respectively (Weru, 2023). In contrast, digital media platforms like mobile phones and social media are predominantly used for entertainment, with 34.6% and 25% of adolescents indicating these as their main motivations for usage.

According to Kabali et al. (2015), media exposure is ‘the extent to which audience members have come across particular messages or media content’ (p. 168). This definition only focuses on simply coming into contact with the content, without necessarily paying attention to it. The spectrum of media available to Gen Z has expanded significantly, encompassing traditional formats such as books, newspapers, and television, and now extending to cable television, videos, video games, and the internet. Social media functions as an influential "super-peer," and the social interactions of adolescent Gen Z are progressively facilitated through electronic devices (Vannucci et al., 2020). Studies have found that adolescent Gen Z in both industrialized and developing countries are immersed in a pervasive electronic media environment, underscoring the global reach and influence of digital media on these digital natives (Livingstone et al., 2017).

Digital devices have become widely accessible to Generation Z, significantly shaping their media consumption habits. In the United States, members of this generation reportedly spend up to 28 hours per week watching television, excluding additional time spent on video games and internet browsing. Chassiakos et al. (2016)

further observed that American adolescents dedicate 27% of their daily media use to computers and gaming consoles, with a notable preference for using computers primarily for gaming. Additionally, approximately 75% of teenagers aged 14 to 17 own smartphones, reflecting the pervasive availability of media devices among adolescents and raising concerns about the potential effects on youth. Similarly, the ownership of tablets among families with younger children (aged 8 and below) increased fivefold between 2011 and 2013, mirroring the rising access to other smart devices (Chang et al., 2018).

The studies conducted in the United Kingdom show that there is a game console in 50% of the households. Youngsters between 8-18 years use different media types for about 5 hours per day (Mullan, 2018). In Canada, research indicates that 93% of 12-year-old children have regular access to television, with an average viewing time of 3 hours per day, surpassing the time devoted to other activities (Fitzpatrick et al., 2019). Furthermore, Summerfield (2023) did a study among youth aged 7 to 17, which indicated that 7 out of 10 children in this age group engage with social media, with significant variations between younger and older demographics. This author further reports that ownership of a personal cellphone significantly influences usage, with 88% of children with their own phone using social media compared to 52% without. This significant media consumption may overshadow other social interactions, potentially countering recommendations that emphasize the importance of adequate social interactions for psychological development.

Asia also reports similar widespread media usage among adolescent Gen Z. For example, research indicates that Indian children aged 11 to 17 spend an average of 2 hours per day consuming media, with 74% using cell phones (Ahmed, 2012). Further, another survey done by Kumar (2022) reveals significant trends in both smart-phone and internet usage in this category of age. This study revealed that 85% of Generation Z individuals aged 11 to 17 own a smartphone, with 80% of them actively using social media. Additionally, the average daily time spent on online media among this group is 5 hours.

The contemporary adolescent Gen Z in Africa has universal access to media. Nigeria serves as a notable example, where 96.7% of this population engages with mass media in various ways, including for educational purposes and entertainment (Odofin & Igabari, 2023). Furthermore, a study conducted in South Africa on children aged 6 to 16 investigated their media usage behaviors and found that 31.8% of this age group used mobile phones, with an average daily call time of up to 75 minutes (Chetty-Mhlanga et al., 2020). In addition, 46% of the participants use electronic media devices with a total screen time of up to 7 hours. Further research highlights significant disparities in media access among South African adolescent Generation Z, influenced by factors such as socio-economic status, geography, race, age, education, and gender. In 2011, statistics showed that 64.8% of this population had no internet access. Of those who did, 16.3% accessed it via cell phones, 8.6% from home, 4.7% at work, and 5.6% from other locations. This digital divide favors affluent, urban, and educated populations, deepening inequalities for rural and lower socio-economic groups (Van der Merwe, 2013).

Most data regarding media accessibility in Kenya comes from surveys focused on ecological measures rather than actual usage. However, these statistics indicate significant potential for engaging Generation Z. A report by the Kenya Film Commission in 2010 found that 59% of Kenyan households have access to television, with a strong preference for local programming and peak viewership occurring in the evenings (Njeru, 2023). Additionally, radio remains the most widely accessible media platform, reaching over 85% of households, while mobile phone penetration is rapidly increasing nationwide. Notably, there are disparities in media access within Kenyan households, with rural areas showing significantly lower access rates compared to urban regions, primarily due to economic limitations and differences in electricity availability (Gustafsson & Nielsen, 2017). The Coastal and North Eastern regions, in particular, suffer from inadequate media access. Furthermore, access to media in Kenya, especially through mobile phones, is susceptible to economic fluctuations, leading users to borrow from friends to maintain their internet connection.

The researcher found no studies specifically examining media exposure among youths, particularly the adolescent Generation Z population in Kakamega, Kisii, and Siaya Counties, highlighting a significant research gap. Understanding media exposure during adolescence is crucial for assessing its implications on identity formation, peer influence, and overall well-being of these youths in these regions. Furthermore, existing literature primarily relies on census data that estimate media access based on availability rather than reflecting actual usage patterns. To address these gaps comprehensively, this study employed a mixed-methods approach, integrating ecological studies to assess media availability with survey-based research to gather data on actual usage patterns among adolescent Gen Z in Kakamega, Kisii, and Siaya Counties.

In conclusion, while substantial research has explored media access and consumption among adolescents globally, localized studies focusing on the experiences of youth in Kakamega, Kisii, and Siaya remain limited. This empirical review emphasizes the need for further exploration to understand the unique media contexts and implications for youth development in these regions.

III. METHODOLOGY

3.1 Study Design

The study applied both descriptive and correlational research designs. Self-administered questionnaires which sought to elicit demographic information on the students as well as specific information on the frequency, length and context of exposure to different forms of media were completed by 384 learners who were 15-16 years old. In addition to this, quantitative data was also collected through interviews with 56 parents and 7 focus group discussions conducted with the lead teachers.

3.2 Study Area

The study was conducted in Western Kenya, specifically Kakamega, Kisii and Siaya Counties.

3.3 Study Population and Sampling Techniques

The target population was 14 to 17-year-old learners, drawn from both public and private secondary schools in these three counties, though data collection specifically focused on the median ages of 15 and 16. Using Krejcie and Morgan's formular for determination of sample size (Krejcie & Morgan, 1970), a sample size of 417 was selected. A stratified random sampling technique was employed due to the dispersed nature of the target population across three counties and varying school categories. These factors were expected to influence socioeconomic status, which in turn could affect access to media technologies. Therefore, the learners were initially divided into three strata based on their county (Kakamega, Kisii, or Siaya), then further into four sub-strata based in the school category (National, Extra-County, County, or Sub- County). Finally, a sample was randomly selected from each sub-stratum, proportionate to the respective populations within the larger population of the study. Additionally, 56 parents and 48 teachers were selected using stratified random sampling, with the sample size determined by data saturation.

3.4 Instruments

Data collection employed multiple methods: questionnaires, interviews, and focus group discussions. The questionnaire consisted of 27 questions in Likert scale, multiple choices, and one open-ended format. It gathered self-reported data on both ecological measures of potential exposure and assessments of actual individual media interactions including the type of media consumed, the frequency, duration, and purposes for usage, focusing on the past six months. Respondents also provided details about average daily time spent on media, context of usage, device ownership, internet access, presence of a television in their bedrooms, and their favorite social networking sites and TV shows. This questionnaire was administered face-to-face.

The interview schedule consisted of nine open-ended questions aimed at gathering qualitative data that would help in understanding parents' views on media accessibility patterns among adolescent learners. Both face-to-face and telephone one-on-one interviews were conducted and recorded.

Additionally, six focus groups, each consisting of eight teachers from various school categories (national, extra-county, county, and sub-county), participated in 25-minute discussion sessions, which were recorded. These sessions provided qualitative data on the types of media commonly accessed by students and the purpose for which they were used. The discussions were conducted through a blended approach, combining face-to-face and virtual meetings.

3.5 Validity and Reliability

3.5.1 Validity

Prior to the development of the instruments, an extensive literature review was carried out to identify established scales and conceptualize the main constructs of the study, specifically those measuring media exposure. This provided essential insights that informed the creation of the initial items for the three instruments. Then, the instruments went through an assessment by experts in the field of media studies. This process enabled the identification of any gaps as well as the inclusion of all the essential elements.

3.5.2 Reliability

A pilot study was conducted with 42 learners selected from Vihiga County to assess the reliability of the questionnaire using the test-retest method. The questionnaire was administered to the pilot sample twice, with a two-week interval between administrations. This resulted in a Cronbach's Alpha of 0.87, indicating a satisfactory level of reliability.

3.6 Data Analysis

Quantitative data from the questionnaires was analyzed using descriptive statistics, thus employing percentages, frequency counts, and means. For instance, frequency counts were used to analyze various dimensions of media access. Additionally, percentages were calculated to determine the proportion of learners with access to specific types of media. This data was then presented in tables and graphs.

Qualitative data gathered from interviews and focus group discussions was transcribed and converted into written form. To interpret the data, common patterns, themes, and sub-themes were identified from the transcriptions. These findings were then reported through written summaries and the inclusion of relevant excerpts from the qualitative data.

IV. FINDINGS & DISCUSSION

4.1 Prevalence of Exposure to Media by Type

Participants were required to indicate all the types of media they had interacted with in the previous 6 months with the options being radio, television, mobile phone, and social media. They were allowed to give multiple responses; hence the total number of responses indicating access to some form of media was 486. Results were as summarized in Table 1.

Table 1

Media Access by Type

Type of Media	Number of Users	% (N=486)
Radio	169	35%
Television	37	8%
Social Media	186	38%
Mobile Phone	94	19%

Social media had the highest access (38%) followed by radio (35%). Similar reports were further gathered from the analysis of data obtained from focus group discussions, where social media emerged as the most frequently accessed form of media among this population, as reported by teachers. Radio and television also stood out as significant themes in the data. However, in contrast, references to mobile phone usage were less frequent, and other unspecified forms of media received the least mentions.

These findings, where social media emerged as the preferred platform among adolescents, are consistent with broader research trends. Anderson and Jiang (2018) reported that U.S. teens frequently choose social media for communication and entertainment, with 95% having access to a smartphone. Similarly, Odgers (2018) highlighted the role of social media in adolescent development, noting its popularity for maintaining social connections. In Kenya, Aboagye et al. (2020) found a similar shift towards social media platforms among urban youth, who favor them for information and social interactions, further indicating a decline in traditional media use. On the contrast, some studies challenge this trend, suggesting that traditional media retains relevance in certain contexts. Smahel et al. (2020) emphasized that traditional media, like television, continues to play a key role, especially among younger teens that might have restricted internet use. Likewise, Julius (2020) observed that in rural Kenyan communities, radio and television are still primary media sources due to digital limitations. These findings suggest that while social media is dominant, media consumption patterns can vary significantly based on socioeconomic and regional factors.

The preference for social media and mobile phones highlights a significant trend toward personalized media consumption, granting individuals greater autonomy in how they engage with the media. This shift reflects the adolescents' desire for increased independence and the formation of connections outside their immediate family environment. Such behavior aligns with key developmental milestones of this age group, as they seek to establish a sense of identity and belonging within broader social networks.

In the interviews and discussions, this study noted that adolescent learners rely on cyber cafés as the main source for accessing social media. This is particularly the case given the fact that mobile phone possession is relatively low among these youths given restrictions from schools or homes. The accessibility of cyber cafés poses challenges for adult supervision, as they often provide unrestricted internet access and exposure to potentially inappropriate content. This lack of direct supervision raises concerns regarding the safety, privacy, and responsible digital media use among adolescents.

4.2 Length of Exposure to Different Types of Media

To find out the average daily duration of media consumption over the past 6 months, participants were prompted to specify the number of hours they spent daily on each medium. Options provided included 0 hour, 1-2 hours, 3-4 hours, 5-6 hours and finally above 6 hours. Their responses were as summarized in Table 2.

Table 2

Average Daily Duration of Media Use

Duration	Radio	Television	Mobile Phone	Social Media	Totals
0 hours	169 (35%)	37 (8%)	94 (19%)	186 (38%)	486 (32%)
1 to 2 hours	139(30%)	44 (9%)	142 (31%)	140 (30%)	465 (30%)
3 to 4 hours	64(22%)	107 (37%)	46 (16%)	75 (27%)	292 (19%)
5 to 6 hours	13(11%)	53 (46%)	28 (25%)	21 (18%)	115 (7%)
Above 6 hours	19 (11%)	47 (26%)	56 (32%)	56 (32%)	178 (12%)

Table 2 further indicates that out of the 1536 responses, 178 (12%) indicated that they spent at least 6 hours (25% of the day) on one form of media or the other. Among these, social media and mobile phones equally stood out as the predominant choices, each representing 32% of the responses. Television and radio were used to a lesser degree, at 26% and 11% respectively.

The finding that social media is the most preferred media platform among adolescents who spend more than six hours daily online, aligns with other global studies where social media platforms dominate screen time among teenagers. Pinho et al. (2017) noted a significant rise in adolescents spending extended hours on social media, attributing this trend to platforms like Instagram and TikTok, which offer continuous, personalized content. Additionally, Valkenburg (2022) emphasized that adolescents' preference for social media is driven by the need for peer interaction and validation, contributing to extended daily engagement. However, other studies suggest that traditional media still retains significant usage among adolescents, especially for long periods. Thomas et al. (2021) found that television continues to be a preferred medium for extended viewing, particularly among younger teens who engage with family-friendly content and streaming services. In addition, Coyne et al. (2014) observed that while social media use dominates, a substantial portion of adolescents still spends extended time on TV series and movies accessed through online streaming platforms.

This is a significant length of time, especially considering that at this age there is a range of other competing tasks that need attention, including physical exercise, studies, social interactions and rest. Indeed, parents who participated in the interview were concerned that their adolescent children often 'forgot' to engage in other activities including school homework, personal hygiene, sleep and having meals because they were constantly engrossed in *TikTok*. Worth noting is that the World Health Organization (WHO) recommends a healthy balance between time spent on the media and time spent on these other activities as an important pre-requisite for the overall well-being of adolescents (World Health Organization, 2020).

These findings also suggest a notable trend toward personalized media consumption, which affords individuals greater autonomy in their media engagement. This shift may signify adolescents' pursuit of increased independence and a move towards establishing connections beyond their immediate familial settings, which aligns with typical developmental characteristics of this demographic.

4.3 Context of Use of Different Types of Media

The participants' use of various forms of media in the absence of their parents was assessed to determine the levels of media co-use between parents and their adolescent children. This was measured on a 3-point Likert scale from Always to Never. Results were as indicated in Table 3.

Table 3

Frequency of Media Use in the Absence of Parents

Frequency	Radio	Television	Mobile Phone	Social Media
Always- 468 (35%)	13%	18%	28%	41%
Sometimes- 672 (51%)	29%	25%	26%	20%
Never- 188(14%)	41%	29%	20%	10%

It was observed that 468 responses, representing 35% of the total (N=1328), indicated consistent media use without parental involvement. Among this group, social media was the most commonly used, followed by mobile phones, TV, and then radio. In contrast, 188 responses, or 14% of the total (N=1328), indicated never engaging with

media without parental involvement. Among these, radio was the most frequently used, followed by TV, mobile phones, and social media. Therefore, social media emerged as the leading medium for private media use, with mobile phones closely following.

The study revealed that social media was the most frequently used media platform without parental involvement among adolescents. This observation aligns with Smahel et al. (2020), who found that adolescents often favor social media due to its private and independent nature, allowing them to interact with peers away from parental supervision. Similarly, Corcoran et al. (2022) reported that social media, accessible through mobile phones, is a primary medium where adolescents seek autonomy, leading to reduced parental monitoring. Lauricella and Cigel (2020) also noted that many adolescents prefer social media as it offers a personalized experience, which contrasts with the more controlled environments of traditional media like TV and radio.

This finding carries significant implications, as social media enables individuals to engage with audiences from diverse geographical areas, cultures, and family systems. Without proper regulation, it can significantly shape users' belief systems and behaviors. This is particularly crucial during adolescence, a stage where identities, beliefs, and behaviors are highly pliable and less stable, making them susceptible to influence from various, and often random, online sources. Moreover, the findings underscore the challenges parents face in monitoring social media and mobile phone use, as these platforms are far more difficult to supervise than traditional media like television and radio. Unlike the latter, managing social media and mobile phone use requires more than just parental controls and content filters, as the complexity of these platforms calls for more nuanced approaches to regulation.

4.4 Reason for Use of Different Types of Media

Participants were asked to specify the primary reason for using different types of media. Results were as summarized in Table 4.

Table 4

Reason for Use of Various Forms of Media

Purpose	Radio	Television	Mobile Phone	Social Media	Totals
Education	154 (32%)	153 (32%)	78 (16%)	96 (20)	481 (40%)
Entertainment	66 (23%)	52 (18%)	101 (34%)	73 (25%)	292 (24%)
Both Education and Entertainment	71 (16%)	131(30%)	141 (33%)	92 (21%)	435 (36%)

The findings reveal that 40% of respondents used media primarily for educational purposes, while 29% reported using it mainly for entertainment. Further details indicate that radio and television were the most commonly used platforms for educational purposes (32% each), while mobile phones and social media were the preferred options for entertainment (34% and 25% respectively). On the other hand, parents who participated in the interviews sighted entertainment, specifically, communication with peers, and downloading music and movies.

The study found a clear divide in media usage among adolescents: social media and mobile phones were primarily used for entertainment, while radio and TV were favored for educational content. This trend is supported by Stockdale and Coyne (2020), who noted that platforms like TikTok, YouTube, and Instagram attract adolescents for entertainment due to their engaging and interactive nature. Haqqu et al. (2019) similarly observed that mobile devices cater to adolescents' preference for personalized and recreational content.

Conversely, the use of radio and TV for education aligns with findings from Abbas et al. (2019), highlighting the importance of traditional media in educational programming, particularly in regions with limited internet access. Collins and Halverson (2018) also emphasized that TV remains a reliable source of educational content, trusted by parents and educators, despite the rise of digital media platforms.

However, this contrasts with findings from focus group discussions with teachers, which indicated that learners utilize media devices such as smartphones, computers, and radios primarily for academic purposes. This trend may be particularly relevant in school settings, where media use is often highly regulated and structured, allowing for focused educational engagement. In these environments, the restrictive nature of media access may lead students to adopt a more purposeful approach to media consumption, aligning their usage with academic goals. An example is the initiative by the Kenya Institute of Curriculum Development to introduce educational radio and television broadcasts, allowing students to engage with subjects included in the school curriculum (Weru, 2018).

4.5 Miscellaneous Media Access Patterns

Of interest to the study were ecological factors that could potentially influence media accessibility, such as mobile phone ownership, which was hypothesized to afford greater access compared to borrowing. Moreover, internet

accessibility was posited to facilitate social media access. The presence of a television in a child's bedroom might also elevate usage rates in the absence of parental oversight. These investigations yielded the results indicated in Table 5.

Table 5

Miscellaneous Relevant Ecological Factor

Pattern	Yes	No
Owned a personal mobile phone	40%	60%
Has easy access to the internet	38%	62%
Has a television in the bedroom	8%	92%
Television in the house is always on	34%	66%

The table presents various ecological factors associated with media access and usage. A considerable proportion of respondents reported owning personal mobile phones (40%) and having easy access to the internet (38%), while 34% indicated that television is frequently on in their households. Conversely, only a small percentage (8%) reported having a television in their bedroom. Additionally, the most popular media platforms identified by the respondents were WhatsApp, Facebook, and TikTok.

The study's findings provide valuable insights into media accessibility and usage patterns among adolescent Gen Z. Firstly, the fact that 40% of participants own personal mobile phones indicates a significant level of individual media access, which may enhance their control over media consumption compared to using shared or borrowed devices. Secondly, the finding that 34% of participants have their home televisions continuously on throughout the day suggests a significant presence of a perversive background media.

4.6 Qualitative Data Analysis and Findings

The study aimed to explore various aspects of media consumption among adolescent Generation Z learners in Kakamega, Kisii, and Siaya Counties, Kenya. Qualitative data were gathered through interviews and focus group discussions, focusing on the types of media accessed, the duration of use, frequency of unsupervised access, and the primary reasons for media consumption. Through interviews with parents regarding the types of media accessed by adolescents, the findings are summarized in Table 6.

Table 6

Parents' Responses on Frequently Accessed Form of Media

Theme (Form of media)	Frequency of Mentions (N)	Percentage (%)
T1 (Radio)	37	27%
T2 (Television)	26	19%
T3 (Mobile Phone)	22	16%
T4 (Social Media)	39	28%
T5 (Other)	14	10%
Total	**138**	**100%**

Going by the number of mentions, social media was the most frequently accessed form of media, as highlighted by both parents. Parents identified social media as the leading media choice among adolescents, with 39 mentions, accounting for 28% of total responses. This preference was closely followed by radio, with 37 mentions (27%), and television, with 26 mentions (19%). Mobile phones were mentioned 22 times (16%), while other unspecified forms of media received only 14 mentions (10%). Parents further agreed that adolescents preferred social media for entertainment and social interactions, distinguishing it from the more educational roles of radio and television. This aligns with research by Stockdale and Coyne (2020), which notes that adolescents are drawn to social media for its interactive nature. In contrast, radio and TV are perceived as passive media primarily used for information dissemination.

From the focus group discussions, majority of teachers reported that students accessed social media daily, often utilizing cyber cafés due to limited personal mobile phone ownership. The restricted access to mobile phones was attributed to school rules that limit mobile device possession among adolescents. Interestingly, a distinction emerged in the teachers' perspective about media used for entertainment and that used for academic purposes. While social media and mobile phones dominated leisure activities, majority of teachers indicated that learners primarily utilized media devices like smartphones, computers, and radios for educational content. Teachers also emphasized that 75% of educational media consumption by students occurred under supervision, contrasting with the more independent and unsupervised use of social media platforms at home. For instance, one teacher remarked,

"While they are glued to their phones for social media, many students rely on radio programs for their studies, especially in subjects like mathematics and science."

(Quoted on 23rd October, 2023 in School X during a Focus Group Discussion)

This statement highlights that, while adolescents engage with social media for leisure, they rely on traditional media like radio for educational purposes, particularly in subjects such as mathematics and science. This reliance underscores the ongoing importance of traditional media in academic learning. It suggests that, despite the prevalence of digital platforms, structured educational content remains essential. Research by Abbas et al. (2019) supports this notion, indicating that radio and television continue to play a vital role in providing educational content, especially in developing areas where internet access may not be consistent. Furthermore, Collins and Halverson (2018) emphasize that traditional media forms are still perceived as credible sources for structured learning, reinforcing their significance in the educational landscape.

V. CONCLUSION & RECOMMENDATIONS

5.1 Conclusions

The study explored various facets of media use among adolescent Gen Z learners in the three counties, and revealed a notable prevalence of social media within this demographic. A significant portion of media consumption occurs without parental supervision, particularly involving social media and mobile phones, which are frequently accessed in private settings. Additionally, the primary purposes of media consumption vary significantly; traditional media, such as radio and television, are predominantly used for educational purposes, while digital media, including mobile phones and social media, are mainly utilized for entertainment.

5.2 Recommendations

In light of these findings, the study recommends that educators leverage social media as a tool for enhancing teaching, guidance, and mentorship by providing access to educational resources through videos, podcasts, and articles. For instance, social media can be used for collaborative classroom activities, such as assignments, discussions, and feedback, while recognizing student achievements to motivate them academically. Furthermore, schools should promote normative education programs to teach online safety and ethical conduct, given the widespread use of social platforms among adolescents. There should extend to parents, by offering resources and training, such as workshops or online materials, to help parents effectively engage with and guide their children in navigating media. With an average daily media use of over six hours, schools and parents must establish appropriate media policies to ensure balanced time for physical activity, schoolwork, and interpersonal communication. Additionally, schools should work with content developers to create engaging educational material, such as games and videos for platforms like YouTube and TikTok, that align with adolescents' interests, embedding learning within the digital environments they frequently engage in.

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Updated Vector Autoregressive Model Incorporating new Information Using the Bayesian Approach

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ABSTRACT

Vector Autoregressive (VAR) models have been applied extensively in modeling time series due to their high precision when used to forecast. In the VAR development, if we have information up to time t, then a VAR(p) model is fitted. However, if new information at time $t+1$, is obtained, then a new VAR(p) model has to be fitted which makes one to go through the process again. Therefore, despite their good performance, a need would arise to incorporate new information that could be obtained after the model has been fitted to update the model instead of fitting a new model each and every time a new information is obtained. This study, therefore, considers incorporating the new information to update the vector autoregressive model of order p using Bayesian approach. First, a VAR model of order 1 is formulated after which this is generalized to the VAR model of order p. We assume that the VAR model is the prior while new information is the likelihood. The performance of updated model is compared with corresponding VAR(p) models and the model is found to perform well based on the small values of the root mean square error (RMSE) in the update and in the prediction for the plots obtained.

Mathematics Subject Classification: Primary 35A01; Secondary 65L10.
Keywords: Vector autoregressive, Bayesian approach, Update, Prediction

1 Introduction

The Vector Autoregressive (VAR) models were developed by the macroeconometrician Christopher Sims in 1980 where the main aim was to model the joint dynamics and causal relations among a set of macroeconomic variables and they dominate time series econometrics modeling [13]. The joint dynamics includes how each variable in the model is explained by the past history of every variable and how the innovations may be correlated. The VAR models explain not only the serial dependence within each component series $\{X_{ti}\}$ but also interdependence between the different component series $\{X_{ti}\}$ and $\{X_{tj}\}$, $i \neq j$ as seen in [2, 3, 7, 15, 16]. A v-variate vector autoregressive time series model of order p, VAR(p), is given by

$$\mathbf{Y}_t = \mathbf{A_1 Y_{t-1}} + \mathbf{A_2 Y_{t-2}} + \cdots + \mathbf{A_p Y_{t-p}} + \mathbf{u_t} \tag{1.1a}$$

where $\mathbf{Y}_t$ is a $(v \times 1)$ vector of time series variables, p is the number of lags and $\mathbf{u_t}$ is a $(v \times 1)$ vector of white noise process. In expanded form, equation 1.1a can be written as

$$\begin{aligned}\begin{pmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{v,t} \end{pmatrix} &= \begin{bmatrix} a_{11,1} & a_{12,1} & \cdots & a_{1v,1} \\ a_{21,1} & a_{22,1} & \cdots & a_{2v,1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{v1,1} & a_{v2,1} & \cdots & a_{vv,1} \end{bmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{v,t-1} \end{pmatrix} + \begin{bmatrix} a_{11,2} & a_{12,2} & \cdots & a_{1v,2} \\ a_{21,2} & a_{22,2} & \cdots & a_{2v,2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{v1,2} & a_{v2,2} & \cdots & a_{vv,2} \end{bmatrix} \begin{pmatrix} y_{1,t-2} \\ y_{2,t-2} \\ \vdots \\ y_{v,t-2} \end{pmatrix} \\ &+ \cdots + \begin{bmatrix} a_{11,p} & a_{12,p} & \cdots & a_{1v,p} \\ a_{21,p} & a_{22,p} & \cdots & a_{2v,p} \\ \vdots & \vdots & \ddots & \vdots \\ a_{v1,p} & a_{v2,p} & \cdots & a_{vv,p} \end{bmatrix} \begin{pmatrix} y_{1,t-p} \\ y_{2,t-p} \\ \vdots \\ y_{v,t-p} \end{pmatrix} + \begin{pmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{v,t} \end{pmatrix}\end{aligned} \tag{1.1b}$$

The VAR model provides forecasts which are superior to those obtained from the univariate time series models, [3]. Traditionally, VAR models are widely much useful in describing the dynamic nature of most economic and financial time series. However, recently the vector autoregressive models have gained much application in a wide range of disciplines such as Medicine, Epidemiology, Economics, Biology and Macroeconomics among others. Indeed, Hamzah $et\ al.$ [3] ascertains that VAR models are the mostly used models for modelling multivariate time series data. This is as seen in the works [1, 4, 5, 6, 10, 12, 17].

Despite the fact that the VAR models have been applied extensively in many areas due to their ability to perform well, there is a concern of what happens in the event that new information is obtained. This is due to the fact that, if we have data up to time t, then we can fit a VAR(p) model. If some new information at time $t+1$ is obtained, then it requires that again a new model is fitted. In this study, an approach is proposed where we incorporate new information obtained at time $t+1$ to update the VAR(p) model. To do this we consider the use of the Bayesian approach to cater for new information obtained as time goes on after the model has been developed instead of repeating the process. The fitted VAR model is considered as the prior, new information as the likelihood and the updated VAR model is the posterior. Therefore, the main objective of the study is to provide an approach for updating the VAR(p) model whenever new information is obtained rather that fitting a new model when even a single measurement is obtained. The rest of the paper is structured as follows: first we have model formulation in Section 2 where we begin with VAR(1) model and then generalize to VAR(p) model using the Bayesian approach, then check the performance of the updated model in Section 3 where the root mean square error (RMSE) is computed alongside the plots of the model. The conclusions are then given in Section 4.

2 Model Formulation

In this section we discuss modification of the ordinary VAR(p) model using the Bayesian technique to incorporate new information. The model to be updated is vector autoregressive model of order 1 after which a generalization to the vector autoregressive model of order p is done. However, first, a brief discussion on the existing VAR model is given.

2.1 Existing VAR(p) Model

A general VAR(p) model is as given in equation 1.1a. It should be noted that the VAR models can further be classified into two types namely: the reduced form, equation 1.1a and the structural VAR model as given in [7]. In the reduced form VAR model, each variable is a function of its own past and the past values of the other variables. On the other hand, the structural vector autoregressive models add the restrictions that allow identification of causal relationships beyond those that can be identified with the reduced form [7, 14]. In addition, structural form is used when the error terms are uncorrelated and that the variables can have a contemporaneous impact on other variables [14].
The identification or fitting of an ordinary VAR model involves model specification, estimation of model parameters and model checking to test whether the model is adequate. The order, p, of VAR is chosen which minimizes the Schwartz and Hannan-Quinn criteria as outlined by [8]. The Schwartz criterion is given by

$$SC(p) = \ln|\widehat{\Sigma}_u(p)| + \frac{\ln T}{T} pv^2$$

On the other hand, the Hannan-Quinn criterion is given by

$$HQ(p) = \ln|\widehat{\Sigma}_u(p)| + \frac{2\ln\ln T}{T} pv^2$$

where, for both criteria, $\widehat{\Sigma}_u$ is the estimated white noise covariance matrix, T is the sample size and v is the number of time series components. The criteria compare the residuals of the models and estimate the relative information loss of representing the original data using each of the model. In addition, the criteria weigh the quality of fit (covariance of residuals) against the complexity (number of free parameters) and therefore the model with least criterion value is considered [14]. The parameters of a fitted VAR model can be estimated by ordinary least squares estimation method under the assumptions that error term has mean of zero, the variables are stationary and no outliers. The developed model is then subjected to diagnostic checking for its adequacy and this involves checking whether the residuals are white noise, normally distributed and uncorrelated. Afterwards, the model is used to forecast which is the most important function of the VAR models. Apart from forecasting, the VAR models can be used to give the dynamics that are predicted by the models in addition to estimating the model's parameters which involves Granger-causality statistics, impulse response function and forecast error decomposition as given in [8]. Granger-causality involves testing whether one variable is statistically significant when predicting another variable while impulse response function traces the dynamic path of variables in the system to shocks to other variables in the system. On the other hand, forecast error decomposition separates the forecast error variance into proportions attributed to each variable in the model which enables understanding of how much of an impact one variable has on another variable in the VAR model [8].

2.2 Updated Vector Autoregressive VAR(p) Model

In this section, the updated Vector Autoregressive model is discussed. First, the updated VAR(1) model is discussed after which the proposed updated VAR(p) model is given. A v-variate VAR model of order 1 is given by

$$Y_t = A_1 Y_{t-1} + u_t, \qquad u_t \sim \mathcal{N}(0, Q). \tag{2.1}$$

Now, let the relation between Y_t, which is assumed to be the state at time t, and X_t, measurements at time t, be given by

$$X_t = P_t Y_t + \eta_t, \qquad \eta_t \sim \mathcal{N}(0, R), \tag{2.2}$$

where P is a matrix that may depend on time t and η_t is the measurement error which is white noise. Equation 2.1 is a transition equation giving transition from state t to state $t+1$ while 2.2 is known as measurement equation. Equations 2.1 and 2.2 now form a system of models referred to as state-space models given by

$$Y_t = A_1 Y_{t-1} + u_t, \qquad u_t \sim \mathcal{N}(0, Q) \tag{2.3a}$$

$$X_t = P_t Y_t + \eta_t, \qquad \eta_t \sim \mathcal{N}(0, R) \tag{2.3b}$$

where: Y_t is an $v \times 1$ state vector, X_t is a $n \times 1$ vector of measurement or observable variables, P_t is a $n \times v$ measurement matrix, A_1 is a $v \times v$ state transition matrix which may be time dependent, u_t is a $v \times 1$ vector of transition equation errors and η_t is a $n \times 1$ vector of measurement errors.
The goal is to get the estimates of the states Y_t given the observations X_t for the representation given by 2.3a and 2.3b. To achieve this, we do it in two steps, namely; the prediction and the update step. In the prediction step, we assume that the previous belief $p(Y_{t-1}|X_{t-1})$ is known and we wish to get $p(Y_t|X_{t-1})$.

$$p(Y_t|X_{t-1}) = \int p(Y_t, Y_{t-1}|X_{t-1}) dY_{t-1}$$

From conditional probability we have that

$$p(Y_t|X_{t-1}) = \int p(Y_t|Y_{t-1}, X_{t-1}) p(Y_{t-1}|X_{t-1}) dY_{t-1}$$

But Y_t is independent of X_{t-1} and therefore

$$p(Y_t|X_{t-1}) = \int p(Y_t|Y_{t-1}) p(Y_{t-1}|X_{t-1}) dY_{t-1} \tag{2.4}$$

The probability density functions $p(Y_{t-1}|X_{t-1})$ and $p(Y_t|Y_{t-1})$ are Gaussian, where

$$\begin{aligned} p(Y_{t-1}|X_{t-1}) &= \mathcal{N}\left(E[Y_{t-1}|X_{t-1}], Var[Y_{t-1}|X_{t-1}]\right) \\ &= \mathcal{N}\left(\hat{Y}_{t-1|t-1}, S_{t-1|t-1}\right) \end{aligned} \tag{2.5}$$

and

$$\begin{aligned} p(Y_t|Y_{t-1}) &= \mathcal{N}\left(E[Y_t|Y_{t-1}], Var[Y_t|Y_{t-1}]\right) \\ &= \mathcal{N}\left(A_{1,t-1} Y_{t-1}, Q\right) \end{aligned} \tag{2.6}$$

Substituting equations 2.5 and 2.6 in the prediction posterior, equation 2.4, we have

$$p(Y_t|X_{t-1}) = \int \mathcal{N}\left(A_{1,t-1} Y_{t-1}, Q\right) \mathcal{N}\left(\hat{Y}_{t-1|t-1}, S_{t-1|t-1}\right) dY_{t-1}$$

But $\int \mathcal{N}\left(\hat{Y}_{t-1|t-1}, S_{t-1|t-1}\right) dY_{t-1} = 1$, then

$$\begin{aligned} p(Y_t|X_{t-1}) &= \mathcal{N}\left(A_{1,t-1}\hat{Y}_{t-1}, S_{t|t-1}\right) \\ &= \mathcal{N}\left(\hat{Y}_{t|t-1}, S_{t|t-1}\right) \end{aligned} \tag{2.7}$$

where the predicted mean in equation 2.7 is given by

$$\begin{aligned} \hat{Y}_{t|t-1} &= E[Y_t|X_{t-1}] \\ &= E[A_{1,t-1}Y_{t-1} + u_t|X_{t-1}] \\ &= E[A_{1,t-1}Y_{t-1}|X_{t-1}] + E[u_t|X_{t-1}] \end{aligned} \tag{2.8}$$

But since u_t are independent and identically distributed and not dependent on X_{t-1}, then equation 2.8 becomes

$$\begin{aligned} \hat{Y}_{t|t-1} &= A_{1,t-1}E[Y_{t-1}|X_{t-1}] + E[u_t] \\ &= A_{1,t-1}\hat{Y}_{t-1|t-1} \end{aligned} \tag{2.9}$$

since $E(u_t) = 0$. On the other hand, the predicted covariance $S_{t|t-1}$ is given by

$$\begin{aligned} S_{t|t-1} &= Var[Y_t|X_{t-1}] \\ &= Var[A_{1,t-1}Y_{t-1} + u_t|X_{t-1}] \\ &= Var[A_{1,t-1}Y_{t-1}|X_{t-1}] + Var[u_t|X_{t-1}] \end{aligned} \tag{2.10}$$

But since u_t is independent of X_{t-1}, then equation 2.10 becomes

$$\begin{aligned} S_{t|t-1} &= A_{1,t-1}Var[Y_{t-1}|X_{t-1}]A_{1,t-1}^T + Var[u_t] \\ &= A_{1,t-1}S_{t-1|t-1}A_{1,t-1}^T + Q \end{aligned} \tag{2.11}$$

where $Var(u_t) = Q$. In the update step, the new measurement X_t is used to obtain the posterior $p(Y_t|X_t)$. From Bayes' theorem,

$$\begin{aligned} p(Y_t|X_t) &= \frac{p(X_t|Y_t)p(Y_t)}{p(X_t)} \\ &= \frac{p(X_t, X_{t-1}|Y_t)p(Y_t)}{p(X_t, X_{t-1})} \\ &= \frac{p(X_t|X_{t-1}, Y_t)p(X_{t-1}|Y_t)p(Y_t)}{p(X_t|X_{t-1})p(X_{t-1})} \end{aligned} \tag{2.12}$$

But

$$p(X_{t-1}|Y_t) = \frac{p(X_{t-1}, Y_t)}{p(Y_t)} = \frac{p(Y_t, X_{t-1})}{p(Y_t)} = \frac{p(Y_t|X_{t-1})p(X_{t-1})}{p(Y_t)} \tag{2.13}$$

and therefore substituting 2.13 in 2.12 we have

$$\begin{aligned} p(Y_t|X_t) &= \frac{p(X_t|X_{t-1}, Y_t)p(Y_t|X_{t-1})p(X_{t-1})p(Y_t)}{p(X_t|X_{t-1})p(X_{t-1})p(Y_t)} \\ &= \frac{p(X_t|X_{t-1}, Y_t)p(Y_t|X_{t-1})}{p(X_t|X_{t-1})} \\ &= \frac{p(X_t|Y_t)p(Y_t|X_{t-1})}{p(X_t|X_{t-1})} \end{aligned} \tag{2.14}$$

Furthermore,

$$\begin{aligned} p(X_t|X_{t-1}) &= \int p(X_t, Y_t|X_{t-1})dY_t = \int p(X_t|Y_t, X_{t-1})p(Y_t|X_{t-1})dY_t \\ &= \int p(X_t|Y_t)p(Y_t|X_{t-1})dY_t \end{aligned} \tag{2.15}$$

Substituting 2.15 in 2.14 we have

$$p(Y_t|X_t) = \frac{p(X_t|Y_t)p(Y_t|X_{t-1})}{\int p(X_t|Y_t)p(Y_t|X_{t-1})dY_t} \tag{2.16}$$

From the measurement equation we have that $p(X_t|Y_t) = \mathcal{N}[P_tY_t, R]$ and since $p(Y_t|X_{t-1}) = \mathcal{N}[\hat{Y}_{t|t-1}, S_{t|t-1}]$, then 2.16 becomes

$$\begin{aligned} p(Y_t|X_t) &= \frac{p(X_t|Y_t)p(Y_t|X_{t-1})}{\int p(X_t|Y_t)p(Y_t|X_{t-1})dY_t} \\ &= \frac{\mathcal{N}[P_tY_t, R]\mathcal{N}[\hat{Y}_{t|t-1}, S_{t|t-1}]}{\int \mathcal{N}[P_tY_t, R]\mathcal{N}[\hat{Y}_{t|t-1}, S_{t|t-1}]dY_t} \end{aligned} \tag{2.17}$$

In the numerator to 2.17, we have that

$$\begin{aligned} \mathcal{N}[P_tY_t, R]\mathcal{N}[\hat{Y}_{t|t-1}, S_{t|t-1}] &= \frac{1}{\sqrt{det(2\pi R)}} e^{-\frac{1}{2}(X_t - P_tY_t)^T R^{-1}(X_t - P_tY_t)} \times \\ &\quad \frac{1}{\sqrt{det(2\pi S_{t|t-1})}} e^{-\frac{1}{2}(Y_t - \hat{Y}_{t|t-1})^T S_{t|t-1}^{-1}(Y_t - \hat{Y}_{t|t-1})} \\ &= \frac{1}{2\pi\sqrt{det(R)det(S_{t|t-1})}} e^{-\frac{1}{2}[M]} \end{aligned} \tag{2.18}$$

where $M = (X_t - P_tY_t)^T R^{-1}(X_t - P_tY_t) + (Y_t - \hat{Y}_{t|t-1})^T S_{t|t-1}^{-1}(Y_t - \hat{Y}_{t|t-1})$. But from [**?**], M can be written as

$$\begin{aligned} M &= (X_t - P_tY_t)^T R^{-1}(X_t - P_tY_t) + (Y_t - \hat{Y}_{t|t-1})^T S_{t|t-1}^{-1}(Y_t - \hat{Y}_{t|t-1}) \\ &= (X_t - P_t\hat{Y}_{t|t-1})^T (R + P_tS_{t|t-1}P_t^T)^{-1}(X_t - P_t\hat{Y}_{t|t-1}) \\ &\quad + (Y_t - \hat{Y}_{t|t})^T (S_{t|t-1} + P_t^T R^{-1}P_t)(Y_t - \hat{Y}_{t|t}) \end{aligned} \tag{2.19}$$

From which

$$det(R) \times det(S_{t|t-1}) = det(R + P_tS_{t|t-1}P_t^T) \times det(S_{t|t-1} + P_t^T R^{-1}P_t) \tag{2.20}$$

Substituting equations 2.19 and 2.20 in equation 2.18 we have

$$\begin{aligned}\mathcal{N}[P_tY_t,R]\mathcal{N}[\hat{Y}_{t|t-1},S_{t|t-1}] &= \frac{1}{\sqrt{det(2\pi(R+P_tS_{t|t-1}P_t^T))}}\times \\ &\quad e^{-\frac{1}{2}(X_t-P_t\hat{Y}_{t|t-1})^T(R+P_tS_{t|t-1}P_t^T)^{-1}(X_t-P_t\hat{Y}_{t|t-1})}\times \\ &\quad \frac{1}{\sqrt{det(2\pi(S_{t|t-1}+P_t^TR^{-1}P_t)^{-1})}}\times \\ &\quad e^{-\frac{1}{2}(Y_t-\hat{Y}_{t|t})^T(S_{t|t-1}+P_t^TR^{-1}P_t)(Y_t-\hat{Y}_{t|t})} \\ &= \mathcal{N}[P_t\hat{Y}_{t|t-1},R+P_tS_{t|t-1}P_t^T]\times \\ &\quad \mathcal{N}[\hat{Y}_{t|t},(S_{t|t-1}+P_t^TR^{-1}P_t)^{-1}] \end{aligned} \tag{2.21}$$

The denominator in 2.17 can be expressed as

$$\begin{aligned}\int \mathcal{N}[P_tY_t,R]\mathcal{N}[\hat{Y}_{t|t-1},S_{t|t-1}]dY_t &= \int \mathcal{N}[P_t\hat{Y}_{t|t-1},R+P_tS_{t|t-1}P_t^T]\times \\ &\quad \mathcal{N}[\hat{Y}_{t|t},(S_{t|t-1}+P_t^TR^{-1}P_t)^{-1}]dY_t \\ &= \mathcal{N}[P_t\hat{Y}_{t|t-1},R+P_tS_{t|t-1}P_t^T]\times \\ &\quad \int \mathcal{N}[\hat{Y}_{t|t},(S_{t|t-1}+P_t^TR^{-1}P_t)^{-1}]dY_t \\ &= \mathcal{N}[P_t\hat{Y}_{t|t-1},R+P_tS_{t|t-1}P_t^T] \end{aligned} \tag{2.22}$$

where $\int \mathcal{N}[\hat{Y}_{t|t},(S_{t|t-1}+P_t^TR^{-1}P_t)^{-1}]dY_t = 1$. Therefore, the updated posterior is given by

$$\begin{aligned} p(Y_t|X_t) &= \frac{\mathcal{N}[P_t\hat{Y}_{t|t-1},R+P_tS_{t|t-1}P_t^T]\mathcal{N}[\hat{Y}_{t|t},(S_{t|t-1}+P_t^TR^{-1}P_t)^{-1}]}{\mathcal{N}[P_t\hat{Y}_{t|t-1},R+P_tS_{t|t-1}P_t^T]} \\ &= \mathcal{N}[\hat{Y}_{t|t},(S_{t|t-1}+P_t^TR^{-1}P_t)^{-1}] \end{aligned} \tag{2.23}$$

Defining the inverse-covariance of the update as

$$\hat{S}_{t|t}^{-1} = S_{t|t-1}^{-1} + P_t^TR^{-1}P_t \tag{2.24}$$

then we have that

$$p(Y_t|X_t) = \mathcal{N}[\hat{Y}_{t|t},\hat{S}_{t|t}] \tag{2.25}$$

By definition, see [9],

$$\hat{S}_{t|t}^{-1}\hat{Y}_{t|t} = S_{t|t-1}^{-1}\hat{Y}_{t|t-1} + P_t^TR^{-1}X_t \tag{2.26}$$

Thus to obtain $\hat{S}_{t|t}$, we apply the Woodbury matrix identity given as

$$(E+FGH)^{-1} = E^{-1} - E^{-1}F(G^{-1}+HE^{-1}F)^{-1}HE^{-1} \tag{2.27}$$

see [9]. Hence, applying 2.27 to 2.24 we have that

$$\begin{aligned}[\hat{S}_{t|t}^{-1}]^{-1} = \hat{S}_{t|t} &= (S_{t|t-1}^{-1}+P_t^TR^{-1}P_t)^{-1} \\ &= S_{t|t-1} - S_{t|t-1}P_t^T(R+P_tS_{t|t-1}P_t^T)^{-1}P_tS_{t|t-1} \\ &= (I - S_{t|t-1}P_t^T(R+P_tS_{t|t-1}P_t^T)^{-1}P_t)S_{t|t-1} \\ &= (I-K_tP_t)S_{t|t-1} \end{aligned} \tag{2.28}$$

where $K_t = \frac{S_{t|t-1}P_t^T}{R+P_tS_{t|t-1}P_t^T}$. To obtain the updated state, suppose that 2.26 is multiplied by $\hat{S}_{t|t}$ so that we have

$$\hat{S}_{t|t}\hat{S}_{t|t}^{-1}\hat{Y}_{t|t} = (I - K_tP_t)S_{t|t-1}[S_{t|t-1}^{-1}\hat{Y}_{t|t-1} + P_t^TR^{-1}X_t] \tag{2.29}$$

Thus

$$\begin{aligned}
\hat{Y}_{t|t} &= (I - K_tP_t)\hat{Y}_{t|t-1} + (I - K_tP_t)S_{t|t-1}P_t^TR^{-1}X_t \\
&= \hat{Y}_{t|t-1} - K_tP_t\hat{Y}_{t|t-1} + S_{t|t-1}P_t^TR^{-1}X_t - K_tP_tS_{t|t-1}P_t^TR^{-1}X_t \\
&= \hat{Y}_{t|t-1} + (S_{t|t-1}P_t^T(R + P_tS_{t|t-1}P_t^T)^{-1}(R + P_tS_{t|t-1}P_t^T)R^{-1} \\
&\quad -K_tP_tS_{t|t-1}P_t^TR^{-1})X_t - K_tP_t\hat{Y}_{t|t-1} \\
&= \hat{Y}_{t|t-1} + (K_t(I + P_tS_{t|t-1}P_t^TR^{-1}) - K_tP_tS_{t|t-1}P_t^TR^{-1})X_t - K_tP_t\hat{Y}_{t|t-1} \\
&= \hat{Y}_{t|t-1} + (K_t + K_tP_tS_{t|t-1}P_t^TR^{-1} - K_tP_tS_{t|t-1}P_t^TR^{-1})X_t - K_tP_t\hat{Y}_{t|t-1} \\
&= \hat{Y}_{t|t-1} + K_t(X_t - P_t\hat{Y}_{t|t-1}) \\
&= A_{1,t-1}\hat{Y}_{t|t-1} + K_t(X_t - P_t\hat{Y}_{t|t-1})
\end{aligned} \tag{2.30}$$

Therefore, the equations for the updated VAR(1) model are given as

$$\hat{Y}_{t|t-1} = A_{1,t-1}\hat{Y}_{t|t-1} \tag{2.31a}$$

$$\hat{S}_{t|t-1} = A_{1,t-1}S_{t-1}A_{1,t-1}^T + Q \tag{2.31b}$$

$$K_t = \frac{\hat{S}_{t|t-1}P_t^T}{P_t\hat{S}_{t|t-1}P_t^T + R} \tag{2.31c}$$

$$\hat{Y}_{t|t} = A_{1,t-1}\hat{Y}_{t|t-1} + K_t\left(X_t - P_t\hat{Y}_{t|t-1}\right) \tag{2.31d}$$

$$\hat{S}_{t|t} = S_{t|t-1} - K_tP_tS_{t|t-1} \tag{2.31e}$$

where $K_t = \frac{\hat{S}_{t|t-1}P_t^T}{P_t\hat{S}_{t|t-1}P_t^T+R}$ is known as the gain while the term $\left(X_t - P_t\hat{Y}_{t|t-1}\right)$ is referred to as the innovation, or the residual in the measurement, which is equivalent to the measurement noise. Having obtained the algorithm for the updated VAR(1) model, then we propose that the updated vector autoregressive model of order p, VAR(p) model, is

$$\begin{aligned}
\hat{Y}_{t|t} &= A_{1,t-1}\hat{Y}_{t|t-1} + K_t\left(X_t - P_t\hat{Y}_{t|t-1}\right) + A_{2,t-2}\hat{Y}_{t-1|t-2} \\
&\quad + K_{t-1}\left(X_{t-1} - P_{t-1}\hat{Y}_{t-1|t-2}\right) + \cdots + A_{p,t-p}\hat{Y}_{t-p+1|t-p} \\
&\quad + K_{t-p+1}\left(X_{t-p+1} - P_{t-p+1}\hat{Y}_{t-p+1|t-p}\right)
\end{aligned} \tag{2.32a}$$

and the corresponding covariance is

$$\begin{aligned}
\hat{S}_{t|t} &= \hat{S}_{t|t-1} - K_tP_t\hat{S}_{t|t-1} - K_{t-1}P_{t-1}\hat{S}_{t-1|t-2} - \cdots - \\
&\quad K_{t-p+1}P_{t-p+1}\hat{S}_{t-p+1|t-p}
\end{aligned} \tag{2.32b}$$

This can be used to update the existing VAR(p) model given the new information which is considered as the likelihood. Therefore, the algorithm for the updated generalized vector autoregressive model of order p is

Algorithm 1 Algorithm for generalized updated VAR(p) model

1: Predict the state: $\hat{Y}_{t|t-1} = A_{1,t-1}\hat{Y}_{t|t-1} + \cdots + A_{p,t-p}\hat{Y}_{t|t-p}$ and error covariance: $\hat{S}_{t|t-1} = A_{1,t-1}S_{t-1}A_{1,t-1}^T + \cdots + A_{p,t-p}S_{t-p}A_{p,t-p}^T + Q$

2: Compute the gains: $K_{t-p+1} = \frac{\hat{S}_{t-p+1|t-p}P_{t-p+1}^T}{P_{t-p+1}\hat{S}_{t-p+1|t-p}P_{t-p+1}^T+R}$

3: Update the state: $\hat{Y}_{t|t} = A_{1,t-1}\hat{Y}_{t|t-1} + K_t\left(X_t - P_t\hat{Y}_{t|t-1}\right) + A_{2,t-2}\hat{Y}_{t-1|t-2} + K_{t-1}\left(X_{t-1} - P_{t-1}\hat{Y}_{t-1|t-2}\right) + \cdots + A_{p,t-p}\hat{Y}_{t-p+1|t-p} + K_{t-p+1}\left(X_{t-p+1} - P_{t-p+1}\hat{Y}_{t-p+1|t-p}\right)$

4: Update the error covariance: $\hat{S}_{t|t} = \hat{S}_{t|t-1} - K_tP_t\hat{S}_{t|t-1} - K_{t-1}P_{t-1}\hat{S}_{t-1|t-2} - \cdots - K_{t-p+1}P_{t-p+1}\hat{S}_{t-p+1|t-p}$

3 Testing Performance of the Updated Model

In this section, performance of the updated model is given. First, consider the Bivariate VAR(2) model given by

$$y_t = \nu + \begin{bmatrix} 0.5 & 0.1 \\ 0.4 & 0.5 \end{bmatrix} y_{t-1} + \begin{bmatrix} 0 & 0 \\ 0.25 & 0 \end{bmatrix} y_{t-2} + u_t \tag{3.1}$$

where it is assumed that $\Sigma_u = \begin{bmatrix} 0.09 & 0 \\ 0 & 0.04 \end{bmatrix}$ and ν is assumed to be a null matrix, see [8]. We use equation 3.1 to test the performance of the updated model under **Algorithm 1**.

Setting $A_1 = \begin{bmatrix} 0.5 & 0.1 \\ 0.4 & 0.5 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 0 \\ 0.25 & 0 \end{bmatrix}$, $P_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $Q = \Sigma_u = \begin{bmatrix} 0.09 & 0 \\ 0 & 0.04 \end{bmatrix}$, $R = \begin{bmatrix} 0.09 & 0 \\ 0 & 0.04 \end{bmatrix}$ and $S_0 = \begin{bmatrix} 0.09 & 0 \\ 0 & 0.04 \end{bmatrix}$ we obtain the output in Figs. 1-2 which gives the output for the first variable and the second variable respectively. The first subplot in Figs. 1-2 represents the output for VAR(2), modified VAR(2) estimate and modified VAR(2) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot represents the RMSE in the estimate and prediction, denoted by the blue and the red lines, respectively.

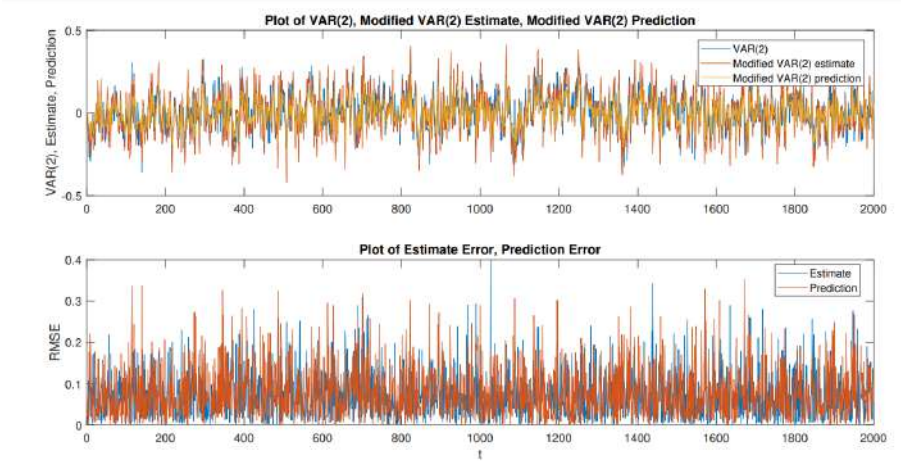

Figure 1: Bivariate VAR(2) - Variable 1

The first subplot gives comparison of the VAR(2), modified VAR(2) estimate and modified VAR(2) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(2) and the modified VAR(2) and between VAR(2) and the modified VAR(2) prediction for variable 1.

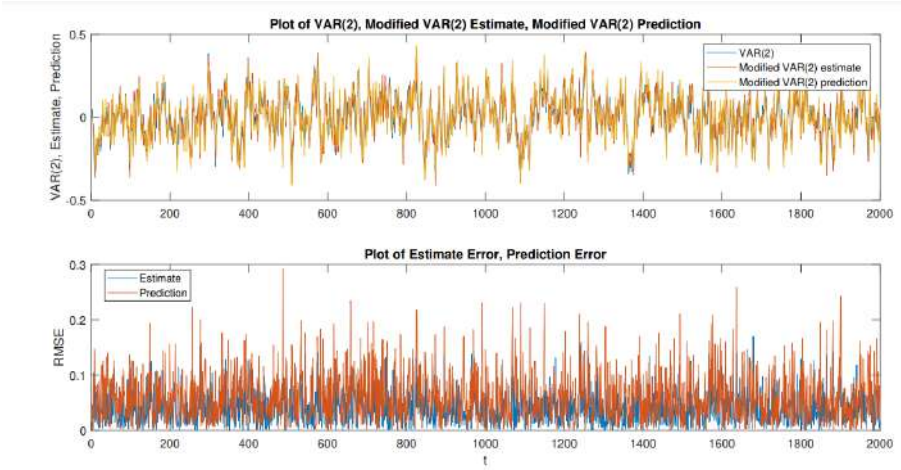

Figure 2: Bivariate VAR(2) - Variable 1

The first subplot gives comparison of the VAR(2), modified VAR(2) estimate and modified VAR(2) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(2) and the modified VAR(2) and between VAR(2) and the modified VAR(2) prediction for variable 2.

From Figs. 1-2, it is observed that the errors between VAR(2) and the modified VAR(2) estimate are less, between 0 and 0.4 for variable 1 and between 0 and 0.3 for variable 2. Furthermore, the errors between VAR(2) and the modified VAR(2) prediction are as well low, between 0 and 0.4 for variable 1 and between 0 and 0.3 for variable 2. This indicates that the updated model performs well due to the small values of RMSE obtained in the estimate and in the prediction.

Next, suppose we consider the tri-variate VAR(1) model given in [**?**] where

$$y_t = \nu + \begin{bmatrix} 0.5 & 0 & 0 \\ 0.1 & 0.1 & 0.3 \\ 0 & 0.2 & 0.3 \end{bmatrix} y_{t-1} + u_t \tag{3.2}$$

where we assume ν is a null matrix, $\Sigma_u = Q = \begin{bmatrix} 2.25 & 0 & 0 \\ 0 & 1 & 0.5 \\ 0 & 0.5 & 0.74 \end{bmatrix}$, $R = \begin{bmatrix} 2.25 & 0 & 0 \\ 0 & 1 & 0.5 \\ 0 & 0.5 & 0.74 \end{bmatrix}$ and $S_0 = \begin{bmatrix} 2.25 & 0 & 0 \\ 0 & 1 & 0.5 \\ 0 & 0.5 & 0.74 \end{bmatrix}$. We test the performance of the updated model under the model given by equation 3.2 whose output is given in Figs. 3-5 for the first, second and third variables respectively. The first subplot in Figs. 3-5 represents the output for VAR(1), modified VAR(1) estimate and modified VAR(1) prediction denoted by the blue line, red line and the yellow line, respectively. The second subplot in Figs. 3-5 represents the RMSE in the estimate and prediction denoted by the blue and the red lines, respectively.

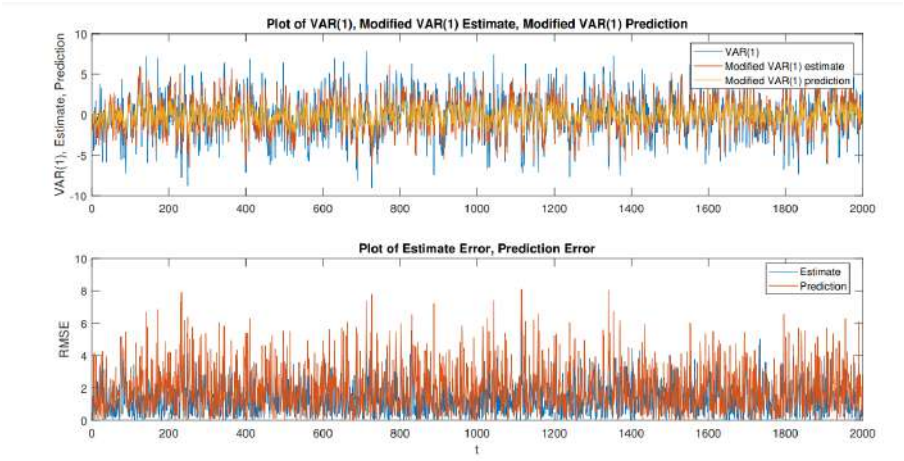

Figure 3: Trivariate VAR(1) - Variable 1

The first subplot gives comparison of the VAR(1), modified VAR(1) estimate and modified VAR(1) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(1) and the

modified VAR(1) and between VAR(1) and the modified VAR(1) prediction for variable 1.

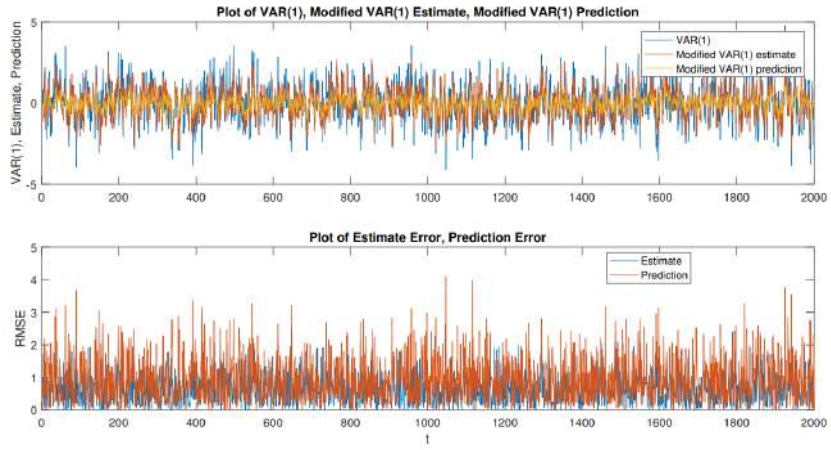

Figure 4: Trivariate VAR(1) - Variable 2

The first subplot gives comparison of the VAR(1), modified VAR(1) estimate and modified VAR(1) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(1) and the modified VAR(1) and between VAR(1) and the modified VAR(1) prediction for variable 2. The first subplot gives comparison

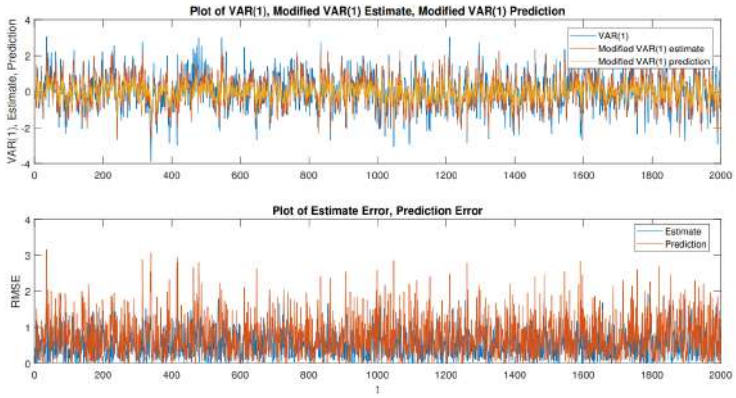

Figure 5: Trivariate VAR(1) - Variable 3

of the VAR(1), modified VAR(1) estimate and modified VAR(1) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(1) and the modified VAR(1) and between VAR(1) and the modified VAR(1) prediction for variable 3.

From Figs. 3-5 it can be seen that the updated model performs well due to the small RMSE values (ranging between 0 and 10) obtained in the update and in the prediction in each of the variables for the model. Furthermore, we check the performance of the updated model by considering the model in five dimensions. In five dimension, then A_1 and P_t are 5×5 matrices. Suppose now that the state space model is given by

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \\ y_{4,t} \\ y_{5,t} \end{pmatrix} = \begin{bmatrix} 0.99 & 0 & 0 & 0 & 0 \\ 0 & 0.99 & 0 & 0 & 0 \\ 0 & 0 & 0.99 & 0 & 0 \\ 0 & 0 & 0 & 0.99 & 0 \\ 0 & 0 & 0 & 0 & 0.99 \end{bmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \\ y_{3,t-1} \\ y_{4,t-1} \\ y_{5,t-1} \end{pmatrix} + \begin{pmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \\ u_{4,t} \\ u_{5,t} \end{pmatrix}$$

$$\begin{pmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \\ x_{4,t} \\ x_{5,t} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \\ y_{4,t} \\ y_{5,t} \end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \\ \eta_{5,t} \end{pmatrix}$$

Upon setting $Q = \begin{bmatrix} 0.001 & 0 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 & 0 \\ 0 & 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0 & 0.001 \end{bmatrix}$,

$$R = \begin{bmatrix} 0.001 & 0 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 & 0 \\ 0 & 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0 & 0.001 \end{bmatrix}$$

and

$$S_0 = \begin{bmatrix} 0.001 & 0 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 & 0 \\ 0 & 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0 & 0.001 \end{bmatrix}$$

we have the plots in Figs. 6-10 which represent the first, second, third, fourth and fifth variables respectively. The first subplot in Figs. 6-10 represents the output for VAR(1), modified VAR(1) estimate and modified VAR(1) prediction denoted by the blue line, red line and the yellow line, respectively while the second subplots represents the RMSE in the estimate and prediction denoted by the blue and the red lines, respectively.

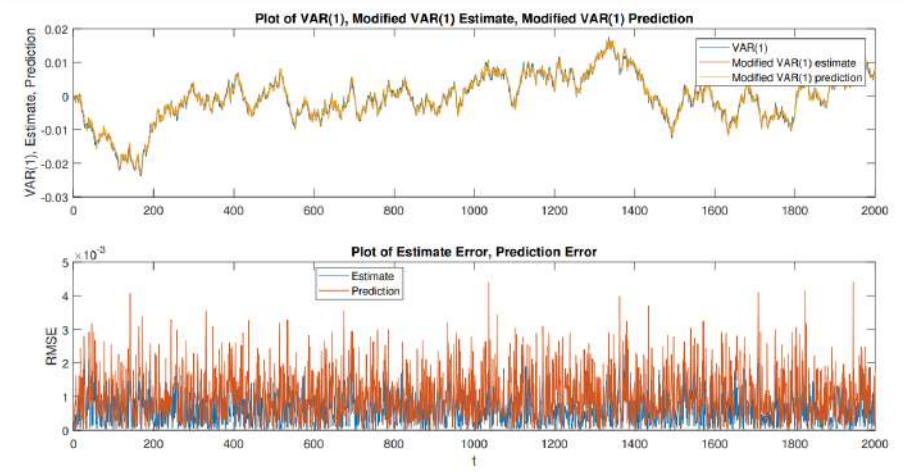

Figure 6: Pentavariate VAR(1) - Variable 1

The first subplot gives comparison of the VAR(1), modified VAR(1) estimate and modified VAR(1) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(1) and the modified VAR(1) and between VAR(1) and the modified VAR(1) prediction for variable 1.

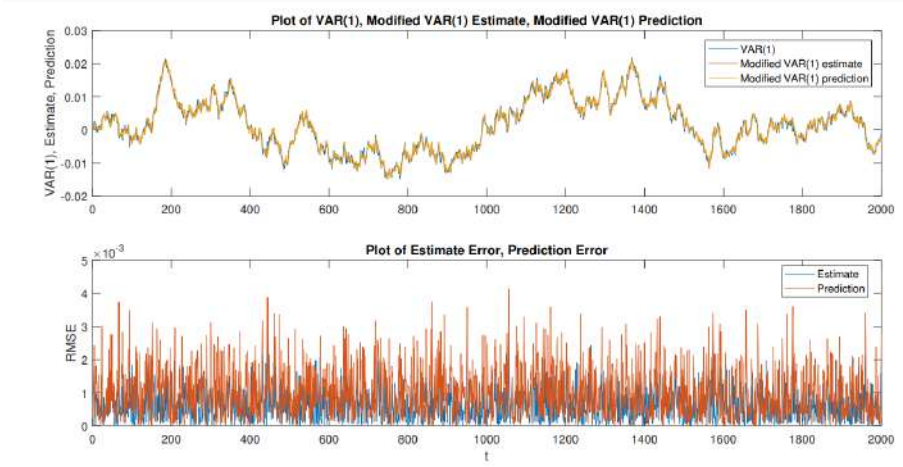

Figure 7: Pentavariate VAR(1) - Variable 2

The first subplot gives comparison of the VAR(1), modified VAR(1) estimate and modified VAR(1) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(1) and the modified VAR(1) and between VAR(1) and the modified VAR(1) prediction for variable 2.

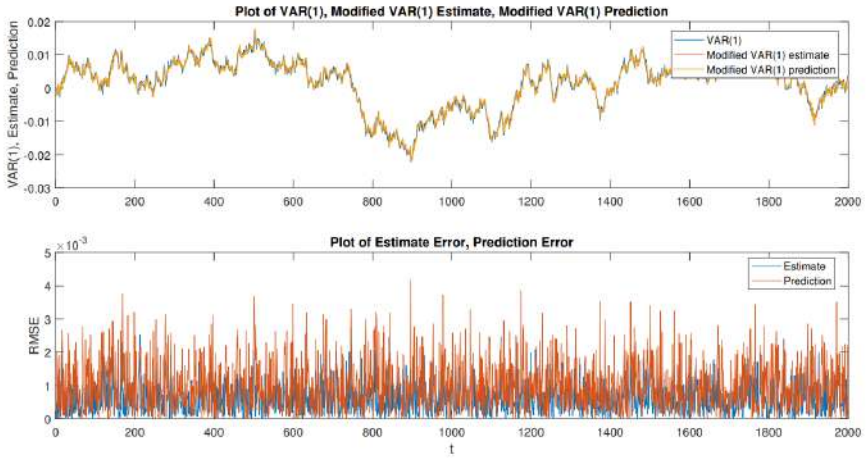

Figure 8: Pentavariate VAR(1) - Variable 3

The first subplot gives comparison of the VAR(1), modified VAR(1) estimate and modified VAR(1) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(1) and the modified VAR(1) and between VAR(1) and the modified VAR(1) prediction for variable 3.

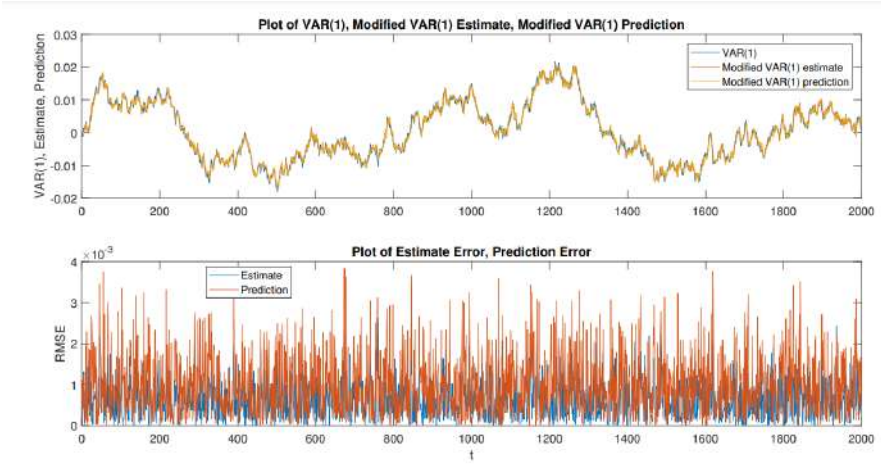

Figure 9: Pentavariate VAR(1) - Variable 4

The first subplot gives comparison of the VAR(1), modified VAR(1) estimate and modified VAR(1) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(1) and the modified VAR(1) and between VAR(1) and the modified VAR(1) prediction for variable 4.

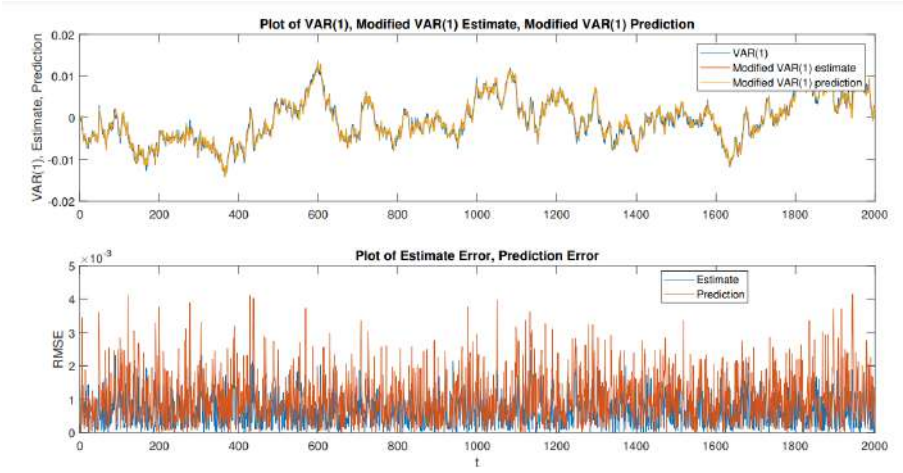

Figure 10: Pentavariate VAR(1) - Variable 5

The first subplot gives comparison of the VAR(1), modified VAR(1) estimate and modified VAR(1) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(1) and the modified VAR(1) and between VAR(1) and the modified VAR(1) prediction for variable 5.

Figs. 6-10 shows that the updated model gives precise estimates as seen from the small value of root mean square error in the update and in the prediction for each variable. The RMSE values range from 0 to 0.005 in the five variables.

3.1 Application to Real Data

We consider testing the performance of the updated model by considering fitting a VAR(p) model to some secondary data. We consider secondary quarterly data for the contribution of the manufacturing, wholesale and retail, and transport and communication sectors to the national GDP obtained from the Kenya National Bureau of statistics (KNBS), quarterly GDP reports from 2009 quarter 1 to 2022 quarter 3. We fitted the VAR(2) model given by

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{pmatrix} = \begin{bmatrix} -0.5 & 0.09 & 0.01 \\ -0.27 & -0.21 & 0.13 \\ 0.1 & 0.05 & -0.21 \end{bmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \\ y_{3,t-1} \end{pmatrix} + \begin{bmatrix} -0.59 & 0.25 & -0.13 \\ -0.1 & -0.34 & 0.04 \\ 0.23 & -0.04 & 0.08 \end{bmatrix} \begin{pmatrix} y_{1,t-2} \\ y_{2,t-2} \\ y_{3,t-2} \end{pmatrix} + \begin{pmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \end{pmatrix} \tag{3.3}$$

We test the performance of the fitted model by associating it with measurement equation given by

$$\begin{pmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \end{pmatrix} \tag{3.4}$$

Using **Algorithm 1** and MATLAB software we obtain the plots in Figs. 11-13 which represent the manufacturing, wholesale and retail, and transport and communication respectively. The first subplot in Figs. 11-13 represents the output for VAR(2), modified VAR(2) estimate and modified VAR(2) prediction denoted by the blue line, red line and the yellow line, respectively. The second subplots in Figs. 11-13 represents the RMSE in the estimate and prediction denoted by the blue and the red lines, respectively. From Figs. 11-13, it is observed that the updated model performs well due to the small RMSE values obtained in the estimate and the prediction. For instance, in the variables, manufacturing and wholesale and retail, the RMSE values in both the estimate and the prediction are less than 0.005 which can be considered to be low. In the transport and communication variable, the RMSE values are less than 0.004 which we may consider to be low thus good performance from the processes of prediction and update.

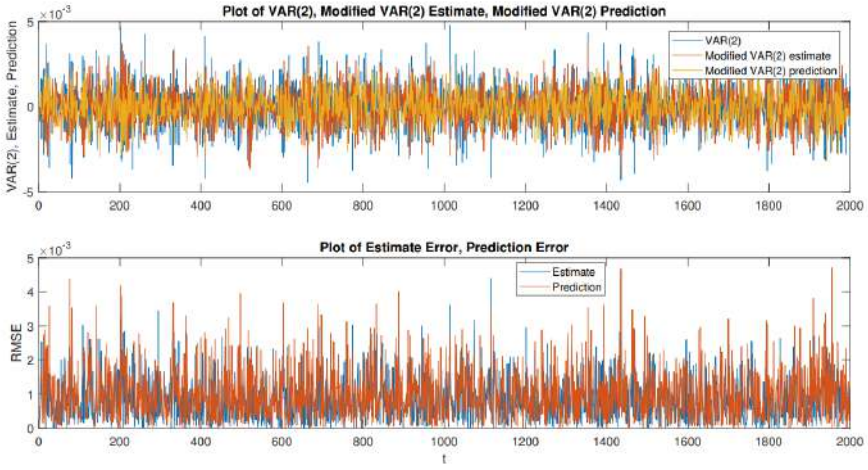

Figure 11: Trivariate VAR(2) - Variable 1 (manufacturing)

The first subplot gives comparison of the VAR(2), modified VAR(2) estimate and modified VAR(2) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(2) and the modified VAR(2) and between VAR(2) and the modified VAR(2) prediction for variable 1.

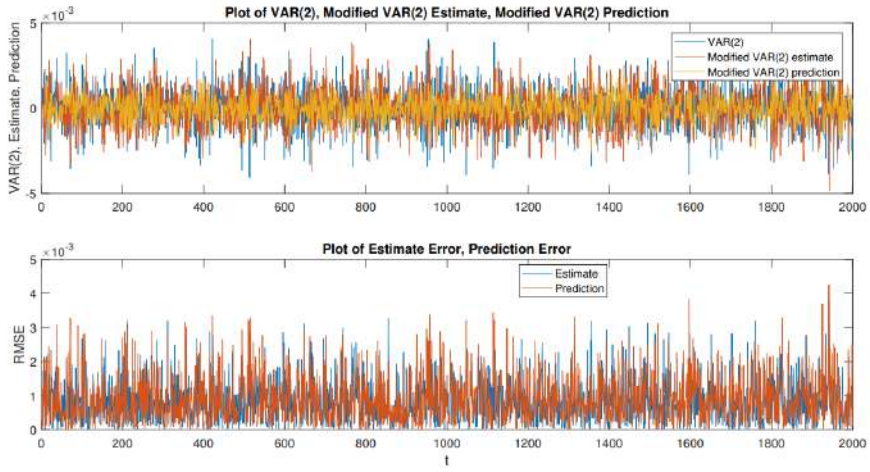

Figure 12: Trivariate VAR(2) - Variable 2 (wholesale and retail)

The first subplot gives comparison of the VAR(2), modified VAR(2) estimate and modified VAR(2) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(2) and the modified VAR(2) and between VAR(2) and the modified VAR(2) prediction for variable 2.

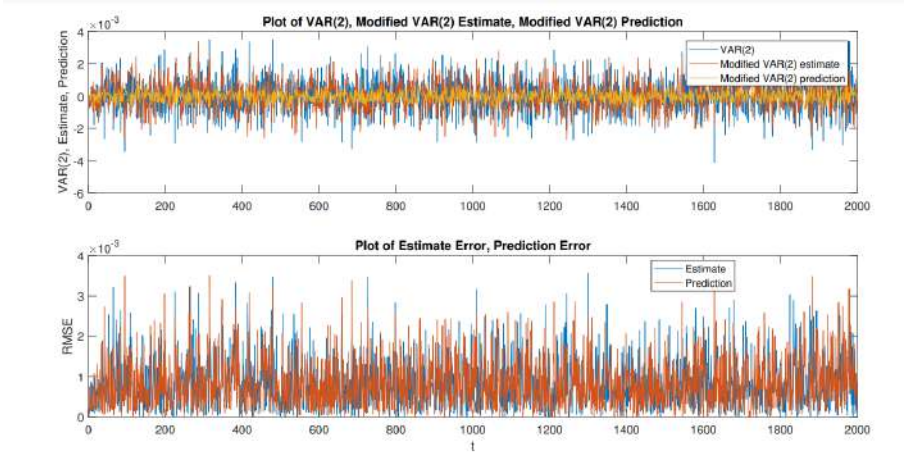

Figure 13: Trivariate VAR(2) - Variable 3 (transport and communication)

The first subplot gives comparison of the VAR(2), modified VAR(2) estimate and modified VAR(2) prediction, denoted by the blue line, red line and the yellow line, respectively while the second subplot shows the errors between VAR(2) and the modified VAR(2) and between VAR(2) and the modified VAR(2) prediction for variable 3.

4 Conclusion

In this paper we developed an updated Vector Autoregressive (VAR(p)) model using the Bayesian approach. The existing Vector Autoregressive (VAR) model is taken to be the prior while new information (measurements) that is gotten is used as the likelihood to update the existing VAR model. After incorporating new information, the proposed updated Vector Autoregressive model of order p is obtained (refer to **Algorithm 1**). The performance of the updated VAR(p) model is then compared with corresponding vector autoregressive models. It is found that the errors between VAR and the modified VAR estimate are less in the models considered. Furthermore, the errors between VAR and the modified VAR prediction are as well low. The plots of the RMSE show that the updated model performs well as indicated by small values of RMSE in the update and in the prediction.

Conflicts of Interest

The authors have no competing interests to declare that are relevant to the content of this article.

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Optimization of Drying Parameters for Minimization of Moisture Content in Black Tea Production

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ABSTRACT

Study focuses on black tea drying process on fluidized bed dryer in tea factories. Tea drying variables considered in the study were air temperature, air velocity and time. The response variable in the experiment was the black tea moisture content. Air velocity was varied between 0.21 m/s and 0.55 m/s. Whereas air temperature was varied between 70 ^{0}C and 130 ^{0}C. Drying time ranged between 0 minute and 20 minutes. Black tea drying experiment was conducted in the macerated tea laboratory at Sotik Tea Company using the miniature FBD Sherwood Tornado model 501. It took 20 minutes experimental time to lower the dhool moisture from 72% to 3.5%. Experimental data was used to develop black tea drying curve and drying rate. The Box Behnken design under response surface design methodology in Minitab software was used to analyze and optimize the black tea drying variables. The optimum variables were found to be at hot air temperature of 100 ^{0}C, hot air velocity of 0.38 m/s and drying time of 12.9 minutes. These drying parameters resulted in a more acceptable black tea moisture content of 3.5% db which falls between the acceptable black tea moisture content of 3% db to 4% db.

Keywords: Dhool, Drying, Fluidized, Moisture, Optimum, Tea

I. INTRODUCTION

Tea is a common aromatic and nonalcoholic beverage consumed globally (Lang'at, Thoruwa *et al.*, 2016). Tea originated from China, and it is the second most popular beverage in the world after water (Pou, Paul *et al.*, 2019). Total world tea production stands at 6012 tons. Major producers are Kenya, China, India, Sri Lanka, Indonesia and Vietnam. Kenya is the leading exporter of tea globally (Hajra, 2021) (Hicks, 2009). Tea is rich of antioxidant owing to Polyphenols, or flavonoids composition and medicinal applications such as cancer prevention (Panda and Datta, 2016) (da Silva Pinto, 2013).

All tea is produced from two leaves and one bud with botanical description of Camellia Sinensis Tea is classified depending on the processing method. Tea is mainly produced in four major types, Cut Tear and Curl (CTC), Orthodox, green and Oolong. Black CTC tea goes through plucking, withering, rotor vane, maceration, fermentation, drying and packaging process. Orthodox tea is processed through, plucking, withering, rolling, fermentation, drying and packaging. Green tea goes through the Plucking withering maceration drying and packaging. Oolong tea is processed through withering, rotor vane, CTC, Oxidation, drying and packaging. Black tea constitutes of 78% of the total tea produced globally, hence the leading produced tea. (Stodt, Blauth *et al.*, 2014). Tea drying is a very vital process. Drying ceases the progression of enzymatic oxidation, inhibits growth of bacteria, gives the final product colour, reduces moisture content from 70% db to 4% db and prolongs products shelf life (Temple and Van Boxtel, 1999). Fluidized bed dryer is considered as the most efficient dryer in tea industries (Özahi and Demir, 2013). The ideal drying temperature for black lays between 70 ^{0}C to 130 ^{0}C. In tea industries, tea drying process is the major consumer of energy about 10% to 15%. Thermal energy constitutes above 75% of the total energy consumption (Chaudhari, Kostha *et al.*, 2018). The thermal energy is directly consumed during green leaves withering and during dhool drying for moisture removal from the leaves (Akhtaruzzaman, Ali *et al.*, 2013). Studies on drying processes has shown that about 30% exergy utilization in fluidized bed dryers (Handayani, Atmanto *et al.*, 2020). Hence, 60% to 70% of total production cost (Das, Mahanta *et al.*, 2021). For sustainable development achievements, energy efficiency is a key factor in industries (bahu RE Energy consumption). In this regard, maximum

energy utilization through process optimization and thermal analysis is foremost (Akbulut and Durmuş, 2010) (Sarker, Ibrahim *et al.*, 2015)

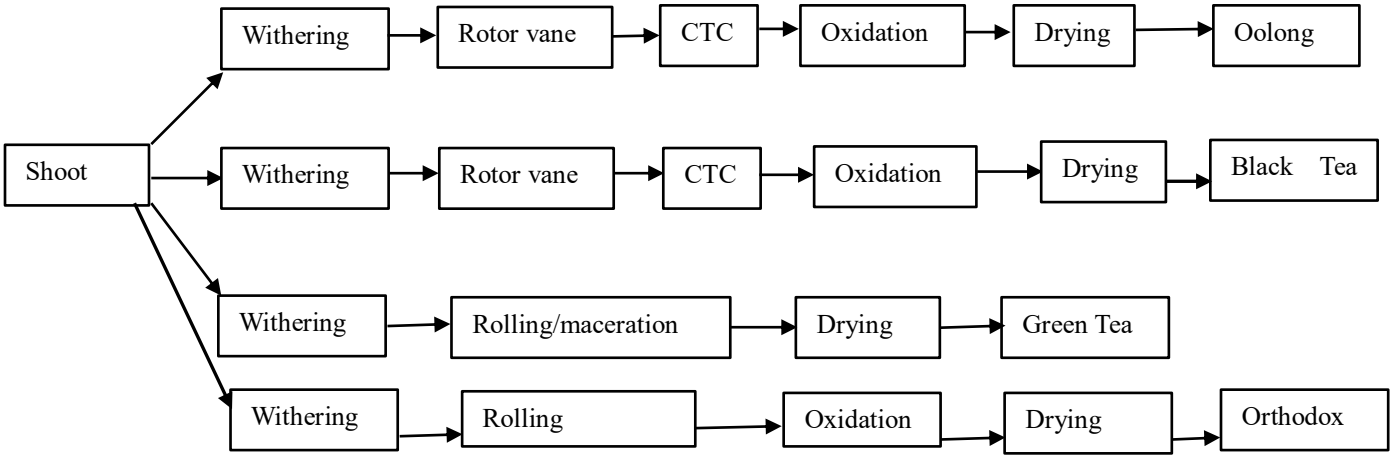

Figure 1
Tea processing flow chart (Pou *et al.*, 2019)

Various techniques are used in tea drying process, this includes Fluidized bed dryer, endless chain pressure dryer, spray dryer, super-heated tea dryer, solar energy assisted dryer and microwave. Fluidized bed dryer is used in tea drying process since dhool is of nonuniform particle and drying air can be modelled to allow for bigger particle fluidization and at the same time the smaller particles are fluidized and maintained the drying chamber (Handayani, Yohana *et al.*, 2023). Hot air inlet in FBDs also provides pneumatic transport of tea particles, increases tea agitation on the drying bed that enhances energy utilization (Temple *et al.*, 1999).

Four distinct process take place at the same time during black tea drying phase.

a. Heat transfer: Hot air transfers heat to the surface of the wet dhool through convection. The rate of heat transfer is function on dhool property and temperature gradient. Transferred heat from the hot air to the room temperature dhool, will be the latent heat of vaporization. Dhool convectional heating will increase the temperature gradient on the surface of the dhool consequently the heat will tend to flow towards the center of the dhool particle through conduction (Chukwunonye, Nnaemeka *et al.*, 2016)
b. Mass transfer: As the hot air blows through the dhool, vaporized moisture will be carried with air resulting in moisture reduction in dhool and increase in air humidity. Transfer of moisture from dhool goes on until when the vapour pressure in the dhool will be equal to the partial pressure of water vapour in the air. This state is known as equilibrium moisture content, and this varies depending with the surrounding ambient (Chukwunonye *et al.*, 2016).
c. Heat transfer: Heat is transferred from the dhool surface to the center on the dhool particle through conduction
d. Mass transfer: Moisture transfer from the center dhool particle to the dhool surface.

Study of moist air is known as psychometry (Singh and Heldman, 2009)**.** The psychometry chart was useful to study black tea convective drying processes. Psychometric chart is defined for a pressure of 101.3 kPa which is the case of black tea drying process. On the psychometric chart, specific humidity is displayed on the Y axis while the X axis displays dry bulb temperature. The curves depict constant wet bulb temperature, constant enthalpy, constant relative humidity and constant specific volume.

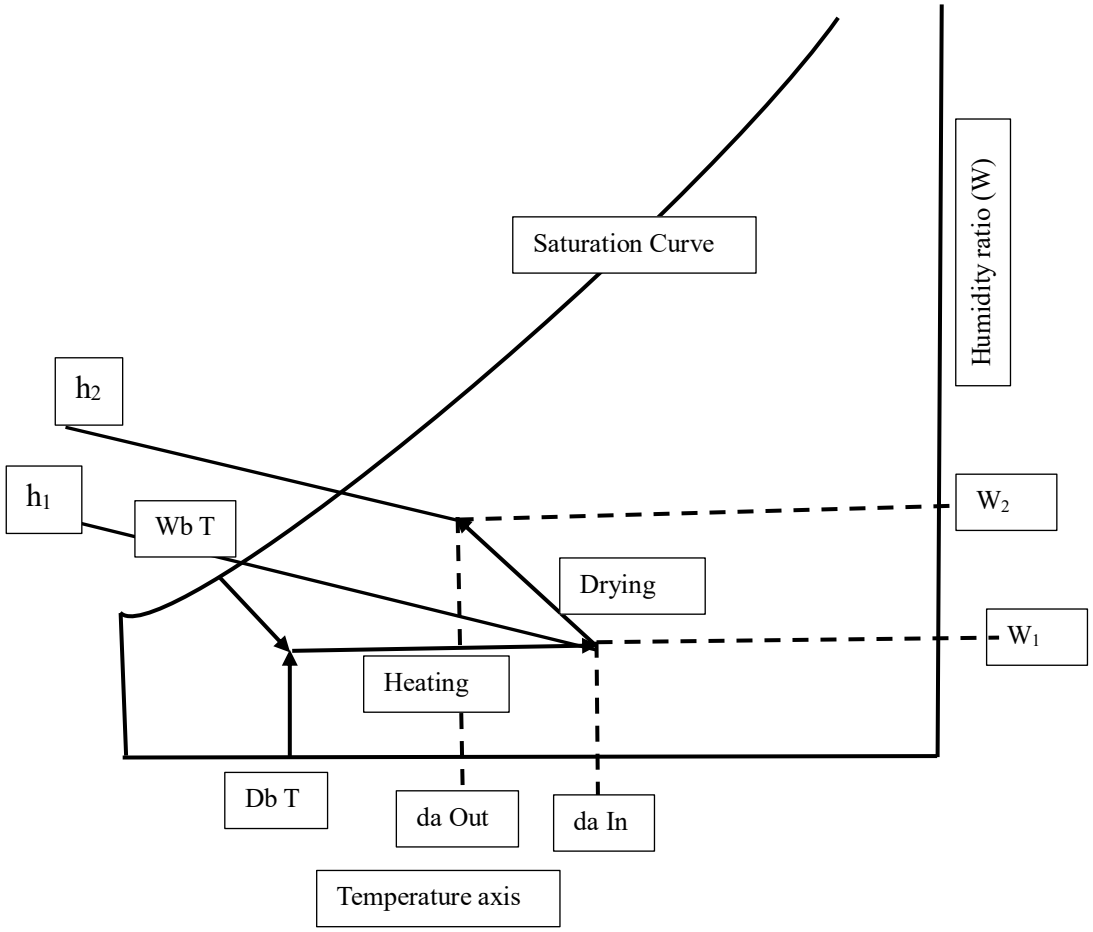

Figure 2
Drying process presentation on Ashae psychometric chart

A number of research works has been done on drying process optimization but, they have all considered only one drying parameter holding other parameters constant. Therefore, this study focused on linear, square and 2-way interaction models for process optimization. Thus, this justifies the objective of this study, to present optimal black tea drying process parameters, determine the black tea drying rate and develop black tea drying curve.

II. MATERIALS AND METHODS

2.1 Materials

Green leaves from the field were plucked (two leaves and a bud). The leaves were withered and then macerated on the CTC to wet macerated leaves referred to as dhool. The dhool from the CTC was fermented at the continuous fermenter's units with conditioned air at 27 ^{0}C. Fermented dhool was taken from the tea processing line for drying at the laboratory using a lab scale fluidizes bed dryer Sherwood 501 model. Figure 3 shows the exploded Sherwood 501 dryer.

2.2 Experiment set up

The fluidized bed dryer used to study tea drying consists of the following components.

1. Blower
2. Electric heater
3. Drying chamber with perforated bed
4. Control panel (Air temperature, air velocity and drying time)

Figure 3. Shows the schematic flow diagram of the laboratory fluidized bed dryer. Fluidized bed dryer experimental study was conducted at the Sotik Tea Company limited using a miniature laboratory quick fluidized bed dryer Sherwood Tornado model 501. Dhool weighing 300 grams was used in conducting the drying experiment. The initial moisture content of the fermented dhool was determined (W_w). Fluidized bed dryer was switched on pre-run for 5 minutes in order to achieve the initial experimental start conditions of FBD. This phase reduced the heat loss through the FBD wall which was at room temperature. After the FBD pre heating was complete, dhool was put into the FBD to start the drying experiment. After time (t), dry dhool was weighed and weight recorded as (W_d.). Tea moisture content X, was then calculated in dry base db using equation 1. Referenced to experimental researches by (Tasirin, Kamarudin *et al.*, 2007) and (Akpinar, Bicer *et al.*, 2003).

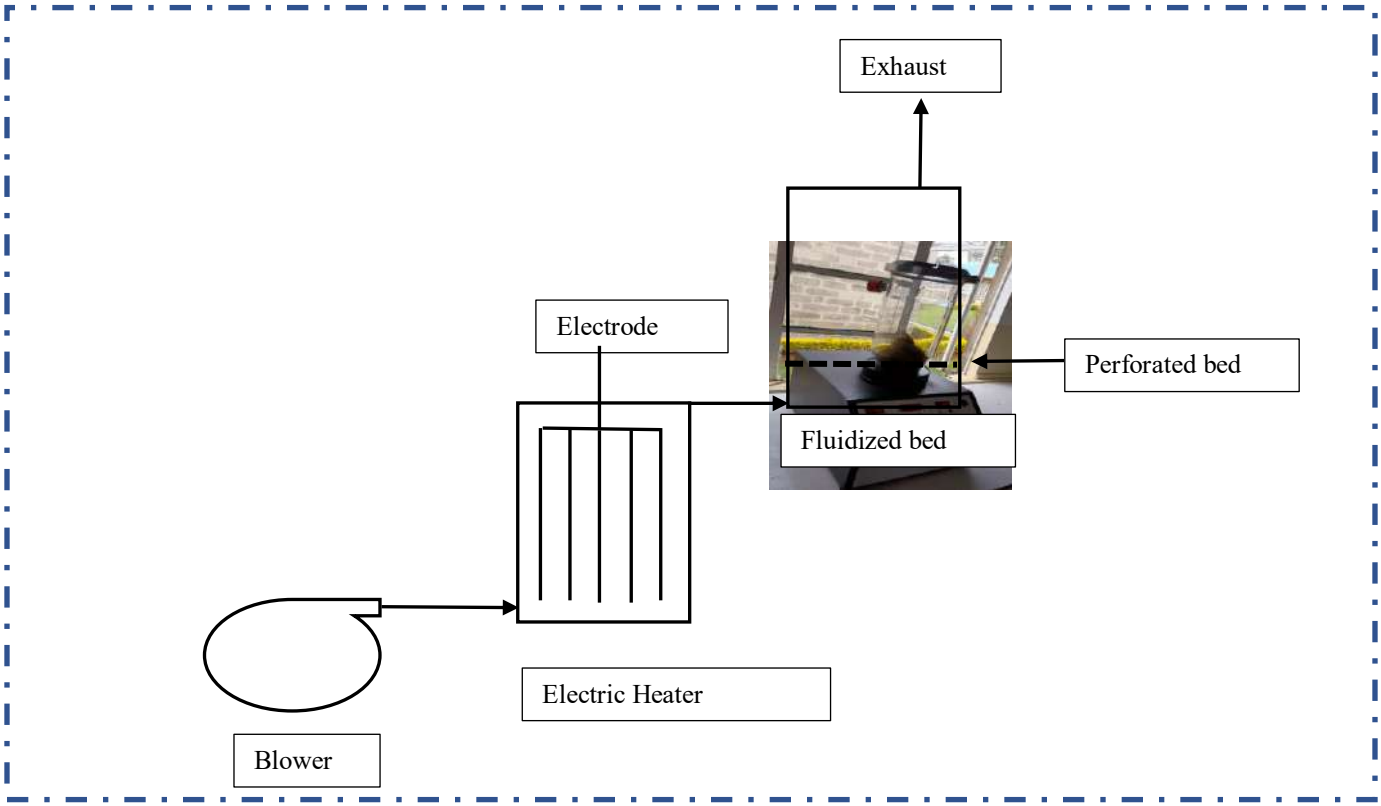

Figure 3
Schematic illustration of fluidized bed dryer

The moisture content in this study was calculated on dry base using the equation 1.

$$X = \frac{(W_w - W_d).W_d}{100} \quad (1)$$

Figure 4
Laboratory KERN 1000-2 weighing scale

2.3 Design of experiment

Minitab software was used for design of experiment and analysis of experimental data. Box Behnken design of the response surface methodology with 3 factors and two levels was used to design experimental models, analyze and optimize the drying process. Table 1. Shows levels and factors that were used in the experiment.

Table 1
Experimental Levels and Factors

Symbol	Intermediate variables	Actual levels uncoded factors	
		Low	High
da_T	Air temperature (^{0}C)	70	130
da_V	Air Velocity (m/s)	0.21	0.55
t_d	Drying Time (min)	3	20
D	Dhool	300 (g) and 4 cm height	

2.4 Experimental models

Table 2. Illustrates the experimental models that were developed from the Box Behnken methodology.

Table 2
Statistical summary of models

Std Order	Run Order	Blocks	Temperature	Velocity	Time
1	1	1	70	0.21	11.5
2	2	1	130	0.21	11.5
3	3	1	70	0.55	11.5
4	4	1	130	0.55	11.5
5	5	1	70	0.38	3
6	6	1	130	0.38	3
7	7	1	70	0.38	20
8	8	1	130	0.38	20
9	9	1	100	0.21	3
10	10	1	100	0.55	3
11	11	1	100	0.21	20
12	12	1	100	0.55	20
13	13	1	100	0.38	11.5
14	14	1	100	0.38	11.5
15	15	1	100	0.38	11.5

III. RESULTS & DISCUSSION

3.1 Experimental results

Table 3 below, shows the moisture content experimental data obtained during dhool drying process using laboratory scale Sherwood 501 fluidized bed dryer. Tea samples were taken manually intermittently from the dryer, tea weight measured using KERN 1000-2 laboratory weigh scale and tea moisture content determined in dry basis.

Table 3
Moisture content at different drying time

Moisture Content (kg H_2O / kg DM)	72	32	21	15	11	9	7	5	3.5
Time (min)	0	3	5	8	10	13	15	17	20

3.2 Drying curve

Figure 5 shows the moisture content profile of black tea drying process verses drying time. The initial moisture content of the fermented macerated tea was 72% db. The moisture content of fermented tea decreases with time in the fluidized bed dryer as hot air was blown through the bed. The final black tea moisture content of 3.5% db was attained after 20 minutes.

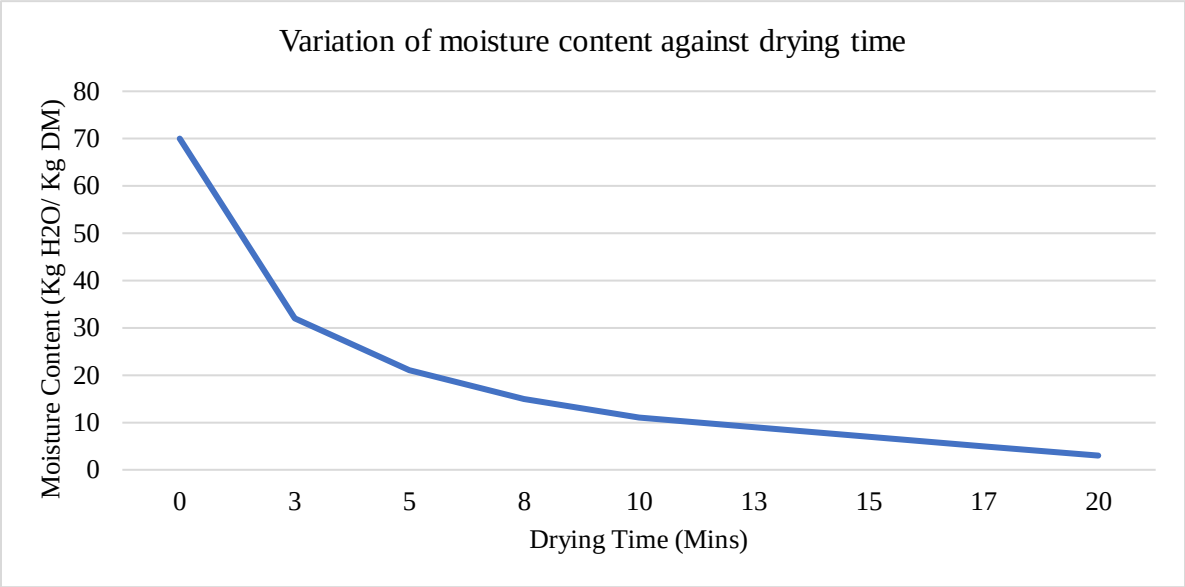

Figure 5
Black tea drying curve

3.3 Drying rate curve

Figure 6 exhibits four regions during the black tea drying process. The first phase is the initial drying phase (A' - A) also referred to as the preheating phase. During this phase, dhool surface was heated to ambient air temperature. Region A to B on the curve is the constant rate period. At the beginning of the constant rate point A, the dhool was wet and fully covered with superficial water. During drying period at constant rate, the unbound water around the wet dhool was evaporated whereas the temperature of dhool remained unchanged. At point B, all superficial water is observed to have been removed and the dhool is at critical moisture content. The critical moisture content of the dhool transitioned to falling rate point B to C. At this point, the dhool drying rate started to decrease. Region C to D is the second falling rate. At this stage of drying, the amount of water removed from the dhool was relatively small compared to other regions. Dhool attained a dynamic equilibrium moisture content of 3.5 kg H_2O/ kg DM at point D.

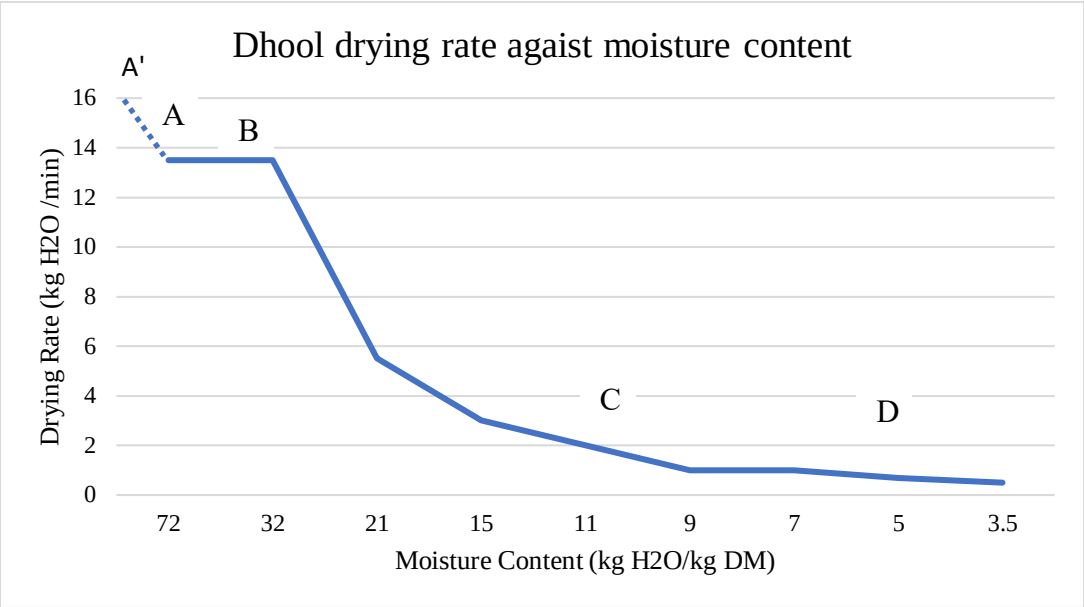

Figure 6
Black tea drying rate curve

3.4 Statistical Data

Table 4. Shows the response parameter (moisture content) at various input variable combinations for each of 15 experimental runs. The data obtained from the experiment were analyzed using Box Behnken under surface response method. Drying air temperature, drying air velocity and drying time were the experiment factors while moisture content was the response variable for the experiment.

Table 4
Experimental and response moisture content at different drying conditions

Std Order	Run Order	Blocks	Temperature (^{0}C)	Velocity (m/s)	Time (mins)	Moisture (%)
1	1	1	70	0.21	11.5	55
2	2	1	130	0.21	11.5	32
3	3	1	70	0.55	11.5	25
4	4	1	130	0.55	11.5	15
5	5	1	70	0.38	3	68
6	6	1	130	0.38	3	21
7	7	1	70	0.38	20	10
8	8	1	130	0.38	20	4
9	9	1	100	0.21	3	50
10	10	1	100	0.55	3	32
11	11	1	100	0.21	20	11
12	12	1	100	0.55	20	9
13	13	1	100	0.38	11.5	7
14	14	1	100	0.38	11.5	3.5
15	15	1	100	0.38	11.5	8

3.5 Model Summary

Model at the confidence level of 95% and 5% significant, resulted to coefficient of determinants R^2 and adjusted R^2 of 96.98% and 91.54% respectively. This shows that the model is significant and adequate. The R-sq value of 96.98%, means that the model's response which is the dhool moisture content can explains tea drying variables of study.

Table 5

R – Squared summary

Co-efficient of determinant	Value
S	5.88926
R-square	96.98%
Adjusted R- squared	91.54%
Predicted R- Squared	54.35%

3.6 Analysis of variance

Analyze response surface design tool under response surface methodology was used to analyse the experimental data shown in Table 4. Table 6, illustrates analysis of variance of the model at 95% confidence level and 5% significant level. Analysis yieded a F-value and a P-value of 17.87 and 0.003 respectively. This depicts that the model was significant. It is also evident from the ANOVA table that all variable of study are significants as because they are all less than the significant value of 0.05.

Table 6

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	9	5567.82	618.65	17.84	0.003
Linear	3	3831.75	1277.25	36.83	0.001
Air Temperature	1	924.50	924.50	26.66	0.004
Air velocity	1	561.13	561.13	16.18	0.010
Drying Time	1	2346.12	2346.12	67.64	0.000
Square	3	1209.57	403.19	11.62	0.011
Air Temperature*Air Temperature	1	616.03	616.03	17.76	0.008
Air velocity*Air velocity	1	592.41	592.41	17.08	0.009
Drying Time*Drying Time	1	164.10	164.10	4.73	0.082
2-Way Interaction	3	526.50	175.50	5.06	0.056
Air Temperature*Air velocity	1	42.25	42.25	1.22	0.320
Air Temperature*Drying Time	1	420.25	420.25	12.12	0.018
Air velocity*Drying Time	1	64.00	64.00	1.85	0.232
Error	5	173.42	34.68		
Lack-of-Fit	3	162.25	54.08	9.69	0.095
Pure Error	2	11.17	5.58		
Total	14	5741.23			

3.7 Regression Equation in Uncoded Units

Equation 2 shows the overal regression equation which relates the moisture content and the three factors of study.

Moisture Content =
385.4- 3.933 Air Temperature- 478 Air velocity- 9.21 Drying Time
+ 0.01435 Air Temperature*Air Temperature+ 438 Air velocity*Air velocity
+ 0.0923 Drying Time*Drying Time+ 0.637 Air Temperature*Air velocity
+ 0.0402 Air Temperature*Drying Time + 2.77 Air velocity*Drying Time **(2)**

3.8 Modified Regression Equation in Uncoded Units

The significant variables were evaluated as shown in the ANOVA Table 5. The model was therefore improved with elimination of non-significant variables. The regression equation 3, consequently shows the improved regression equation with only significant terms in the study.

Moisture Content =
385.4- 3.933 Air Temperature- 478 Air velocity- 9.21 Drying Time
+ 0.01435 Air Temperature*Air Temperature+ 438 Air velocity*Air velocity
+ 0.0402 Air Temperature*Drying Time **(3)**

3.9 Optimization of drying variables

Response optimizer tool of surface response design methodology was used to optimize the black tea drying process parameters. The response optimizer gave the optimal process variables values of 100 ^{0}C drying air temperature, 0.38 m/s drying air velocity/s and 12.9 minutes dhool retention drying time. The resultant moisture content of black tea was 3.5 kg of water/ kg DM.

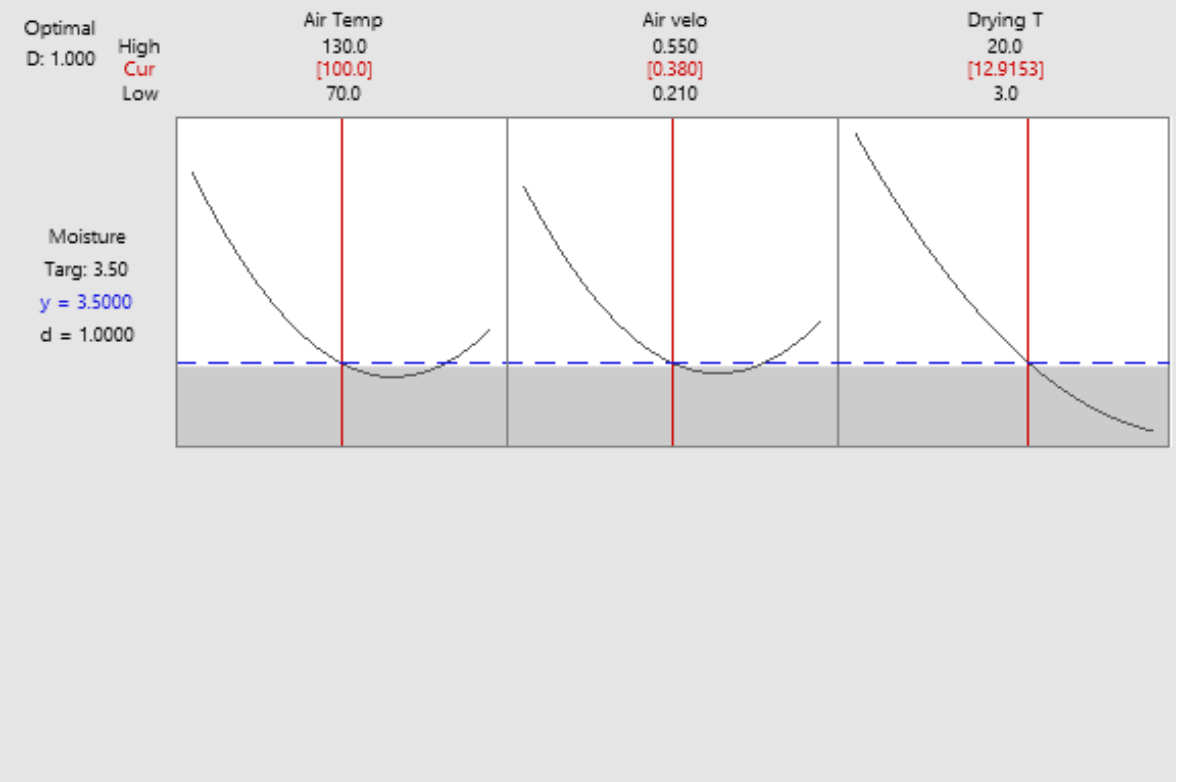

Figure 7
Optimal drying process variable

3.10 Surface and contour plots

Figure 7, show the surface plot of moisture content vs drying air temperature and drying air velocity. From the surface plot, it is observed that the dhool moisture content reduces with increase of both drying air velocity and drying air temperature.

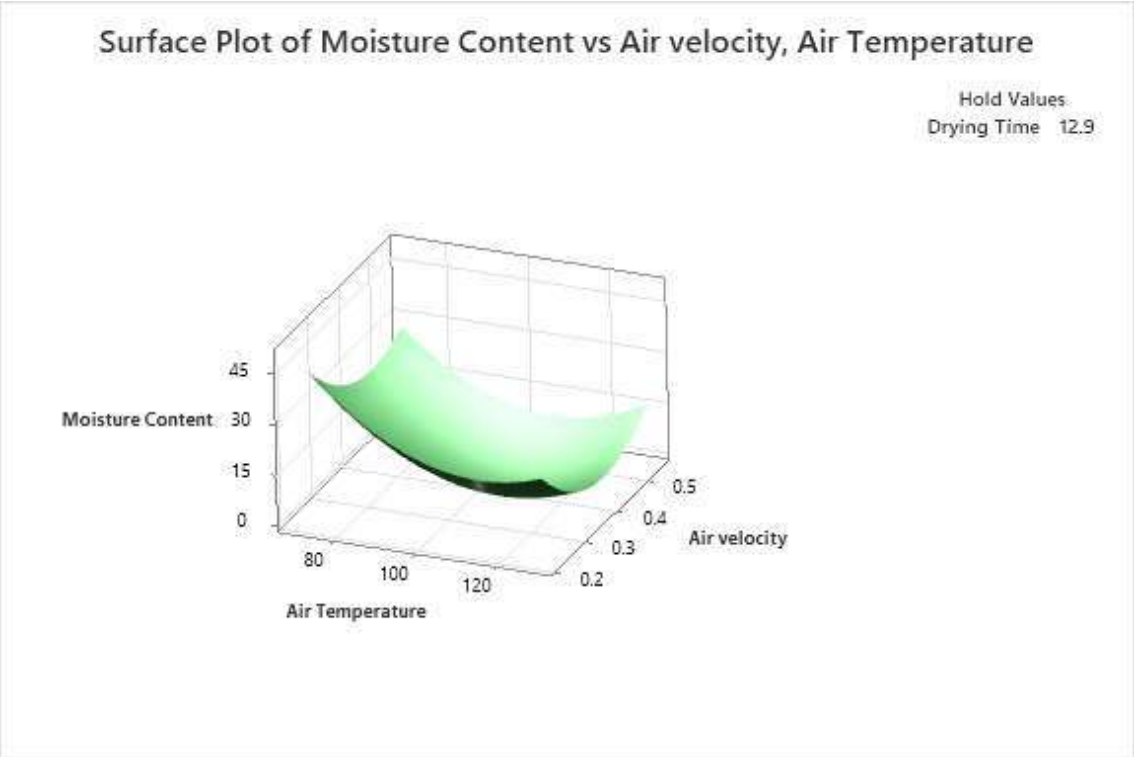

Figure 8
Surface Plot of moisture vs velocity, temperature

Figure 8 shows contour plot of dhool moisture content visa vee drying air velocity and drying air temperature. Drying air temperature between 88 ^{0}C to 130 ^{0}C and air velocity between 0.3 m/s to 0.55 m/s yielded moisture content of below 10 kg of water / kg MD. The most acceptable moisture falls within this region. This is validates the Minitab response optimizer results of, 100^{0}C drying air temperature and 0.38 m/s drying air velocity.

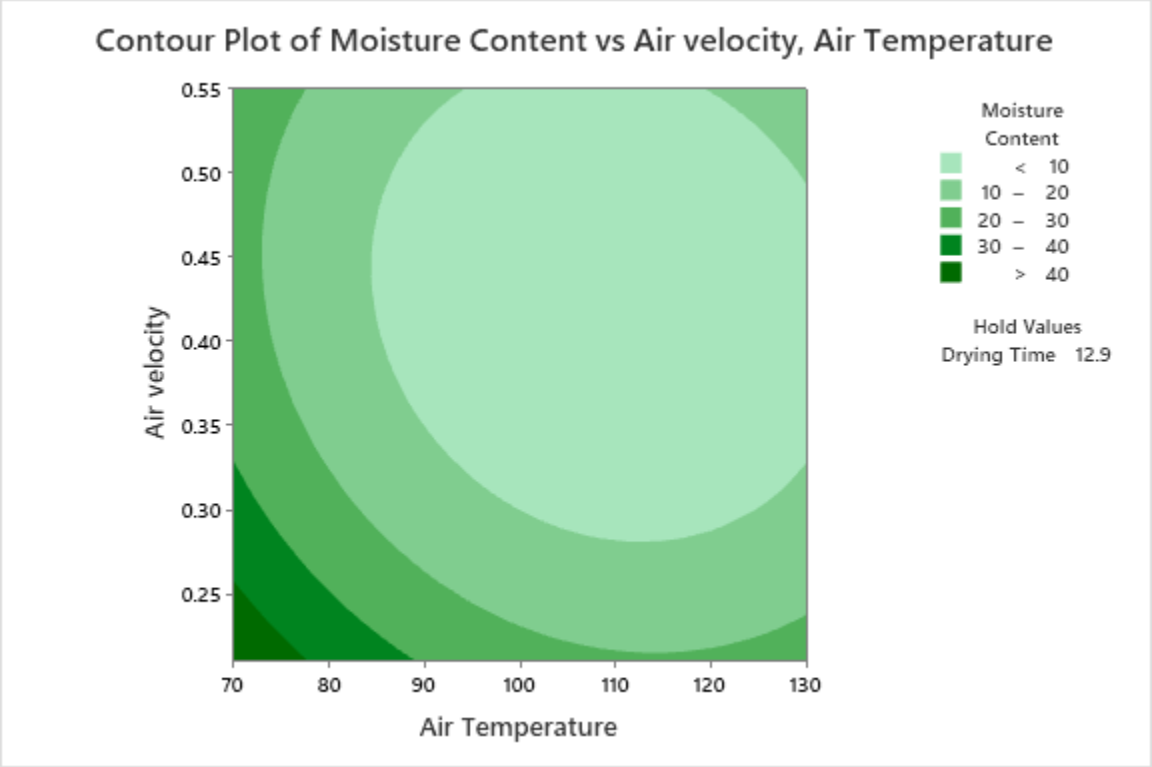

Figure
Variables contour plot of moisture Vs velocity, Temperature

IV. CONCLUSION

The drying process of black tea is key since it ceases enzymatic reactions and growth of bacteria in tea. Drying also enhances tea leaves color and sensory. Drying also reduces tea leaves moisture content to fall between 3 - 4 kg of water / kg of DM, with intention of increasing shelf-life. In fluidized bed dryer, most dhool drying process happened in the falling rate period. The experimental analysis of tea drying resulted to a dynamic equilibrium moisture content of the black tea of 3.5 kg of water / kg DM was attained after 12.9 minutes of drying time. Drying process optimization gave optimal drying parameters of, drying air velocity of 0.38 m/s, drying air temperature of 100 ^{0}C and drying time of 12.9 minutes. Experimental results in this study is projected to be helpful in optimizing tea industry fluidized bed dryers and for future design and production of dryers.

Recommendation
Since dryers are energy intensive and they consume much of the energy produced in the tea factories. Study ought to be explored on dhool pre-treatment prior to drying process with the intention of reducing dryer's energy consumption and tea leaves quality.

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Promoting Logistics Visibility in Supply Chain and Its Influence on Performance of Manufacturing Industry: A Study on Kenya's Food and Beverage Manufacturing Firms

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ABSTRACT

This paper sought to examine the relationship between logistics visibility and performance of large food and beverage manufacturing firms in Kenya. The study was anchored on transaction cost analysis theory. The study utilized a cross-sectional research design. The target population was 561 respondents drawn from the 187 large food and beverage manufacturing firms in Kenya. The sampling frame for the study was the human resource managers in each of the food and beverage manufacturing firms since they are the custodians of all the employees in the firms. Using a stratified random sampling, a sample of 228 respondents was selected. A structured and semi-structured questionnaire was used to collect data for the study. Using a 10% of the sample size, the questionnaire was pilot-tested to assess validity and reliability before the actual data was collected. The data was analyzed using SPSS version 27. The data was presented using tables. The findings revealed that logistics visibility significantly and positively influenced performance of food and beverage manufacturing firms. It was concluded that as a result of ineffective embrace of logistics visibility, the manufacturing firms failed to optimize their supply chain towards enhancing performance. The study recommended the essence of logistics visibility as a way of ensuring control of logistics processes among the food and beverage manufacturing firms for better performance.

Keywords: Firm Performance, Food and Beverage Manufacturing Firms, Logistics Visibility, Supply Chain Visibility

I. INTRODUCTION

Supply chain visibility (SCV) is the process of tracking down the flow of goods/products from the supplier to the customer through the manufacturer. It involves putting across measures and systems that ensure proper tracking of the customer orders and harmonizing them with the suppliers' ability to deliver and their planned delivery time (Moshood et al., 2021). This ensures that the suppliers are able to deliver goods on time based on the customers' orders, and the manufacturer is able to meet the needs of the customers. In the modern business market, leading organizations particularly those in the manufacturing sector are embracing supply chain visibility as a way of enhancing their efficiency in supply chain processes. According to Swift et al. (2019), companies have suffered losses over long waiting hours for the suppliers to deliver the merchandise while in other occasions the suppliers deliver early than expected thus making the companies incur extra costs for holding idle inventory.

Logistics refers to the movement of inventory from the supplier to the area of storage where need be, and finally to the final destination (Xu et al., 2020). Logistics visibility means the stakeholders in the goods are able to see details of the goods in transit. Nowadays, companies increasingly see themselves as part of a supply chain to compete against other supply chains, than a single firm competing against other individual firms. In the past two decades, there has been a trend that manufacturers, distributors and logistics service providers in join hands to collaborate in business operations and decision-making (Zhou, 2015). Enhancing logistics visibility means that the company is in a position to fully control its logistics and have a well-framed flow of its goods from the point of origin to the final consumers (Rogerson & Parry, 2020). Logistics visibility has been argued to be one of the integral critical success factors in supply chain, where organizations are able to ensure that their logistics and long-standing the changing dynamics, and well monitored to achieve the intended goals.

Logistics visibility, as a component of supply chain visibility, encompasses the planning, execution, and supervision of the smooth flow and storage of goods, services, and associated information from the point of origin to the point of consumption, with the aim of meeting customer needs (Kalaiarasan et al., 2022). Companies often engage in partnerships within the supply chain to enhance product distribution and achieve cost savings in logistics visibility (Ahmed et al., 2021). Presently, many companies rely on external logistics providers to collaborate with business

partners and customers, facilitating information exchange throughout the supply chain. Given that logistics constitutes a cornerstone activity in the supply chain, it is imperative to optimize it and improve its efficiency to enhance firm performance (Kharlamov & Parry, 2018). This can be accomplished through continuous monitoring of goods flow, ensuring seamless processes for efficient delivery to end consumers. Furthermore, organizations must establish well-defined logistics processes to monitor the flow effectively, facilitating easier management (Mwangeka, 2020).

Logistics visibility is also addressed in terms of transportation. The process of managing transportation and ensuring that it is aligned to the best possible route ensures that the logistics are more visible and manageable (Jermsittiparsert & Srisawat, 2019). Transportation is the main practice in logistics, thus its effectiveness is a key determinant to the success of the supply chain process. To enhance supply chain visibility, it is integral for organizations to have a means of managing the transportation process to ensures effectiveness and enhance performance (Saqib & Zhang, 2021). Lead time is another aspect that ought to be considered in the logistics visibility (Mwangeka & Odok, 2020). This is guided by the urge to have a more significant and effective logistics process that saves on time and enhances customer satisfaction. Logistics visibility through lead time implies that the organization is not only aiming at enhancing visibility of the delivery process, but also ensuring that lead time is significantly reduced for better customer satisfaction.

The manufacturing sector in Kenya has been applauded as a one of the major economic backbones, playing a fundamental role in supporting other sectors such as the agricultural sector, real estate sector, infrastructure development, as well as the overall job creation in the country (Muthoni & Mose, 2020). One of the sub-sectors of the manufacturing sector in Kenya is the food and beverage manufacturing firms. The food and beverage manufacturing firms in Kenya make majority of the manufacturing sector, with over 40% of the firms listed under the Kenya Association of Manufacturers being from this sub-sector. Kenya association of manufacturers (KAM) describes this category of manufacturing sector as the most vulnerable while at the same time most critical, owing to the customer's ability to change tastes and preferences frequently, entry of international players into the market, and increased dynamism in the entire manufacturing sector, which exposes the firms to a major threat of losing markets. The food and beverage manufacturing firms are the firms that deal with processing, value addition, packaging and distribution of food related products such as cooking oil, maize and wheat flour among others and packaged beverages such as carbonated drinks, fresh juices among others (Muteshi et al., 2018). Most of these firms rely on varied and extensive supply chain processes, whereby they incorporate both push and pull processes, thus supply chain visibility is major requirement for the continued performance.

1.1 Statement of the Problem

The manufacturing sector in Kenya has over the recent past facing a declining trend, calling for robust interventions that do not focus on a single perspective but all-rounded in areas such as finance, accountability, divestment, research and development, and most importantly, the supply chain process. Recently, some of the leading food manufacturing companies like Unga group recorded an 88% decline in profits in the year ended June 2020 and others like Del Monte have recorded over 45% decline in revenues (Muiruri, 2021). In the year 2020, it was reported that Chemilil sugar company had an accumulated losses of over Kshs.6.05billion (Igadwah, 2019) while others like Nzoia Sugar were ruled insolvent by the auditor general after recording over Kshs.49.7 billion of accumulated debts (Ndanyi, 2021). These among other statistics reveal that the food and beverage manufacturing sector is at stumble, prompting the question on what could be lacking that is making the companies sink into losses over the years, while others in the same industry are rapidly growing and recording increased profits.

Available literature shows that logistics visibility is fundamental in steering organizational performance. A study by Khalifa et al. (2021) revealed that logistics visibility had a significant impact on the seamless flow of goods across the supply chain thus promoting organizational performance. Rogerson and Parry (2020) on the other hand revealed that logistics visibility was essential for enhancing organization's control of distribution processes thus steering its effectiveness. A study by Leończuk (2021) showed a weak relationship between logistics visibility and organizational performance. These studies show mixed results and also they have been carried out in different contexts. This motivated this study to examine the relationship between logistics visibility and performance of food and beverage manufacturing firms in Kenya.

1.2 Research Objectives

i. To examine the extent to which large food and beverage manufacturing firms in Kenya have embraced logistics visibility in their supply chain
ii. To assess the performance status of large food and beverage manufacturing firms in Kenya
iii. To evaluate the relationship between embrace of logistics visibility and performance status of large food and beverage manufacturing firms in Kenya.

II. LITERATURE REVIEW

2.1 Theoretical Review

2.1.1 Transaction Cost Analysis (TCA) Theory

This paper was anchored on Transaction Cost Analysis (TCA) theory by Coase (1937). The theory expounds on the economic benefits of the firm having prolonged relationship with its partners. The theory explains the need for visibility of multiple firms as a way of reducing costs and promoting efficiency in the operations. According to Rindfleisch and Heide (1997), the TCA theory best fits in the context of supply chain visibility and visibility, in that it points out to when a firm should consider having expansion of its boundaries to incorporate other key players in the industry such as the suppliers. A company can reduce its total transaction costs by integrating suppliers into their operation framework and ensuring the suppliers fully understands what the customers want (Yu & Goh, 2014). Through this theory, the study will assess the role played by supplier visibility in enhancing firm performance.

For a business to choose whether to perform a particular activity, transactional cost analysis becomes very important. The theory uses transaction as the unit of analysis and divides transaction costs into production and co-ordination costs (Bergman et al., 2019). According to the theory, transaction costs arise at contracting (drafting, negotiation and safeguarding) or at implementation (maladoption, haggling and establishment, operational and bonding costs). Decision makers must weigh and compare the costs associated with executing a transaction within their firms (in-house) and outsourcing. The foundations of TCA were laid down by Coase (1937) and were further developed by Williamson (1991; 1994). Fundamentally, TCA suggests that transaction costs related to make or buy decision impact the choice between the firm and the market. The transaction costs analysis helps in deciding whether to perform activity in-house or outsource from third party. According to TCA, there are five determinants of transaction costs, namely transaction frequency, asset specificity, uncertainty, bounded rationality, and opportunistic behavior.

In supply chain management, when dealing with customers the company has to anlayse its transactional costs and determine the best ways to enhance the satisfaction of the customers through the minimal costs possible (Rogerson & Parry, 2020). The transaction costs include the costs incurred when handling customers and when meeting their needs and requirements. It is essential for companies to ensure that they analyse what costs will be required in meeting varied needs of their customers, thus planning effectively for continued effectiveness and performance (Hamadneh et al., 2021). The customer visibility is one of the aspects of supply chain visibility that requires a proper analysis of the costs for the company to continue supporting the customer-oriented goals. The transactional cost theory, therefore, aligns with the need for customer visibility in supply chain processes, thus the theory was used to highlight on the need for role played by logistics visibility on the performance of food and beverage manufacturing firms

2.2 Conceptual Framework

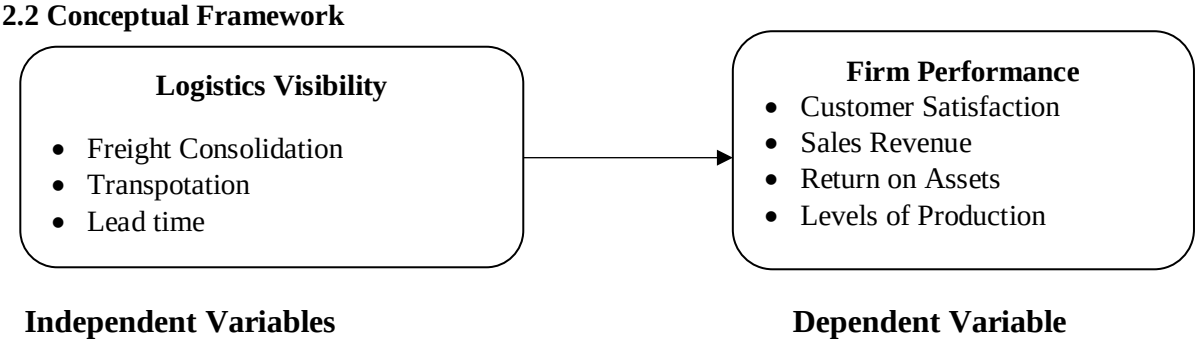

Figure 1
Conceptual Framework

2.2 Empirical Review

Given the complexity of supply chain issues even in a research context, it is understandable that research collaboration, either between scholars or between academia and industry is needed to meet publication outcomes (Papert et al., 2016). Recently, research collaboration has become a focal sub-domain of interest among academics. Logistics visibility, which straddles several traditional learned disciplines, is one such subject matter. Indeed, research collaboration is intense and fast becoming a requisite for success in the publication arena, which often requires multi-

disciplinary expertise. A key indicator for the success of such collaborations is the extent of co-authorship in scholarly output.

One intended outcome of collaboration is an improvement in scientific productivity. However, the literature has also highlighted the collaboration paradox which claims that collaboration seems has a positive impact on publishing productivity, particularly when it is research undertaken in developing areas such as logistics visibility in Asia. The extant literature is also replete with theoretical lenses such as the popular resource based view, and transaction cost economics theory which have been used to show that international research collaborations are more successful when complementary resources are increased and transaction costs reduced (Tang *et al.,* 2016). Clearly, this points to the choice of social networks as an important collaboration mechanism.

Empirical studies have established that logistics visibility helps manufacturers in reducing inventory and improving delivery speed, quality, and customer service whenever they share information and work with suppliers and customers. Though empirical studies on the direct relationship between logistics visibility and customer satisfaction are somewhat rare, some studies indicate the existence of indirect effects. The study by Srinivasan and Swink (2018), for instance, found that logistics visibility, when mediated by manufacturing competitive capabilities, is positively associated with customer satisfaction. Again, Biggs et al. (2017) found out that firms with higher levels of logistics visibility achieve better customer service. Whenever there is high-level logistics visibility, manufacturers usually become satisfied with materials or services provided by suppliers.

III. METHODOLOGY

3.1 Research Design

This study used a cross-sectional research design. According to Sekaran and Bougie (2016), a cross-sectional research design seeks to establish the relationship between two or more variables. Cross-sectional research design answers the what, when, how and why questions, thus giving the study a deeper meaning. The cross-sectional research design was also deemed appropriate in this study since it make it possible to employ the regression model and correlation analysis, thus enabling the testing of the hypotheses of the study.

3.2 Target Population

The target population for this study was 561 respondents drawn from the large food and beverage manufacturing firms in Kenya. According to the Kenya Association of Manufacturers, there are 187 large food and beverage manufacturing firms in Kenya. These are firms that deal with processing, value addition, packaging and distribution of food products and carbonated drinks and other ready-to-drink drinks (KAM, 2020). The unit of analysis was the 187 firms, while the units of observation were the 561 employees drawn from these firms.

3.3 Sampling and Sampling Technique

The study used a stratified random sampling technique where the respondents were categorized into strata comprising of procurement department, finance department and the production/operations department. Respondents were picked randomly from each stratum. The study utilized a sampling formula by Kothari (2017) to establish the appropriate sample size. The formula has been applauded by Taherdoost (2016); Berndt (2020); and Etikan and Bala (2017). The formula is as follows:

$$n = \frac{z^2 \cdot p \cdot q \cdot N}{e^2(N-1) + z^2 \cdot p \cdot q}$$

Where:

$\boldsymbol{n}$ = desired sample size

N = the total population (561)

$\mathbf{e}^2$ = acceptable error (the precision level at 0.05)

p = the proportion in the target population that assumes the characteristics being sought. In this study, a = 50:50 basis is assumed which is a probability of 50 percent (0.5).

q = The balance from p to add up to 100 percent. That is 1-P (1 - 0.5), which in this case is 100 - 50 percent (0.5)

$\mathbf{z}^2$ = number of standard deviation units of the sampling distribution corresponding to the desired confidence level of 95% which is 1.96.

Sekaran and Bougie (2016) recommended that if there are no estimates in the target population assumed to have interest, 50% should be used as the proportion of the target population with characteristics being measured. At 95% confidence level and a 5% level of significance, the sample size (n) was as follows:

n = (1.96 x 1.96) x (0.5 x 0.5) x 561
(0.05 x 0.05) x 561 + (1.96 x 1.96 x 0.5 x 0.5)
n = 228

The sample size therefore comprised of 228 respondents. Through stratified random sampling technique, the respondents were randomly picked from each stratum. This ensured equal representation of the population, where every respondent had an equal chance to be selected in the study (Taherdoost, 2016).

3.4 Data Collection Procedure

The study utilized a questionnaire to collect primary data. The questionnaire was deemed appropriate for this study since it was free from bias and it allowed the respondents adequate time to handle and respond to the questions. The questionnaires were administered by the researcher, with the help of four competent research assistants to collect data.

3.5 Data Analysis

The study used both descriptive and inferential statistics to analyze the quantitative data through the help of Statistical Package for Social Sciences (SPSS) version 27. Linear and multiple regression analyses were conducted to test the hypotheses. In adopting the 95% confidence interval, the hypotheses were considered significant if the p-value is below 0.05.

IV. FINDINGS & DISCUSSIONS

4.1 Response Rate

Out of the 205 questionnaires issued, one hundred and eighty three (183) questionnaires were dully filled returned back for analysis. This represented a response rate of 80.3% and a non-response rate of 19.7%. According to Sekaran and Bougie (2016), when a study obtains a response rate of more than 60%, it implies majority of the sampled respondents have been represented, and this could be a good representation of the population. Therefore, the study embraced the 80.3% response rate and the data was considered adequate for analysis.

Table 1
Response Rate

Response	**Frequency**	**Percent**
Successful	183	80.3%
Unsuccessful	22	19.7%
Total	**205**	**100%**

4.2 Logistics Visibility among the Manufacturing Firms

The study sought to establish the extent to which logistics visibility had been embraced by the surveyed food and beverage manufacturing firms in Kenya. To measure logistics visibility, the study used freight consolidation, transportation and lead time planning as the main sub-constructs. The findings are as shown in Table 2. The findings portrayed that majority of the respondents disagreed that their respective organizations often consolidated freight in transit to minimize time and cost (Mean = 2.44; Std. Dev. = 0.794). The respondents also disagreed that their respective organizations were proactive in planning their transportation and distribution processes to ensure proper planning of product movement (Mean = 2.31; Std. Dev. = 0.861). The findings imply that most of the surveyed food and beverage manufacturing firms in Kenya have not effectively enhanced visibility of their logistics processes, and this as elaborated by Orji et al. (2022), limits the organization' control of their logistics and their ability to optimize them for enhanced performance. As noted by Purwanto and Juliana (2022), logistics visibility is responsible for ensuring that the manufacturing entities were in a position to reduce lead time and through this enhance the satisfaction of their customers for better performance. Also the results of Rogerson and Parry (2020), they observed that the extent to which the supply chain process is aligned to conform with the logistics processes determines the effectiveness of the company's supply chain and eventually leads to enhanced performance.

Table 2

Descriptive Statistics on Logistics Visibility

Statements	Mean	Std. Dev.
Our organization often consolidates freight in transit to minimize time/cost	2.44	0.794
There are formulated frameworks for consolidating freight in our company	2.06	0.831
The management of freight in our organization is tasked with establishing which freight can be consolidated and those not to be consolidated	2.54	0.865
Transportation process has been enhanced to minimize delivery time	2.75	0.878
Our organization is proactive in planning its transportation and distribution processes to ensure proper planning of product movement	2.31	0.861
Our company has integrated modern transport management systems across its supply chain network	2.77	0.904
Our organization has embraced lead time management to minimize the period between customer order and fulfillment of the order	2.55	0.872
Our organization has made significant lead time planning efforts to enhance customer satisfaction	2.70	0.769
The organization has effective logistics policies and frameworks that enable the reduction of lead time	2.78	0.861
Through a visible logistics process, the firm stands a greater chance to meet customer needs and maximize performance	2.64	0.893

4.3 Performance of Food and Beverage Manufacturing Firms

The study sought to assess the performance of food and beverage manufacturing firms in Kenya. This was measured using: Sales Revenue, Profitability, and productivity of the food and beverage manufacturing firms.

4.3.1 Sales Revenue

The study compared the sales turnover for the entire manufacturing sector and the food and beverage manufacturing company. From the findings on Figure 2, it was established that the sales revenue for the food and beverage manufacturing companies decreased from Kshs.272.6 million in 2018 to Kshs.191.9 billion in 2019. During the same period, the entire manufacturing sector had increased its sales from Kshs.2.2 billion in 2018 to Kshs.2.3 billion in 2019. Between 2020 and 2021, the food and beverage manufacturing sub-sector had their total sales reduce from Kshs.354.1 billion to Kshs.281.9 million, while the entire manufacturing industry had recorded an increase in sales turnover from Kshs.2.37 billion to 2.70 billion in 2021. A further decline was recorded in the food and beverage manufacturing sub-sector to Kshs.260.03 million in 2022, whereas the entire sector grew their sales to Kshs.3.175 billion. This signifies that the food and beverage manufacturing firms have been recording a decline in their sales revenue, despite the entire manufacturing sector growing as far as sales revenue is concerned.

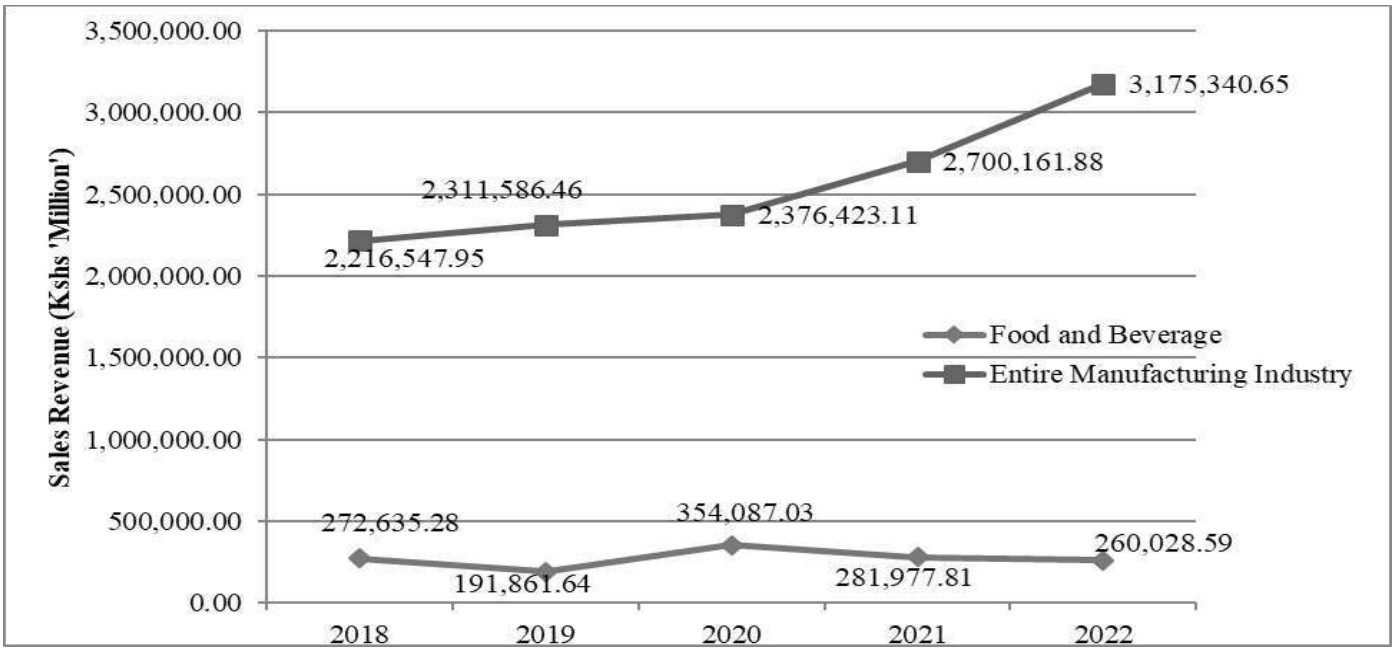

Figure 2

Annual Sales Turnover

4.3.2 Profit Margin

The research assessed the profit margins of the firms and results presented in Figure 3. The findings show that the average profit margin in the firms have been decreasing in a fluctuating way for the five consecutive years. The profit margin represents the total percentage of sales that result in a profit. Further, the profit margin gives the measure of a company's earnings (or profits) relative to its revenue.

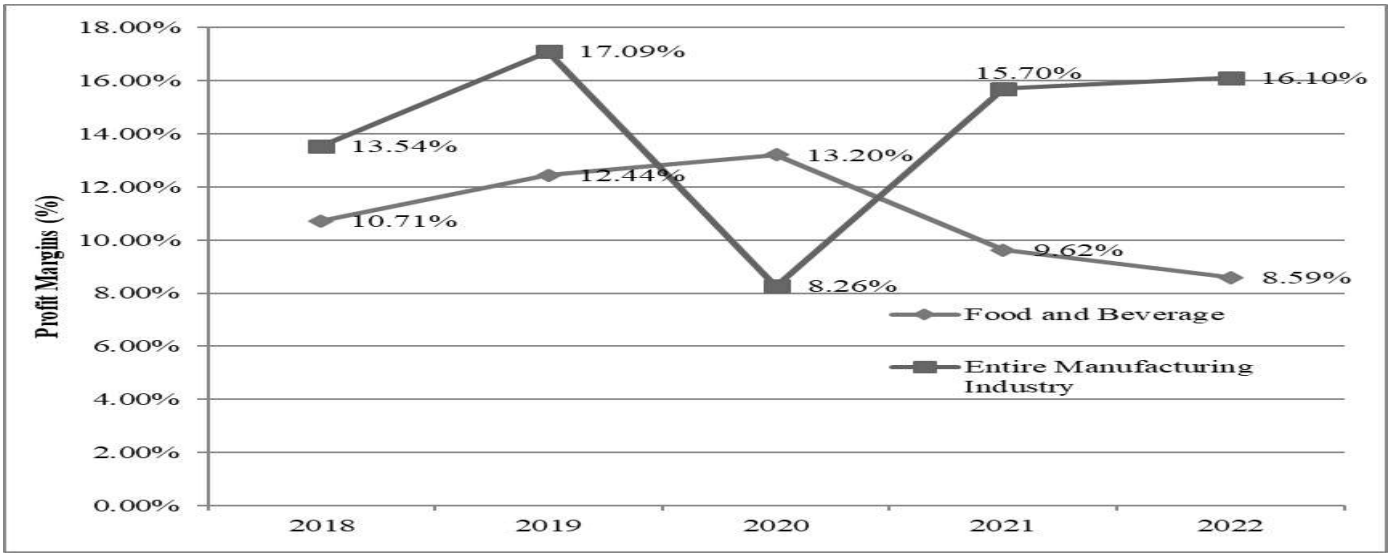

Figure 3

Annual Profit Margins

4.3.3 Productivity

On the level of productivity, it was established that while the entire manufacturing industry had a quantum index of 6.7% in 2018 while the food and beverage manufacturing firms had a 10.2%, in 2019, the entire sector declined to 3.0% while the food and beverage manufacturing firms had their production decline to negative 0.20%. In 2020, the food and beverage manufacturing sub-sector increased its production to 56% but declined to 3.0% in 2021, while in the same period (2021), the entire manufacturing sector had its productivity grow to 6.5%. The food and beverage manufacturing companies further recorded a decline of 1.1% in productivity in 2022, while the entire sector had a production index of 3.8%. From the results, it can deduced that as much as the entire manufacturing sector is facing decline in production, the food and beverage sub-sector has more decline in production, implying that the sector is indeed ailing.

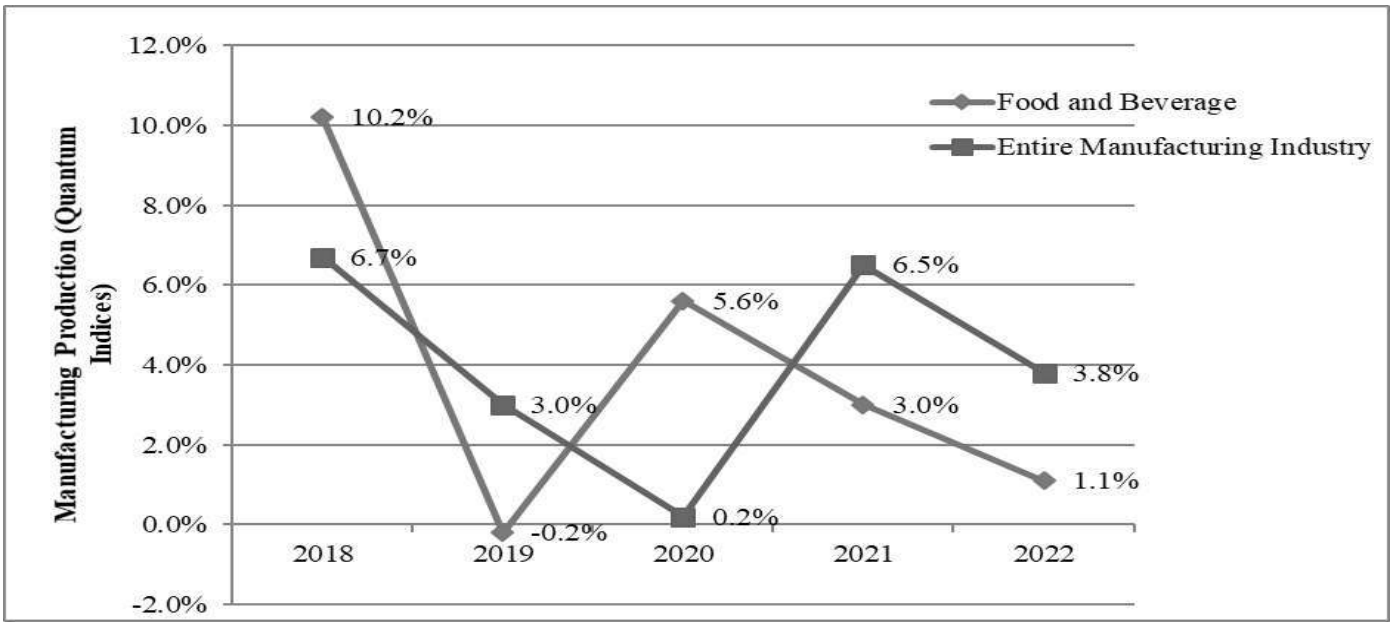

Figure 4

Production Rate for the Food and Beverage Manufacturing Firms

4.4 Correlation between Logistics Visibility and Performance of F&B Manufacturing Firms

Correlation analysis was carried out to establish the relationship between independent variable (logistics visibility) and the dependent variable (performance of large food and beverage manufacturing firms). According to Gogtay and Thatte (2017), correlation analysis tells the extent to which variables relate thus giving a glimpse on the probability of a linear relationship between the variables. The study carried out a correlation analysis to establish the relationship between logistics visibility and performance of large food and beverage manufacturing firms in Kenya. Table 3 shows that the Pearson correlation coefficient was 0.718. These findings indicate that there is a strong positive linear relationship between supply chain planning visibility and performance of large food and beverage manufacturing firms in Kenya.

Table 3

Correlation Analysis for Logistics Visibility

Variable		Performance Manufacturing Firms	Logistics Visibility
Performance of manufacturing firms	Pearson Correlation	1	.718**
	Sig. (2-tailed)		.000
	N	183	183
Logistics Visibility	Pearson Correlation	.718**	1
	Sig. (2-tailed)	.000	
	N	183	183

**. Correlation is significant at the 0.01 level (2-tailed).

4.5 Hypotheses Testing

The hypothesis was tested using a univariate regression model as shown below:

H_O: Logistics visibility has no significant influence on performance of large food and beverage manufacturing firms in Kenya.

$Y = \beta_0 + \beta X + e$

The summary of the linear regression model used for this specific objective indicates a coefficient of determination, $R^2 = 0.749$ which means that about 74.9% of the change in the performance of large food and beverage manufacturing firms in Kenya can be explained by logistics visibility. The results are presented in Table 4.

Table 4

Model Summary of Logistics Visibility

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.865[a]	.749	.748	.44181

a. Predictors: (Constant), Logistics visibility

b. Dependent Variable: Performance of Large food and beverage manufacturing firms

Table 5 shows the ANOVA result of the regression of performance of large food and beverage manufacturing firms on logistics visibility. The result indicates that the significance of the F-statistic is less than 0.05 ($F=17.036$, $p<0.05$), an implication that the model is statistically significant to predict the relationship between logistics visibility and performance of large food and beverage manufacturing firms.

Table 5

ANOVA of Logistics Visibility

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	105.474	1	105.474	540.348	.000[b]
	Residual	35.331	181	.195		
	Total	140.805	182			

a. Dependent Variable: Performance of Large food and beverage manufacturing firms

b. Predictors: (Constant), Logistics Visibility

Shown in Table 6 are the coefficients and t-statistic obtained from the model. The constant term ($\beta_0 = 0.525$) is interpreted to mean that if logistics visibility is held constant, then there will be a positive performance of large food and beverage manufacturing firms in Kenya by 0.525. The regression coefficient for logistics visibility was positive

and significant (β_1 = 0.893; P = 0.000<0.05), with a t-value of 23.245. This implies that a unit increase in logistics visibility is predicted to increase the performance of manufacturing firm s by 0.893 units.

Table 6

Coefficients of Logistics Visibility

Model		Unstandardized Coefficients		Standardized Coefficients	T	Sig.
		B	Std. Error	Beta		
1	(Constant)	.525	.112		4.701	.000
	Logistics visibility	.893	.038	.865	23.245	.000

a. Dependent Variable: Performance of Large food and beverage manufacturing firms

V. CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

The study concluded that logistics visibility had a positive significant influence on performance of large food and beverage manufacturing firms in Kenya. Key components of logistics visibility—freight consolidation, transportation, and lead time planning—were inadequately addressed, contributing to inefficiencies in the supply chain. Most firms did not consolidate freight to minimize time and costs, indicating a lack of structured frameworks and decision-making processes for freight management. This results in higher operational costs and delayed deliveries. Additionally, transportation systems in these organizations lack proactive planning and the use of modern transport management technologies, leading to further delays and suboptimal distribution processes. The study, therefore, concluded that while logistics visibility posed a major potential in stimulating effectiveness of supply chain processes and ultimate performance among the food and beverage manufacturing firms, the firms' inability to embrace visibility of their logistics affected their performance.

5.2 Recommendations

The management team at the food and beverage manufacturing firms in Kenya have a duty to streamline their logistics processes as a way of enhancing the visibility of logistics. This would be an integral approach towards gaining more control of their logistics through which they can contain costs, operate more efficiently and enhance customer satisfaction. There is need for the supply chain managers to come up with ways through which the firms can consolidate their freight so as to reduce costs of transportation while optimizing delivery schedules.

It is also recommended that the supply chain managers in food and beverage manufacturing firms ought to modernize their transportation systems to enable real-time tracking of trucks and cargo. Proactive transportation planning, including predictive route optimization, can minimize delivery delays and ensure timely product distribution.

The study also recommend the need for food and beverage manufacturing firms to adopt lead time management practices by implementing effective logistics policies that reduce the time between customer orders and delivery. This will not only increase customer satisfaction but also boost operational efficiency. The firms should also ensure that their supply chain processes are transparent and aligned with logistics operations. This will enhance the overall effectiveness of the supply chain, ensuring that customer needs are met promptly and improving the company's competitiveness.

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Second order Extended Ensemble Kalman Filter with Stochastically Perturbed Innovation

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ABSTRACT

Studies have shown several forms of non-linear dynamic filters. However, Extended Kalman filters have proved to provide more accurate values of the state of dynamic systems over period of time. Though, the results of estimation by use extended Kalman filters are accurate, there is involvement of computation of high dimension covariance matrix that are very expensive. Although Bayesian methods offer a robust and accurate approach, they are often hindered by the computational complexity involved in computing high-dimensional matrices. This study introduces a new filter, the Second Order Extended Ensemble Filter with pertubed innovation (SoEEFPI), designed to numerically address the inversion of high-dimensional covariance matrices and then stochastically perturbing the innovation. The SoEEFPI is derived from the numerical expansion of the expected values of non-linear terms in the stochastically perturbed Kushner-Stratonovich equation, utilizing a second-order Taylor series expansion. Validation of the SoEEFPI is conducted on a three-dimensional stochastic Lorenz 63 model, with simulations performed using MATLAB software. In the validation process , SoEEKFPII is compared with First Order Extended Ensemble Filter (FoEEF), First Order Extended Kalman Bucy Filter (FoEKBF), Second order Extended Ensemble Filter (SoEEF), Bootstrap Particle Filter, and Second Order Extended Kalman Bucy Filter (SoEKBF). Results indicated that SoEEFPI outperformed the other filters (KBF, FoEEF, SoEEF) across all three variables of the Lorenz 63 model: x_1, x_2 and x_3. While SoEKBF exhibited the lowest root mean square error (RMSE), its computational cost is significantly higher due to the integration of high-dimensional covariance, making SoEEFPI a more desirable option since its covariance computation is performed empirically.

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Keywords: Non-linaer filtering, Non-linear state space dynamic models, Bayesian methods

1 Introduction

The introduction of a First Order Extended Ensemble Filter (FoEEF) offers an alternative approach, calculating covariance matrices empirically instead of relying on matrix multiplication, thus expediting the convergence process [12]. However, the first-order term used in the Taylor series expansion of nonlinear terms may yield less accurate results compared to filters developed using a second-order Taylor polynomial. Recently, Kevin Midenyo developed a second Order Extended Ensemble Filter which proved more efficient in estimation than the First Order Extended Ensemble Filter [9]. However there is a need for a new filter which incorporates the noisy components of the dynamic system. Therefore, this study has developed a Second Order Extended Ensemble Filter (SoEEF) with stochastic perturbed innovation, potentially more efficient for initializing neural network weights. The SoEEFPI utilizes an empirical method to estimate the inverse of covariance matrices, addressing the computational challenges and costs associated with previous Kalman filters. As a second-order estimate, the SoEEFPI provides greater accuracy compared to first-order estimations. Its efficient implementation will benefit various sectors, such as manufacturing, image recognition, healthcare, and transportation in estimations.

2 Literature Review

[12] Proposed the first-order extended ensemble filter to solve the computationally intensive initialization of weights in artificial neural networks. Unlike the Kalman filter-based approach, this method avoids the direct computation of the matrix products and their inverses, thus improving the learning rate. The first-order extended ensemble filter uses an empirical estimate of the matrix products and inverses through a weighted ensemble of samples. The algorithm first generates a set of particles, each representing a possible value of the weight parameters. The computation of the matrix products and inverses in the first-order extended ensemble filter is replaced by the empirical estimates of the mean and covariance of the particles. The mean and covariance are updated using the weighted samples, and the covariance is modified to account for the resampling process. This modification ensures that the covariance of the new set of particles is equal to the covariance of the previous set of particles, which is an essential property for maintaining the diversity of the particles [10].

2.1 Non-Linear Filters

Kalman filters have been applied in many areas and more significantly in the non-linear filtering methods that are just but the extension of the Kalman filter. When working with the non linear models, it is prudent to make an assumption that the probability densities and the conditional densities are Gaussian. In the non-linear filters, the Extended Kalman Filters (EKF) has been utilized and it is just an extension of the Kalman filters. In EKF, the evolution of the time state vector is well described by a dynamical model. In this case, we have different

equations and the observation mode amd the noice part as shown below;

$$y_k = f(y_{k-1}) + w_{k-1} \tag{2.1}$$

where; $w_k \sim N(0, Q)$

$$x_k = h(x_k) + A_k \tag{2.2}$$

where; $A_k \sim N(0, R)$,

The initial state y_0 is a random vector and the mean is;

$$\mu_0 = E[y_0]$$

and the covariance is given by;

$$\rho_0 = \left[(y_0 - \mu_0)(y_0 - \mu_0)^T\right]$$

where;

y_k is $n \times 1$ state vector,
w_k is $n \times 1$ process noice vector,
x_k is $m \times 1$ measurement vector,
A_k is $m \times 1$ measurement noice vector,
$f(y-k)$ $n \times 1$ process non-linear vector,
$h(y_k)$ $m \times 1$ observation non-linear vector finction,
k is the index of time.

The EKF algorithm is shown in the following steps.

Step 1: Initializing estimated initial state vector

$$\hat{y}_0^n = \mu \tag{2.3}$$

The error covariance matrix ρ_0

Step 2: Predicting the state vector.

$$\hat{y}_{k+1} = f(\hat{y}_k, \mu_k) \tag{2.4}$$

then predict error covariance ahead;

$$P_k = V_k P_k V^T + Q_k \tag{2.5}$$

Step 3: Calculation of the observation matrix.

$$H_{y_{k+1}} = \frac{\partial h}{\partial y} \tag{2.6}$$

which is the Jacobian of the non-linear measurement function $h(y_k)$.

Step 4: Acquiring new measurement vector.

$$x_{(k+1)} \tag{2.7}$$

Step 5: Calculating the Kalman gain matrix.

$$M_k = P_k H (H_k P_k H^T + T)^{-1} \tag{2.8}$$

the correction of predicted results, that is, updated state vector with measurement Z_k is given by,

$$y_k = \hat{y}_k + K_k\Big(Z_k - H(\hat{y}_k)\Big)$$

the update, error covariance

$$P_k = (1 - K_k H_k) P_k \tag{2.9}$$

Step 5: Corrected $\hat{y}_k$ (output) now becomes the previous state in the next iteration and the process is then repeated.

The EKF does not perform well when noise is non-Gaussian and the dynamic system must be linear. Though applied widely, the high computational cost makes it more expensive.

2.2 Kalman-Bucy Filter

The weak form of the Kushner-Stratonovich equation, when integrated over y, gives the estimate $\hat{y}$:

$$d\hat{y} = F(t)\hat{y}_t dt + Q_t H^T(t)(dx_t - H(t)\hat{y}_t dt) \text{ [3]} \tag{2.10}$$

The equation of covariance is described as:

$$dQ_t = F(t)Q_t dt + Q_t F^T(t)dt + G(t)G^T(t)dt - Q_t H^T(t)R^{-1}(t)H(t)Q_t dt \tag{2.11}$$

The optimal estimate of y given x is provided by equations2.10 and 2.11 above equations. The prediction steps involve the mean estimate:

$$d\hat{y} = F(t)\hat{y}_t dt \tag{2.12}$$

and the covariance:

$$dQ_t = F(t)Q_t dt + Q_t F^T(t)dt + G(t)G^T(t)dt$$

The new measurement is captured by the additive term in the predicted estimator:

$$Q_t H^T(t)\big(dx_t - H(t)\hat{y}_t dt\big)$$

2.3 Bootstrap Particle Filter

In this approach, Monte Carlo methods are utilized to numerically approximate estimates of mean and covariance. According to [15] [8] ,the posterior distribution is approximated as:

$$q(y_t|x_{1:t})$$

The set of particles is denoted as:

$$M_t = (y_t^i, w_t^i); \quad i = 1, 2, ..., M.$$

Here y_t^i represents the dynamic state, w_t^i signifies the weight of the particle at time t, and $x_{1:t}$ denotes the measurements. The filtering probability density is defined as:

$$Q\big(y_t|x_{1:t}\big) = \sum_{i=1}^{M} w_t^i \delta(y_t - y_t^i) \tag{2.13}$$

2.4 Ensemble Kalman-Bucy Filter

The Ensemble Kalman-Bucy Filter (EKBF) is based on the particle filter and is designed for modeling continuous dynamic processes. The covariance is estimated using Monte Carlo techniques [3]. The evolution of the state is represented as:

$$dy_t^i = F(t)y_t^i dt + Z(t)dB_t^i + Q_t^M H^T(t)R^{-1}(t)\big(dx_t + R^{\frac{1}{2}}(t)\eta_i^t - H(t)y_t^i dt\big) \tag{2.14}$$

The mean of the ensemble is given by:

$$\hat{y}_t = \frac{1}{N}\sum_{i=1}^{N} y_t^i \tag{2.15}$$

The covariance is estimated as:

$$Q_t = \frac{1}{N-1}\sum_{i=1}^{N}(y_t^i - \hat{y}_t)(y_t^i - \hat{y}_t)^T \tag{2.16}$$

2.5 First Order Extended Kalman-Bucy Filter (FoEKBF)

In scalar forms, the first-order model equations are given as follows:

$$\text{signal}: \ dy_t = f(y_t)dt + g(t)(t)dB_t, \quad x_{t_0}, t_0 < t$$
$$\text{measurement}: \ dx_t = h(y_t)dt + R^{\frac{1}{2}}(t)d\eta_t, \quad t_0 \leq t$$

Substituting $g(y,t)$ with $g(t)$ in corresponding derivatives and using Taylor series expansion around the mean, we arrive at:

$$d\hat{y}_t = f(\hat{y}_t)dt + Q_t\nabla[h](\hat{y}_t)R^{-1}(t)\big(dx_t - h(\hat{x}_t)dt\big)$$

2.6 Second Order Extended Kalman-Bucy Filter (SoEKBF)

The Second Order Extended Kalman-Bucy Filter (SoEKBF) is derived from a second-order Taylor expansion around the mean [5]. The equations are as follows:

$$d\hat{y}_t = f(\hat{y}_t)dt + \frac{1}{2}\Delta[f]\Big(\hat{y}_t Q_t dt + Q\nabla[h]^T(\hat{y}_t)R^{-1}(t)(dx_t - \big(h(\hat{y}_t) + \frac{1}{2}\Delta[h](\hat{y}_t q_t)dt\big)\Big) \tag{2.17}$$

$$\begin{aligned} dQ_t =& Q_t\nabla[f]^T(\hat{y}_t)dt + \nabla[f](\hat{y}_t)Q_t dt + g(t)g^T(t)dt - Q_t\nabla[h]^T(\hat{y})R^{-1}\nabla[h](\hat{y}_t)Q_t dt \\ &+ \frac{1}{2}Q_t\Delta[h]^T(\hat{y}_t)R^{-1}(t)\big(dx_t - (h(\hat{y}_t)) + \frac{1}{2}\Delta[h](\hat{y}_t Q_t)dt\big)Q_t \end{aligned} \tag{2.18}$$

where $\delta[f]$ and $\delta[h]$ are second-order derivatives.

2.7 First Order Extended Ensemble Filter (FoEEF)

The First Order Extended Ensemble Filter (FoEEF) is based on an ensemble of weights for a neural network model [12]. The process involves:

1. Initializing ensemble weights w, filter object $\hat{w}_0$, and covariance matrix P_0.
2. Projecting state forward: $f(w_t^i)df + g(w_t^i)q^{\frac{1}{2}}(t)dw_t^{*i}$.
3. Computing Kalman gain: $ph_w(w_i^i)r^-1(t)$.
4. Updating state with measurement information and iterating over the ensemble to update weights $\hat{w}_t$ and covariance matrix P_t [11].

2.8 Second Order Extended Ensemble Filter (SoEEF)

The Second Order Extended Ensemble Filter (SoEEF), developed by Kevin Midenyo in 2023 [9], improves on the First Order version by incorporating second-order Taylor expansions and Monte Carlo methods [5]. The filter equation is:

$$dx_t^i = f(x_t^i,\theta)dt + g(x_t^i)q^{\frac{1}{2}}(t)B_t + Mh_x(x_t^i)r^{-1}(t)(d(z_t) - (\frac{1}{2}M_t(h_{xx}(xx_t^i) - h(x_t^i))dt; \quad t_0 \le t \tag{2.19}$$

Empirical approximations of mean $\hat{x}_t$ and covariance M_t are given by:

$$\hat{x}_t = \frac{1}{N}\sum_{i=1}^{N} x_t^i; \quad M_t = \frac{1}{N-1}\sum_{i=1}^{N}(x_t^i - \hat{x}_t)^2 \tag{2.20}$$

This method was validated using the Lorenz 63 system, showing improved performance with increased ensemble size. Numerous Filters have been discussed and it was established that Kalman Bucy filters were very accurate in their applications in non-linear dynamic systems, however the cost of computing of the high dimensional covariance is very high [11]. The first-order extended ensemble filter [12, 11] proved to be a

better filter as it solved the computational costs by estimating the expected covariances empirically . However, it applied Taylor series to the first order thus slightly less accurate as compared to filters where the estimation of expected covariance id done to second order. KevIn Midenyo developed a new filter, second order extended ensemble filter and in application it proved better than FoEEF as the expected values of the covariances was approximated by Taylor series to the second order.This thesis is thus building on the past work by expanding Kushner Stratonovich equation to the second order around the mean and stochastically perturbing the innovation to take care of any other uncertainties by developing second order extended ensemble filter with stochastically perturbed innovation. The new filter performs better than SoEEF and FoEEF and it is computationally cheaper than SoEKBF and other filter.

3 Development of Filter

3.1 Stochastic calculus

There is a lot of underlying mathematics when developing filters. The method of derivation involves dynamic state-space model hence a need for a deeper discussion on stochastic calculus. This section gives an introduction , step by step derivation , explanation of stochastic calculus needed to derive Second Order Extended Ensemble Kalman Filter with stochastically perturbed Innovation.

Assuming that a state of a system is according to

$$dy_t = f(y_t, t)dt + g(y_t, t)dB_t;\ t_0 \leq t$$

and a noisy measurements of the system given by

$$dz = h(y_t, t)dt + \theta dv;\ t_0 \leq t$$

Where f is the drift function, g refers to the volatility , B and v are the independent Brownian motion at time t. Then the conditional probability density $p(y,t)$ of the state at time t is given by the Kushner-Stratonovich equation;

$$dp(y,t) = M[p(y,t)]dt + p(y,t)\big(h(y,t) - E_t(y,t)\big)\theta^{-T}\theta^{-T}\big(dz - E_t h(y,t)dt\big) \tag{3.1}$$

where

$$M[p] = -\sum \frac{\partial(f_i p)}{\partial y_i} + \frac{1}{2}\sum (gg^T)_{ij}\frac{\partial^2 p}{\partial y_i \partial y_j}$$

is the foward Kolmogorv operation and $dp(x,t) = p(x, t+dt) - p(x,t)$ gives the variation of conditional probability and the innovation is given by $dz - E_t h(x,t)$, that is the difference between the measurement and the expected value. Throughout this paper, because the derivation of Second Order Extended Ensemble Filter with stochastically perturbed innovation involves dynamic processes, stochastic calculus will be utilized in solving differential and integral equations to help come up with the filter [2].

3.2 Itô formula

The Taylor expansion of the KS model is given by

$$\begin{aligned} h(T_{t+\delta t}, t+\delta t) = h(y_t, t) + \partial(y_t, t)]\delta_t + \partial_{xy}[h(y_t, t)]\delta y_t \\ + \frac{1}{2}(\partial_{tt}[h(y_t, t)]\delta t^2 + 2\partial_{tt}[h(y_t, t)]\delta_t \delta y_t \\ + \partial_{yy}[h(y_t, t)]\delta y_t^2) + \cdots \end{aligned}$$

In the above equation, values with powers greater than 2 are insignificant hence ignored;

$$\delta y_t = f(y_t, t)\delta t + g(y_t, t)\delta B_t \tag{3.2}$$

Then,

$$\delta h(y_t, t) = \partial(y_t, t)]\delta t + \partial y[h(y_t, t)]\delta y_t + \frac{1}{2}\partial_{yy}[h(y_t, t)]g^2(y_t, t)\delta_t + 0\delta t \tag{3.3}$$

The derivation of equation 3.3 is borrowed from the work of [1, 6].

From the above equation, terms containing orders greater than 2 are assigned zero because they are negligible. Substituting equation 3.2 and 3.3 and assuming the higher order terms are negligible, we will get:

$$\begin{aligned} \delta h(y_t, t) &= \partial_t[h(y_t, t)]\delta_t + \partial_y[h(y_t, t)](f(y_t, t)\delta_t) + g(y_t, t)\delta B_t) + \frac{1}{2}\partial_{yy}[h(y_t, t)]g^2(y_t, t)\delta t. \\ &= \partial_t[h(y_t, t)]\delta_t + \partial_y[h(y_t, t)](f(y_t, t)\delta_t) + \frac{1}{2}\partial_{yy}[h(y_t, t)]g^2(y_t, t)\delta t \\ &+ \partial_y[h(y_t, t)]g(y_t, t)\delta_t B_t \end{aligned} \tag{3.4}$$

when we extend the equation 3.4 to vector form through substituting n dimensional column vector, y_t, in the place of scalar y_t and by use of differential operators we get;

$$\nabla = (\frac{\partial}{\partial y_1}\frac{\partial}{\partial y_2}\vdots\frac{\partial}{\partial y_n})$$

and

$$\begin{bmatrix} \frac{\partial^2}{\partial {y_1}^2} & \frac{\partial^2}{\partial y_1 \partial y_2} & \cdots & \frac{\partial^2}{\partial y_1 \partial y_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2}{\partial y_n \partial y_1} & \frac{\partial^2}{\partial y_n \partial y_2} & \cdots & \frac{\partial^2}{\partial {y_n}^2} \end{bmatrix}$$

Then equation 3.2 will yield

$$\delta h = \partial_t[h]\delta t + \nabla[h]^T f(y_t, t)\delta t + \frac{1}{2}tr[g(y_t, t)g^T(y_t, t)]\Delta[h]\delta t + (\nabla[h]^T g(y_t, t))\delta_t B_t \tag{3.5}$$

Taking the limit in the mean of δh as δt and δt tending to zero, we get:

$$\delta h = \partial_t[h]dt + \nabla[h]^T dy_t + \frac{1}{2}tr[g(y_t, t)g^T(y_t, t)]\Delta[h]dt \tag{3.6}$$

equation 3.4 is referred to as Itô formulae. This formula is significant in the calculation of derivatives of stochastic calculus [7]. In fact,it is equivalent to chain rule in deterministic calculus [7].

3.3 Fokker-Plank equation

By considering the scalar of the Itô stochastic differential equation 3.4. It can be shown that the process for $\{y_t, t \in [t_o, I]\}$ generates equation 1 is a molecular process.
The density function of the Marker process is given by

$$\Phi_t(y) = \Phi(y,t), \forall\, t \in [t_o, T] \tag{3.7}$$

together with the transition probability density function we have;

$$\Phi_{t|T}(y/z) = \Phi(y,t)(Z,T), \forall\, T < t \in [t_o, T] \tag{3.8}$$

Fokker plank is an equation of the evolution of the density function, $\Phi_t(y)$ and the conditional density, $\Phi_{t|T}(y/Z) \forall\, t > T \in [t_o, T]$. The Process $\{y_t, t > 0\}$ that is generated by equation 3.4 is a Markov process, then we can say that, given $t_1 < t_2 < t_3$;

$$\Phi_{t_3|t_1,t_2}(y|z,x) = \Phi_{t_3|t_2}(y/x) \tag{3.9}$$

and the following equation satisfies;

$$\Phi_{t_3|t_1}(y/z) = \int \Phi_{t_3|t_1}(y/x)\Phi_{t_2|t_1}(x/z)dx \tag{3.10}$$

equation 3.10 above is the Chapman-Kolmogor equation and is applying to all Markov processes. We will then use the equation 3.10 and Taylor expansion to obtain the equation of evolution of transition probability density function, $\Phi_{t|T}(y/x) = \Phi(y,t|x,T)$, and is given by

$$\frac{\partial \Phi_{t|T}(y/x)}{\partial t} = \frac{-\partial(\Phi_{t|T}(y/x)f(y,t))}{\partial y} + \frac{\frac{1}{2}\partial^2(\Phi_{t|T}(y/x)g^2(y_t,t))}{\partial y^2} \tag{3.11}$$

Equation 3.11 is the Fokker plank equation and also referred to as Kolmogorov's forward equation.
Then we take expectations to equation 3.11 with respect to $\Phi_t(x)$;

$$E_T[\Phi_{t|T}(y/x)] = \int \Phi_{t|T}(y/x)\Phi_T(x)dx = \Phi_t(y) \tag{3.12}$$

The equation of each the evolution of probability density function, $\Phi_t(y)$ is thus;

$$\frac{\partial \Phi_t(y)}{\partial t} = \frac{-\partial(\Phi_t(y/x)f(y,t))}{\partial y} + \frac{\frac{1}{2}\partial^2(\Phi_t(y_t)g^2(y,t))}{\partial y^2} \tag{3.13}$$

By use of the vector equation 2, the corresponding Fokker plank is got by the evolution of every element in equation 3.13.

$$\frac{\partial \Phi_t(y,t)}{\partial t} = -\sum_{k-1}^{n} \frac{\partial \Phi_t(y,t)f_k(y,t)}{\partial y_k} + \frac{1}{2}\sum_{k,1=1}^{n} \frac{\partial^2(\Phi_t(y,t)(g(y,t)g^T(y,t))_{k1})}{\partial y_k \partial y_i} = \ell\Phi_t(y,t) \tag{3.14}$$

we therefore have;

$$\ell\partial\Phi_t(y,t) = -\sum_{k-1}^{n}\frac{\partial\Phi_t(y,t)f_k(y,t)}{\partial y_k} + \frac{1}{2}\sum_{k,1=1}^{n}\frac{\partial^2(\Phi_t(y,t)(g(y,t)g^T(y,t))_{k1})}{\partial y_k\partial x_i} \tag{3.15}$$

The solution of the equation 3.14 gives the probability density function $\Phi_t(y)$. It should be noted that most SDE for nonlinear drafts are cumbersome to solve hence numerical solutions are preferred. We then derive a form of Fokker- plank equation forming the basis of numerical methods.

3.4 Weak form of Fokker plank equation

Considering the scalar Fokker Plank equation 3.13. Let $\Theta(y) \in_c^\infty (\mathbb{R},\mathbb{R})$ meaning $\Theta(y)$ is differentiable function to infinity from $\mathbb{R}$ to $\mathbb{R}$, and the compact support is set $\mathbb{R}$. The value of

$$\Phi_{to}(y) = \Phi_o(y) \tag{3.16}$$

Multiplying equation 3.13 by $P(x)$ and intergrity over domain;

$$\int\frac{\partial\Phi_t(y)}{\partial t} = -\int\frac{\partial\Phi_t(y)f(y,t)}{\partial y}P(y)dy + \frac{1}{2}\int\frac{\partial^2(\Phi_t(y)g^2(y,t))}{\partial y^2}P(y)dy \tag{3.17}$$

when we intergrate 3.17 by parts we obtain

$$\int\frac{\partial\Phi_t(y)}{\partial t}P(y)dy = \quad -\int\partial\Phi_t(y)f(y,t)\tfrac{\partial P(y)}{\partial y}dy + \tfrac{1}{2}\int\tfrac{\partial(\Phi_t(y)g^2(y,t))}{\partial y^2}\tfrac{\partial P(y)}{\partial y}dy \tag{3.18}$$

$$= \qquad \int\tfrac{\partial P(y)}{\partial y}(\Phi_t(y)f(y,t) - \tfrac{1}{2}\tfrac{\partial(\Phi_t(y)g^2(y,t))}{\partial y})dy \tag{3.19}$$

By intergrating the R.H.S of equation 3.19, we have;

$$\int\frac{\partial\Phi_t(y)}{\partial t}P(y)dy = \int\frac{\partial P(y)}{\partial y}\Phi_t(y)f(y,t)dy + \frac{1}{2}\int\frac{\partial^2P(y)}{\partial y^2}\Phi_t(y)g^2(y,t)dy \tag{3.20}$$

Getting expected values of 3.20,

$$\frac{\partial/E_t[P(y)]}{\partial t} = /E_t[\frac{\partial P(y)}{\partial y}f(y,t)] + \frac{1}{2}/E_t[\frac{\partial^2P(y)}{\partial y^2}g^2(y,t)] \tag{3.21}$$

Then defining $/E_t[P]$ as

$$/E_t[P] = \qquad \Phi[P] \tag{3.22}$$

$$= \quad \int\Phi_t(y)P(y)dy \tag{3.23}$$

we have,
$d\Phi_t[P] = \Phi_t[\ell^*P]dt$ but the operator

$$\ell^* = f\frac{\partial}{\partial y} + \frac{1}{2}g^2\frac{\partial^2}{\partial y^2} \tag{3.24}$$

The above eqaution 3.24 is the weck form Fokker plank equation. We can now obtain different numerical methods based on choice of the test function $P(y)$

3.5 Equation of Evolution of the Mean

Given the equation of the mean is

$$d\Psi_t(\hat{x}_t) = \hat{f}dt$$

Let;
$\hat{x}_t = \Psi_t(x) = \int x\Psi_t(x)dx$ and
$\hat{f}_t = \Psi_t[f(x,t)] = \int f(x,t)\Psi_t(x)dx$ then

$$a_t = \Psi_t[x_t^2] - (\hat{x}_t)^2 \, and \, da_t = d\Psi_t[x_t^2] - d(\hat{x})^2$$

using its formulae:

$$d\Psi_t(x_t^2) = 2\Psi_t[fx_t]d\hat{x} + \Psi[q^2]$$

hence

$$da_t = 2\Psi_t[fx_t]d\hat{x} + \Psi[q^2] + 2\hat{x}_t\hat{f}_t dt \tag{3.25}$$

3.6 Kushner stratonovich

We can describe Kushner-stratonovich equation as the purtabation of Fokker Plank equation through the addition of the knowledge from measurement by use of Bayesian approach.
The equation therefore estimate $\hat{y}_t$ at time t through combination of the noisy dynamic with the noisy measurement.

$$dP(y,t) = L[P(y,t)]dt + P(y,t)[h()g,t - E_t h(y,t)]^T \tag{3.26}$$

$$\eta\eta[d2 - E_t h(y,t)dt] \tag{3.27}$$

where

$$\ell P = -\sum \frac{\sigma\partial(\delta_i, P)}{\partial y_i} + \frac{1}{2}\sum(\delta\delta^T)_{ij}\frac{\partial^2 P}{\partial y_i \partial y}$$

which is Kolmogorov formed operation and the

$$dP(y,t) = P(y,t+dt) - P(y,t)$$

By apply as

$$q(\delta_t, \delta y_r) = 1 + (h-h)^T R^{-1}(\delta y_t - \hat{h}_i \delta t) + \cdots$$

3.7 Evolution of the Mean and Covariance

Mean refers to the first moment whereas covariance refers to the second moment.

$$d\hat{y}_t = \hat{f}dt + (\widehat{y_t h^T} - \hat{y}_t\hat{h}^T)R_{(t)}^{-1}(dz_t - \hat{h}d(t)); \;\; y_{t_0} = y(0) \tag{3.28}$$

$$
\begin{aligned}
(dM_t)_{ij} = (\widehat{y_i f_i} - \hat{y}_i \hat{f}_i)dt + (\widehat{f_i y_j} - \hat{f}_i \hat{y}_j)dt + (\widehat{GQG^T})_{ij}dt - \\
(\widehat{y_i h} - \hat{y}_i \hat{h})^T R^{-1} (\widehat{hy_j} - \hat{h}y_j - \hat{h}\hat{j})dt + (\widehat{y_i y_j h} - \\
\widehat{y_i y_j}\hat{h} - \hat{y}_i \widehat{y_j h} - \hat{y}_j \widehat{y_i h} + z\hat{y}_i \hat{y}_j \hat{h})^T R^{-1}(t)(dz_t - \hat{h}dt); \quad M_{t_0} = M(0)
\end{aligned} \tag{3.29}
$$

The estimate $\hat{y}$ is solution to the equation 3.28 and M_t is the solution of equation 3.29. The two equations give exact equation of the evolution of mean and covariance. To simplify the expressions, we can use scalar and write as;

$$
signal: \quad dy_t = f(y_t, \theta)dt + g(y_t)q^{\frac{1}{2}}dB_t; \quad y_{t_0} = y(0), t_0 \leq t \tag{3.30}
$$

$$
measurement: \quad dz_t = h(y_t)dt + r^{\frac{1}{2}}d\eta_t; \quad t_0 \leq t \tag{3.31}
$$

The equations for evolution of the conditional mean and variance for scalar are given by;

$$
d\hat{y}_t = \hat{f}dt + (\widehat{y_t h} - \hat{y}_t \hat{h})r^{-1}(t)(dz_t - \hat{h}dt); \quad y_{t_0} = y(0) \tag{3.32}
$$

$$
\begin{aligned}
dM_t = 2(\widehat{yf} - \hat{y}\hat{f})dt - \Big(\widehat{qg^2}dt - (\widehat{yh} - \hat{y}\hat{h})^2 r^{-1} \\
+ (\widehat{y^2 h} - \hat{y}^2\hat{h} - 2\hat{y}y\hat{h} + 2\hat{y}^2\widehat{yh} + 2\hat{y}^2\hat{h})r^{-1}(dz_t - \hat{h}dt)\Big); \\
dM_0 = M(0)
\end{aligned} \tag{3.33}
$$

where,

$$
\hat{f} = \int f(y)M(y|Z_t)dy_t \tag{3.34}
$$

solving 3.30 and 3.31 will give exact filter. Here the solution involves the calculation of conditional expected values which is difficult as they involve integration over non-linear functions. We then resort to approximating the expected values. The second order approximation of exact filter is made by neglecting the third and higher order

3.7.1 Second-Order Approximate Filter

3.7.2 Theorem 1: Second-Order Approximate Filter

Given continuous functions $f(y)$ and $h(y)$, with existing first and second derivatives f_y, f_{yy}, h_y, h_{yy}, the second-order approximation of the exact filter neglecting higher-order terms is:

$$
d\hat{y}_t = f(\hat{y})dt + \frac{1}{2}M_t f_{yy}(\hat{y}dt + Mh_y(\hat{y})r^{-1}(t)\big(dz_t - (h\hat{y}) + \frac{1}{2}M_t h_{yy}(\hat{y})\big) \tag{3.35}
$$

$$
\begin{aligned}
dM_t = 2Mf_y(\hat{y})dt + \big(q(t)g^2(\hat{y}) + Md(t)g_y^2(\hat{y})\big)dt - \big(M_t h_y(\hat{y})\big)^2 r^{-1} \\
+ \frac{1}{2}M_t^2 h_{yy}(\hat{y})r^{-1}(t)\big(dz_t - (h(\hat{y}) + \frac{1}{2}M_t h_{yy}(\hat{y}))dt\big)
\end{aligned} \tag{3.36}
$$

Proof: By applying a Taylor expansion for $f(y)$ and $h(y)$ around $\hat{y}$, and taking expectations up to the second order, we have:

$$f(y) \approx f(\hat{y}) + (y - \hat{y})f_y(\hat{y}) + \frac{1}{2}(y - \hat{y})^2 f_{yy}(\hat{y}) \tag{3.37}$$

$$h(y) \approx h(\hat{y}) + (y - \hat{y})h_y + \frac{1}{2}(y - \hat{y})^2 h_{yy}(\hat{y}) \tag{3.38}$$

The approximations for the remaining terms can be similarly derived using Taylor expansions. Taking expectations and using covariance terms leads to the final second-order equations.

3.8 First-Order Approximate Filter

3.8.1 Theorem 2: First-Order Approximate Filter

For continuous functions $f(y)$ and $h(y)$, with existing first derivatives f_y and h_y, the first-order approximation of the exact filter is:

$$dy_t = f(\hat{y})dt + Mh_y(\hat{y})r^{-1}(t)(dz_t) - h(\hat{y})dt \tag{3.39}$$

$$dM_t = 2Mf_y(\hat{y})dt + \Big(q(t)g^2(\hat{y}) + Mq(t)g_y^2(\hat{y})\Big)dt - \Big(M_t h_y(\hat{y})\Big)^2 r^{-1} \tag{3.40}$$

Proof

This is derived from the second-order filter by setting the second-order derivatives f_{yy} and h_{yy} to zero, leading to first-order terms only.

3.9 Second-Order Extended Ensemble Filter (soEEF)

The second-order extended ensemble filter (soEEF) is developed based on the nonlinear model with second-order approximation:

$$dy_t^i = f(y_t^i, \theta)dt + g(y_t^i)q^{\frac{1}{2}}(t)B_t + M_t h_y(y_t^i)r^{-1}(t)(d(z_t) - (\frac{1}{2}M_t(h_{yy}(y_t^i) - h(y_t^i))dt; \quad t_0 \leq t \tag{3.41}$$

the empirical mean $\hat{y_t}$ and covariance M_t are estimated as:

$$\hat{y}_t = \frac{1}{N}\sum_{i=1}^{N} y_t^i; \quad t_o \leq t \tag{3.42}$$

$$M_t = \frac{1}{N-1}\sum_{i=1}^{N}(y_t^i - \hat{y}_t)^2; \quad t_0 \leq t \tag{3.43}$$

3.10 SoEEF with Stochastically Perturbed Innovation, S0EEFPI

In this section, you introduce the SoEEFPI (Second Order Extended Esemble Filter with Stochastically Perturbed Innovation) model, derived by adding a stochastic component to the deterministic perturbation term of SOEEF. This results in the following McKean-Vlasov Stochastic Differential Equation (SDE) for the system's state evolution, using mean-field theory (MFT) [13] as an approximation tool. In MFT, all interactions are replaced by an effective interaction with the mean field.

The key stochastic differential equation is:

$$d(\bar{y}_t) = f(\bar{y}_t)dt + g(\bar{y}_t)q^{\frac{1}{2}}(t)d\beta_t + Mh_y(\bar{y}_t)r^{-1}(t)\Big[dz_t - 2h(\bar{y}_t)dt + R^{\frac{1}{2}}(t)d\bar{u}_{(t)}]\Big] \tag{3.44}$$

This describes the evolution of the system's state with stochastic innovations. In the finite system, where M independent copies of the state variables interact, the equation becomes:

$$d(y_t^i) = f(y_t^i)dt + g(y_t^i)q^{\frac{1}{2}}(t)d\beta_t^i + Mh_y(y_t^i)r^{-1}(t)\Big[dz_t + -2h(y^i{}_t)dt + R^{\frac{1}{2}}(t)du^i{}_{(t)}]\Big] \tag{3.45}$$

This captures the stochastic interaction hypothesis for each state in the system.

The SoEEF evolution equation in its controlled Stratonovich form is then:

$$dy_t = f(y_t)dt + g(y_t)q^{\frac{1}{2}}(t)d\beta_t + Mh_y(y_t)r^{-1}(t)\big[dz_t + R^{\frac{1}{2}}(t)du_{(t)} - 2h(y_t)dt\big] \tag{3.46}$$

This equation is eventually transformed into the Itô form, and the corresponding Fokker-Planck equation describes the evolution of the filtering density $\pi(y_t|Z_t)$.

3.11 Exactness of SoEEF with Stochastically Perturbed Innovation

We begin with filtering posterior

$$\pi_{t_0}(y \mid Z_0) = \Pi^*_{t_0}(y \mid Z_0) \tag{3.47}$$

$\pi^*_{t_0}(y \mid Z_0)$ is true posterior at initial time t_0 and filter posterior matches time posterior at all times, t and hence we prove that filtering in equation (3.8.5) is exact. It makes sense to show that the equations of evolution of true posterior and the filtering are the same. By multiplying equaton (3.9.0) by $-\pi_t$ we get;

$$-\pi_t N = \pi_t Mh - 2\pi_t q \tag{3.48}$$

$$-\pi_t M(h - \hat{h}_t) - \pi_t M\hat{h}_t + 2\pi_t q \tag{3.49}$$

by introducing $\pi_t M\hat{h}_t - \pi_t \alpha \hat{h}_t$ from equation (3.8.9) we get,

$$-\pi(h - \hat{h}_t) = R(t)\nabla \cdot (\pi_t \alpha_k) \tag{3.50}$$

substituting equation (3.9.3) to equation (3.9.2) we get;

$$-\pi_t N = MR(t)\nabla \cdot (\pi_t M) - \pi_t M\hat{h}_t + 2\pi_t q \tag{3.51}$$

But

$$\nabla \cdot \big(\pi[MRM^T]big) = \pi MR\nabla \cdot (M) + MR\nabla \cdot (\pi M) \tag{3.52}$$

and applying in 3.9.4 and noting that

$$q(y,t) = \frac{R}{2}\sum_{k=1}^{n} M_k(y_t)\frac{\partial k_j}{\partial y_t} \tag{3.53}$$

we get

$$-\Pi N = -\nabla\big(\pi[MR\alpha^T]\big) + \Pi M\hat{h}_t \tag{3.54}$$

Taking divergence on both sides of equation 3.9.5 we get;

$$-\nabla \cdot (\pi N) = -\sum_{i,k=1}^{n} \frac{\partial^L}{\partial_{y_i}\partial y_k}\big(\pi[MRM^T]_{IK}\big) + \nabla \cdot (\pi M\hat{h}_t) \tag{3.55}$$

we substitute equation (3.8.6) and equation (3.9.6) for;

$$-\nabla \cdot (\pi_t N) + \sum_{i,k=1}^{n} \frac{\partial^L}{\partial_{y_i}\partial_{y_t}}\big(\pi[MRM^T]_i k\big) \tag{3.56}$$

and $\nabla cdot(\pi, M)$ in equation 3.8.7 to get

$$d\pi_t = \mathcal{L}(\pi_t)dt + (h - \hat{h}_t)^T R^{-1}(t)(dy_t - \hat{h}_t dt) \tag{3.57}$$

Thus proving exactness.

4 Filter Validation

The Lorenz 63 system, first introduced by Edward Lorenz [4], is a set of three ordinary differential equations that exhibit chaotic behaviour for specific parameter values and initial conditions [14]. The system is given by:

$$\frac{dx_1}{dt} = a(x_2 - x_1) \quad \frac{dx_2}{dt} = x_1(b - x_3) - x_2 \quad \frac{dx_3}{dt} = x_1x_2 - cx_3 \tag{4.1}$$

where $a = 10$, $b = 28$ and $c = \frac{8}{3}$. These parameter values determine the system's chaotic behavior, which is highly sensitive to initial conditions. Small changes in these conditions can result in significantly different trajectories over time, a hallmark of chaos.

Lorenz 63 is widely used as a testbed for data assimilation because it exemplifies how small initial perturbations lead to large changes in outcomes, making it an ideal model for testing filters [4]. In this case, a new filter, the Stochastically Perturbed SoEEKF, was applied to the Lorenz 63 model and compared with several other filters, including:

1. First Order Extended Ensemble Kalman Filter with perturbed Innovation (FoEEKPi)
2. First Order Extended Kalman-Bucy Filter (FoKBF)
3. Second Order Extended Kalman-Bucy Filter (SoEKBF)
4. The Bootstrap Particle Filter (KBF)
5. First Order Extended Ensemble Filter (FoEEF)
6. Second Order Extended Ensemble Filter (SoEEF)

4.1 Stochastic Lorenz 63 Model

The stochastic version of the Lorenz 63 model incorporates randomness through Brownian motion and is given by:

$$dy_t = f(y_t, \theta)dt + G(y_t)Q^{\frac{1}{2}}(t)dB_t; \quad x_{t_0} = x(0), t_0 \leq t$$

where $f(y)$ represents the Lorenz 63 dynamics, B_t is a 3-dimensional standard Brownian motion, and G and Q represent the noise covariance. The measurement equation associated with this model is:

$$d\bar{x}_t = h(y_t)dt + R^{\frac{1}{2}}(t)d\eta_t; t_0 \leq t \tag{4.2}$$

where $h(y)$ describes the observation process, and η_t is also a 3-dimensional Brownian motion.

4.2 Simulation Setup

The simulation of the Lorenz 63 system used the following setup:

1. Time increment: $dt = 0.001$
2. Simulation time: $T = 0$ to $T = 30$
3. Initial condition: $y_{t_0} = [-5.91652, -5.52332, 24.5723]^T$
4. The system ran for $30,000$ iterations with nine ensemble sizes: $10, 15, 22, 26, 29, 34, 41, 46, 49$

4.3 Perturbed SoEEKF

The stochastic SoEEKF (Second Order Extended Ensemble Kalman Filter) perturbed version used for the experiment is described by:

$$dy_t = f(y_t)dt + g(y_t)q^{\frac{1}{2}}(t)d\beta_t + Mh_y(y_t)r^{-1}(t)\big[dz_t + R^{\frac{1}{2}}(t)du_{(t)} - 2h(y_t)dt\big] \tag{4.3}$$

4.4 Performance Evaluation

The performance of the newly developed SoEEKF filter was tested using the Lorenz 63 system and compared with several other filters. These include: Bootstrap Particle Filter (BPF) First Order Extended Ensemble Kalman Filter (FoEEF), First Order Extended, Kalman-Bucy Filter (FoEKBF), Second Order Extended Kalman-Bucy Filter (SoEKBF), Second Order Extended, Ensemble Kalman Filter (SoEEF)

By evaluating the accuracy and robustness of these filters on the Lorenz 63 system, the new filter's performance was validated. The chaotic nature of the Lorenz 63 system made it an ideal candidate to test the filter's ability to handle sensitivity to initial conditions, noise, and nonlinearity.

5 Results

In Figure 1, the Root Mean Square Error (RMSE) is plotted on the y-axis against the reciprocal of the ensemble size M on the x-axis. The graph compares the performance of six different filters: Bootstrap Particle Filter (BPF): represented by a thin blue line, First Order Extended Ensemble Filter (FoEEF): represented by a green line, First Order Extended Kalman-Bucy Filter (FoEKBF): represented by an orange line, Second Order Extended Kalman-Bucy Filter (SoEKBF): represented by a purple line, Second Order Extended Ensemble Kalman Filter (SoEEF): represented by a sky blue line and Stochastically Perturbed SoEEF (SoEEFPI): represented by a maroon line

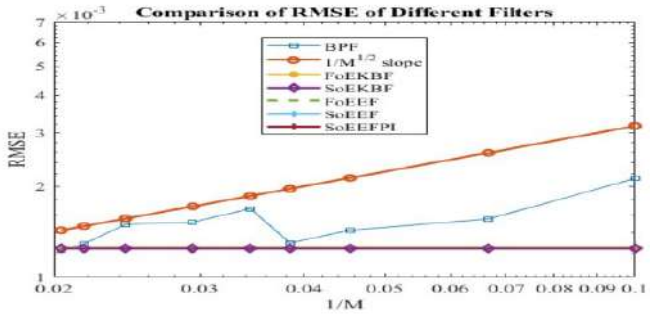

Figure 1: Root mean square error for the Reciprocal of Ensemble.
The sizes of the Ensamble used are $10, 15, 22, 26, 29, 34, 41, 46, 49$ Other settings; $dt = 0.001,\ R = 0.17,\ and\ G = 0.2000$.

From the graph, several observations can be made; As the ensemble size increases, the RMSE of the Bootstrap Particle Filter (BPF) rises, indicating that this filter may become less effective with larger ensembles.

In contrast, the RMSE for the other filters—FoEKBF, SoEKBF, FoEEF, SoEEF, and SoEEFPI—decreases as the ensemble size increases, suggesting improved performance with larger ensembles.

Among these filters, SoEKBF shows the lowest RMSE, reflecting its strong performance in reducing estimation error. However, it is important to note that the computation of the inverse of covariance matrices for SoEKBF is quite expensive, which raises concerns about its practicality for larger applications.

The Stochastically Perturbed SoEEF (SoEEFPI) consistently demonstrates lower RMSE compared to the other filters, making it a compelling option. Although SoEKBF has the best performance in terms of RMSE, its high computational cost underscores the need for an efficient alternative. SoEEFPI effectively balances performance and computational efficiency, proving to be a superior choice in this analysis.

Figure 2 effectively illustrates the performance of the SoEEFPI filter in comparison to traditional filters in the context of chaotic systems. The close alignment of SoEEFPI's trajectory with the true state, combined with its error performance, underscores its effectiveness as a reliable data assimilation method within the Lorenz 63 model framework. The findings emphasize the importance of selecting filters that not only provide accurate estimates but also maintain computational efficiency in chaotic environments.

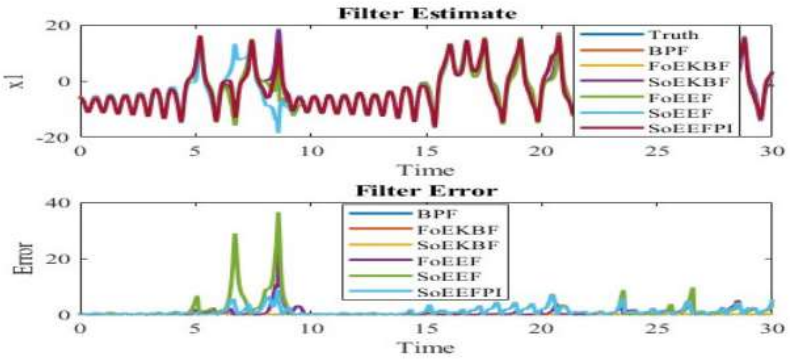

Figure 2: Filter Estimate and Filter Error for first variable, $x1$ in Lorenz 63 model 94

Figure 3 illustrates the comparison between the true trajectory of the second variable x_2 of the Lorenz 63 model and the estimates produced by six different filters. The plot reveals that there is no noticeable deviation between the output of SoEEFPI and the true trajectories of the Lorenz 63 system, indicating that SoEEFPI outperforms the other filters in accurately tracking the true state. This highlights the effectiveness of SoEEFPI in providing reliable estimates in chaotic systems like the Lorenz 63 model.

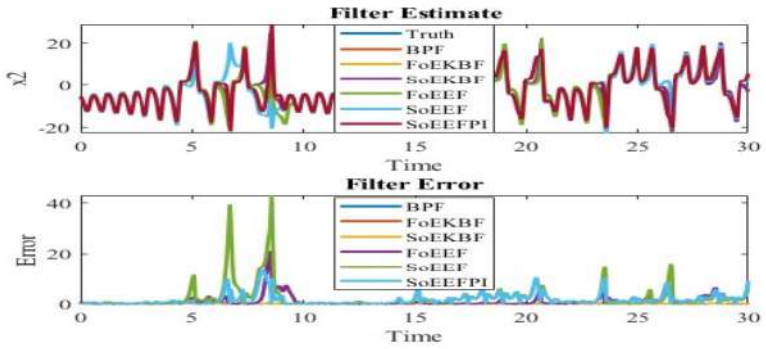

Figure 3: Filter Estimate and Filter Error for $x2$ in Lorenz 63 Model 94

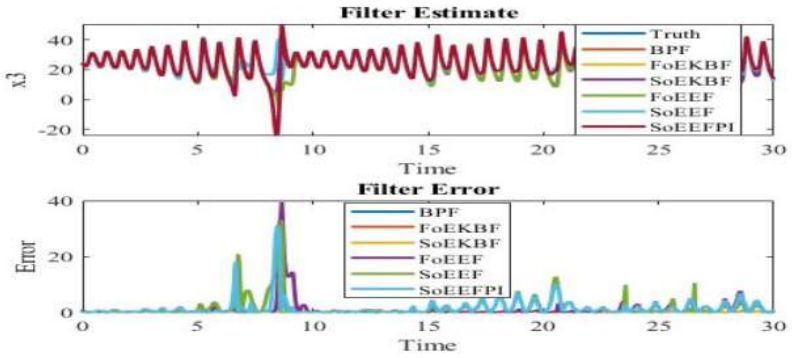

Figure 4: Filter Estimate and Filter Error for $x3$ in Lorenz 63 Model 94

Figure 4 consists of two graphs. The first graph displays the comparison of the trajectories of evolution for the new filter SoEEFPI alongside SoEEF, SoEKBF, FoEKBF, and BPF, against the true state generated by the third variable in the Lorenz 63 model. These trajectories are obtained using nine ensembles measured between times $T = 0$ and $T = 30$. The second graph presents the filter errors measured across the same time interval for all six filters. This comprehensive analysis provides insight into the performance and accuracy of each filtering approach in relation to the true state of the Lorenz 63 model.

6 Conclusion

The complexity of integrating the covariance of high-dimensional data and the associated computational costs have led to the emergence of ensemble Kalman filters. The plots presented compare six filters: First Order Extended Ensemble Filter (FoEEF), First Order Extended Kalman Bucy Filter (FoEKBF), Second Order Extended Kalman Bucy Filter (SoEKBF), and Second Order Extended Ensemble Kalman Filter (SoEEF). The experiment was conducted on a three-dimensional stochastic Lorenz 63 model, with simulations performed using MATLAB software. Results indicate that SoEEFPI outperformed the other filters (KBF, FoEEF, SoEEF, and FoEKBF) across all three variables of the Lorenz 63 model: x_1, x_2, and x_3. While SoEKBF exhibited the lowest root mean square error (RMSE), its computational cost is significantly higher due to the integration of high-dimensional covariance, making SoEEFPI a more desirable option since its covariance computation is performed empirically. The graphs further demonstrate that SoEEFPI is advantageous for parallel computing, as it does not require resampling, unlike particle filters. Additionally, the performance of SoEEFPI improves with increasing ensemble sizes.

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Investigating Final-Year Senior High School Students' Academic Performance Dynamics across Demographics: The Case of Students in Cape Coast, Ghana

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ABSTRACT

The academic performance of senior high school (SHS) students has garnered significant attention, particularly in understanding the various factors influencing it across diverse demographics. This study investigated the academic performance dynamics of final-year SHS students in Ghana, focusing on academic self-efficacy (ASE), examination anxiety (EA), and academic cheating behaviours (ACB). The Theory of Performance by Elger guided the study. A cross-sectional survey design used a structured questionnaire to collect data from 370 students sampled out of 8,656 across 10 SHSs in the Cape Coast Metropolis. Descriptive statistics (frequency, percentage, mean and standard deviation) and inferential techniques (independent sample t-tests and one-way ANOVA) were employed to analyse the data. Results indicated that students generally exhibited high ASE, showing confidence in their academic abilities, though time management was a challenge. While moderate levels of EA were reported, significant variability highlighted the importance of emotional intelligence in academic performance. Regarding ACB, students largely disapproved of serious academic dishonesty but were more inclined to engage in less severe behaviours like sharing notes. Gender differences were notable, with female students reporting higher ASE and lower EA than males, while male students were more likely to engage in ACB. Age differences showed that the 16-20 age group had higher ASE and lower ACB compared to older students, although EA did not significantly vary across age groups. It is concluded that students' ASE correlates positively with their academic capabilities, and their emotional intelligence plays a significant role in academic performance. It is recommended that targeted interventions be put in place to enhance students'ASE, manage EA, and promote academic integrity, particularly among male students, to improve overall academic outcomes.

Keywords: Age, Academic Performance Dynamics, Academic Self-Efficacy, Academic Cheating Behaviours, Examination Anxiety, Gender

I. INTRODUCTION

The education sector has been buzzing with debates about performance levels, as shown by a plethora of research and scholarly articles (Kohn, 2000; Popham, 2001; Darling-Hammond, 2010; Reardon, 2018; Nahar, 2023; Paramole & Adeoye, 2024). These conversations touch on many aspects of the educational system, from policy decisions to classroom practices, and they carry important implications for students, teachers, and society as a whole. One of the most controversial topics in educational performance is standardized testing. Supporters believe these tests offer objective measures of student achievement, enabling comparisons across diverse groups (Popham, 2001; Paramole & Adeoye, 2024).

On the other hand, critics argue that standardized tests can narrow the curriculum, heighten student anxiety, and overlook the full range of students' abilities (Kohn, 2000; Nahar, 2023). Another significant debate focuses on how socioeconomic factors influence educational outcomes. Many studies have pointed out persistent achievement gaps between students from different socioeconomic backgrounds (Reardon, 2018). This reality has sparked discussions about the effectiveness of various interventions, like school funding reforms and early childhood education programmes, aimed at addressing these disparities (Darling-Hammond, 2010). Despite the numerous debates made concerning performance levels in education, it is quite important to note that major advancements have been recorded over the years

on the levels of academic performance across educational levels. One of such educational levels that have gained interest in the sight of some researchers is the secondary level of education.

Ghana's secondary level education, known as Senior High School (SHS), typically spans three years and serves students between the ages of 15 and 18. It follows the completion of basic education and is designed to prepare students for higher education or vocational training. The SHS curriculum offers a broad range of subjects, including core areas such as Mathematics, English, Science, and Social Studies, along with elective courses in areas like business, arts, sciences, and technical studies. Admission to SHS is based on performance in the Basic Education Certificate Examination (BECE), and completion of SHS culminates in the West African Senior School Certificate Examination (WASSCE), which determines eligibility for tertiary education (Takyi et al., 2021). Due to WASSCE being an entry requirement for tertiary education, factors influencing the academic performance of final-year SHS students have been an area of interest to researchers.

One of the factors seen as a strong influence on the academic performance of final-year SHS students is Academic Self-efficacy (ASE). Nasa (2014) defined ASE as the belief in one's capacity to plan, carry out, and control performance to achieve certain performance goals. This implies that a high ASE enhances students' academic performance and a low ASE will result in poorer students' academic performance. A study by Alzabidi et al. (2024) found that the majority of students reported high levels of academic self-efficacy and established a strong positive relationship between self-efficacy and academic performance. However, no statistically significant gender differences were observed in academic self-efficacy, although academic performance was significantly influenced by self-efficacy. Also, a study by Hayat et al. (2020), using structural equation modelling, revealed that students' self-efficacy influences their learning-related emotions and metacognitive learning strategies, which in turn affect their academic performance.

Additionally, researchers have also indicated that Examination Anxiety (EA) influences the ASE of final-year SHS students. EA, also known as test anxiety, is defined as the negative emotions, worry, physiological arousal, and behavioural responses associated with fears of failure or perceived lack of competence in an exam or similar evaluative situation (Matthews et al., 2006). It can deduced from the definition that a high EA leads to lower academic performance, while a low EA is associated with higher academic performance. This was corroborated by Khalid's (2024) study, which found a substantial inverse relationship between secondary school students' academic success and test anxiety (including physical, emotional, behavioural, and cognitive anxiety). Ray and Negi (2024) found that girls experienced higher levels of EA compared to their male counterparts, highlighting that EA is a significant issue affecting many students preparing for board exams. Javed and Abiodullah (2021) also found a negative relationship between test anxiety and academic performance, with female students experiencing higher levels of test anxiety compared to males. Additionally, their study showed no statistically significant difference in test anxiety across different age groups.

Aside from ASE and EA, Academic Cheating Behaviour (ACB) is also seen as a factor that influences final-year students' academic performance. ACB, also known as Academic dishonesty, is intentionally carrying out forbidden behaviours to gain an unfair advantage in an academic context (Zhao et al, 2021). This suggests that when ACB) go unchecked; students may experience temporary improvements in their academic performance, while a lack of such behaviours could result in comparatively lower performance. Lacanlale et al. (2022) demonstrated a positive association between students' academic performance and academic cheating. In a related study, Ossai et al. (2023) identified a significant predictive relationship between age and academic integrity during exams, as well as differences between male and female students. The results indicated that female students had marginally higher academic integrity scores compared to males.

Ideally, it is assumed that their ASE, EA, and ACB likely influence final-year SHS students' academic performance dynamics. Students with high academic self-efficacy are expected to exhibit better academic performance, as they are confident in their ability to succeed without resorting to unethical practices. In contrast, those with low academic self-efficacy may be more prone to academic cheating, as they doubt their capabilities. Similarly, students with lower examination anxiety would likely perform better, as they are more composed during exams and less inclined to cheat. Regarding demographics, it is expected that gender may influence these dynamics, with potential differences in how male and female students experience ASE, EA, and ACB. Age is also anticipated to play a role, with older students possibly demonstrating higher academic self-efficacy and lower examination anxiety, thereby reducing their likelihood of engaging in academic cheating.

1.1 Statement of the Problem

Ghana's final-year Senior High School (SHS) students saw notable increases in their academic performance in the West African Senior School Certificate Examination (WASSCE) in 2023, which was the best performance in four years. A notable proportion of students obtained grades ranging from A1 to C6 in all major subjects. In English Language, 73.11% of applicants received grades between A1 and C6, which is a significant increase from prior years. In Core Mathematics, 62.23% of candidates received the same grades. Similar to this, a consistent upward trend in performance

was shown by the grades that 66.82% of applicants received in Integrated Science and 76.76% in Social Studies (Bonney, 2023; Frimpong, 2023). Despite these achievements, the examination results were not without challenges, as 235 schools had their results withheld due to the alleged use of artificial intelligence-generated answers, and the results of some students were cancelled for possessing unauthorized materials and mobile phones in examination halls (Bonney, 2023; Osei, 2023).

These patterns offer a crucial background for comprehending the dynamics of academic success for senior high school students, particularly when considering ASE, EA, and ACB. The overall improvements in WASSCE performance suggest that students' confidence in their abilities, reflected in higher ASE, and may have contributed to their academic success. However, concerns over the prevalence of ACB among students are raised by the persistent problem of academic misconduct, which includes the ownership of foreign resources and the use of AI-generated responses (Bonney, 2023; Osei, 2023). Since cheating is correlated with exam anxiety, it may also be explained by the academic pressure to perform well, which is increased by the examination atmosphere. Are ASE, EA, and ACB of final-year SHS students' factors that affect their overall academic dynamics? Do gender and age differences provide further insights into these academic performance dynamics?

Studies done on academic performance dynamics (ASE and ACB) have revealed conflicting results. For instance, Alzabidi et al. (2024) revealed no statistically significant gender differences were observed in academic self-efficacy, although academic performance was significantly influenced by self-efficacy. However, Huang's (2013) meta-analysis, which reviewed 187 studies with a total of 247 independent samples (N = 68,429), identified a small effect size of 0.08 in gender differences in academic self-efficacy, with a slight advantage for males. In addition, Webb-Williams (2017) found significant gender differences in self-efficacy among primary school pupils in England, with boys exhibiting lower self-efficacy and poorer performance compared to girls.

Also for ACB, gender and age were not found to be major predictors of academic cheating behaviours, according to a study by Sarkar (2022). However, a study by Isakov and Tripathy (2017) indicated that there was a gender difference in ACB with males exhibiting high levels compared to females. The contrast between these studies highlights the need for further research to clarify the role of gender in ACB. Apart from contradictory findings, the study will address a geographical and population gap as no research in Ghana has collectively examined academic performance dynamics (ASE, EA, and ACB) across demographics. This clearly suggests that research on academic performance dynamics across demographics in Ghana is still in its early stages.

1.2 Research Objectives

The study was guided by the following objectives:

i. To determine the level of Academic Performance Dynamics (ASE, EA, and ACB) of final year SHS students in Cape Coast.
ii. To examine the difference in Academic Performance Dynamics among final-year SHS students in Cape Coast based on gender.
iii. To examine the difference in Academic Performance Dynamics among final-year SHS students in Cape Coast based on age.

II. LITERATURE REVIEW

2.1 Theoretical Review

2.1.1 Theory of Performance

Elger (2007) propounded the Theory of Performance. The theory offers a thorough framework for comprehending and enhancing both individual and team performance in a variety of fields. For educators, managers, and other professionals looking to improve their own or others' performance, this theory has important ramifications. Elger's theory is fundamentally based on several important elements. First, it makes the assumption that performance can be divided into discrete levels, from poor to high, which are not fixed but may be raised with practice and intentional effort. The theory also specifies six fundamental components that contribute to total performance: knowledge, skills, identity, personal variables, fixed elements, and emotional aspects. The theory also stresses the significance of performance conditions or the setting and circumstances in which a performance takes place (Elger, 2007).

The theory describes tactics for raising performance, such as practice, immersion, and introspection. Elger's theory is applicable in a wide range of fields. Educators may use this approach to create curriculum and instructional strategies that cover every aspect of performance, perhaps improving student results. Organisations can use this idea in the field of professional development to design training initiatives and performance improvement plans that are more successful. Individuals may evaluate themselves using Elger's framework and make focused goals to enhance their performance on a personal level (Elger, 2007). This study, which focuses on academic performance dynamics (ASE,

EA, and ACB), may be successfully related to Elger's Theory of Performance. This link emphasises how applicable the idea is to educational settings. Elger's emphasis on different aspects of performance, especially emotional and personal elements, is in line with the study's analysis of ASE and EA. The conviction a student has in their capacity to complete academic tasks, or ASE, is correlated with Elger's concept of identity and personal aspects in performance.

Likewise, Elger's identification of emotional elements affecting performance is connected to EA, as investigated in this study. Elger's performance criteria, which highlight how environmental influences and outside pressures may affect a student's decision to participate in academic dishonesty, can be used to analyse this study's focus on ACB. Age and gender can play crucial roles in how individuals interact with and respond to Elger's (2007) Theory of Performance. From a developmental perspective, age influences the accumulation of knowledge and skills, two of the core components outlined in the theory. Younger individuals may have less experience but greater adaptability, while older individuals often bring more depth in knowledge but may face challenges in adapting to new performance conditions. Gender, on the other hand, can shape how emotional aspects and personal variables such as identity and confidence are expressed and developed. Elger's theory used in this study will help educators and researchers have a better understanding of the complex dynamics influencing students' academic achievement. This understanding might result in more effective interventions and support techniques.

2.2 Empirical Review

2.2.1 Perceived Level of Academic Performance Dynamics

Mata (2020) examined grit and ASE as predictors of the academic performance of SHS students. The study used a descriptive-correlational design with 303 SHS students at Carmen and Cantumog National High Schools. The results showed that students demonstrated an above-average level of Academic Self-Efficacy (ASE), with a mean score of 4.00 and a standard deviation of .46. Also, Villas (2019) assessed the self-efficacy of Filipino senior high school students using a ten-point self-efficacy scale ($\alpha = .77$). Administered to a sample of 150 students from three public and three private schools in Quezon City, Metro Manila, the study found that students demonstrated a moderate level of self-efficacy, with a mean score of 7.88. Alzabidi et al. (2024) examined the relationship between academic performance and self-efficacy among Malaysian pre-university students using cross-sectional survey data. The sample included 171 students, aged 18 to 20, in Kuala Lumpur. Findings indicated that most students reported high levels of academic self-efficacy ($M = 1.7$, $SD = 0.44$).

Khalid's (2024) study employed a quantitative research design to explore the relationship between EA and academic achievement among secondary school students in Pakistan. Using a stratified sampling technique to ensure demographic representation, the sample included 210 students. The findings revealed that many students experienced elevated levels of EA ($n = 122$, 58.08%). In addition, Koramoah and Danyoh (2022) conducted a study to assess examination anxiety among final-year students in public Junior high schools in Ghana. Utilizing a descriptive survey design and a multistage sampling technique, the study sampled 261 final-year JHS students. Data were collected through a questionnaire with a Cronbach's alpha reliability of 0.84. Results indicated that students reported moderate examination anxiety ($M = 31.57$, $SD = 7.83$) concerning the 2020 Basic Education Certificate Examination.

Lacanlale et al. (2022) examined the effects of ACB on the academic performance of grade 12 students. Employing a descriptive-correlational approach, the researchers used standardized and self-designed questionnaires for data collection. Findings indicated that students engaged in academic cheating to a moderate extent ($M = 2.69$). In addition, Amua-Sekyi and Mensah (2016) investigated student-teachers' perspectives on cheating during examinations using a mixed-methods approach, incorporating surveys and focus group interviews. The study surveyed 900 undergraduate education students from a public university and three colleges of education in Ghana, with focus group interviews conducted with six students from each institution. A total of 942 students participated. Findings indicated that the majority of respondents ($n = 766$, $M = 1.9$, $SD = 1.4$) disagreed with engaging in cheating during exams.

2.2.2 Difference in Academic Performance Dynamics Based on Gender

Alzabidi et al. (2024) investigated the relationship between academic performance and self-efficacy among pre-university students through a cross-sectional survey. The sample consisted of 171 undergraduates, aged 18 to 20 in Kuala Lumpur. The findings revealed no statistically significant gender differences (p=.18) in ASE. Huang's (2013) meta-analysis investigated gender differences in ASE by reviewing 187 studies comprising 247 independent samples ($N = 68,429$). The analysis identified a small effect size of 0.08, indicating a slight advantage in ASE for males. Webb-Williams (2017) explored children's self-efficacy beliefs in science through a mixed-methods study involving 182 children aged 10 to 12. Data were collected in the classroom using focus groups, individual interviews, and surveys. The findings revealed significant gender differences in self-efficacy among primary school pupils in England, with boys demonstrating lower self-efficacy and performance compared to girls.

Ray and Negi (2024) assessed gender differences in test anxiety among Indian students preparing for board exams. 105 students participated in the study, including 40 males and 65 females. The findings revealed that male students exhibited higher levels of exam anxiety compared to female students, aligning with the results suggesting that boys are more prone to stress related to exams. Javed and Abiodullah (2021) investigated the effects of test anxiety on students' academic achievement at the secondary school level in Lahore. Using a causal-comparative research design, the study aimed to determine the underlying causes of existing differences within the population. The sample comprised 840 students from 30 schools in the Lahore district, representing all secondary school students enrolled in the area. The findings indicated that female students experienced higher levels of test anxiety than their male counterparts did.

In Nigeria, Ossai et al. (2023) identified a significant predictive relationship between age and academic integrity during exams, along with differences between male and female students. The study utilized a survey research design focused on final-year high school students nationwide. A sample of 3,214 students was selected from six geopolitical zones and the Federal Capital Territory using a multistage proportionate random sampling technique, resulting in 1,274 males and 1,940 females. The findings indicated that female students had slightly higher academic integrity scores compared to their male counterparts. It is evident from the literature that, no study has been conducted in Ghana, looking at the differences in students' performance dynamics from the perspective of their gender.

2.2.3 Difference in Academic Performance Dynamics Based on Age

Villas (2019) assessed the self-efficacy of Filipino senior high school students using a ten-point self-efficacy scale ($\alpha = .77$). Administered to 150 students from three public and three private schools in Quezon City, Metro Manila, the study found no statistically significant difference ($p=.177$) between age and ASE. In addition, Abusalehi et al. (2019) investigated ASE and related factors among students at the Tehran University of Medical Sciences. The cross-sectional study sampled 385 students using a stratified sampling method. Pearson correlation results indicated a statistically significant relationship between age and academic self-efficacy ($P= 0.001$, $r =.17$). In addition, Javed and Abiodullah (2021) investigated the effects of test anxiety on students' academic achievement at the secondary school level in Lahore. Using a causal-comparative research design, the study aimed to identify the causes of existing differences within the population. The sample included 840 students from 30 schools in the Lahore district. Findings indicated no statistically significant difference in test anxiety across age groups.

In Nigeria, Ossai et al. (2023) identified a significant predictive relationship between age and academic integrity during exams, alongside differences between male and female students. The survey involved 3,214 final-year high school students selected from six geopolitical zones and the Federal Capital Territory, revealing notable findings regarding age's impact on academic integrity during examinations. In India, Sarkar (2022) investigated an incident of collaborative cheating among postgraduate management students during an online quiz. Using a mixed-methods design, data were collected from 184 postgraduate management students (52.72% women; average age 24.03 years). The findings indicated that gender and age were not significant predictors of ACB. The available literature suggests that no study has been conducted in Ghana on the differences age brings about in students' performance dynamics.

2.3 Conceptual Framework

The conceptual framework in Figure 1 illustrates the hypothesized effect of demographic characteristics (gender and age) on the academic performance dynamics of final-year SHS students. The framework suggests that gender difference (H1) is expected to have a direct impact on student's academic performance dynamics, implying that variations in performance dynamics could be attributed to differences in gender-based experiences, expectations, and opportunities. Additionally, age difference (H2) is hypothesized to influence academic performance dynamics, with the assumption that developmental stages and maturity levels play a role in shaping how students engage with academic tasks. This model aims to explore how these demographic variables contribute to the broader understanding of academic performance dynamics in final-year SHS students.

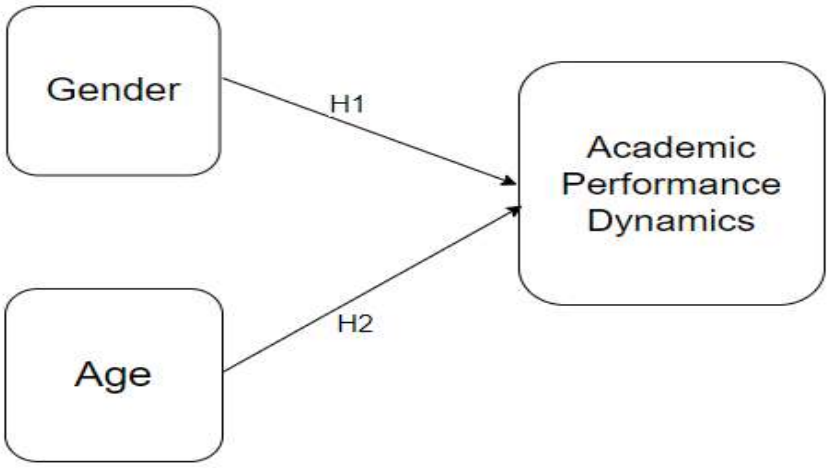

Figure 1
Conceptual Framework Depicting Gender and Age Difference in Academic Performance Dynamics

III. METHODOLOGY

3.1 Procedures

This study utilized a cross-sectional survey design to gather data from final-year senior high school students, aiming to assess the academic performance dynamics across demographics and to determine the differences influenced by these demographic factors. The study population consisted of all final-year students across ten senior high schools in the Cape Coast Metropolis, totalling 8,656 students. These students were chosen as the unit of analysis because they had completed the senior high school curriculum and were preparing to take their final certificate examinations (WASSCE). Their status as imminent exam candidates made them particularly suitable for investigating their academic performance dynamics across demographics.

The study utilized a proportionate simple random sampling technique to select 370 students from the 10 senior high schools in the Metropolis. Table 1 provides a breakdown of the population from each school along with the number of students chosen to participate in the study. Questionnaires were distributed to all selected students, resulting in a 100% return rate for the responses.

Table 1

School	Population	Sample
University Practice SHS	765	33
Wesley Girls SHS	967	41
Mfanstipim SHS	1058	45
St. Augustine's College	750	32
Academy of Christ the King SHS	188	8
Adisadel College	1,400	60
Oguaa Sec. Tech School	464	20
Ghana National College	914	40
Holy Child College	750	32
Aggrey Memorial School	1400	60
Total	**8656**	**370**

To ensure ethical standards were upheld, ethical clearance was obtained from the Institutional Review Board of the University of Cape Coast. Following approval, an introductory letter was presented to the respective schools to request permission from school authorities for data collection. With the consent of the Headmasters from all ten schools, access was granted to the final-year students' classes. The purpose of the study was clearly explained to the students, and sampling was conducted within each school to select participants. Before distributing the questionnaires, students received instructions on how to respond to the items, and they were assured of anonymity and confidentiality. Participation was voluntary.

3.2 Instrumentation and Data Collection

The study employed an adopted scale developed by Gafoor and Ashraf (2006) to measure ASE and an adapted scale by Cassady and Johnson (2002) to assess EA. ACB was measured using a scale sourced from Greene and Saxe (1992). The reliability of the data collected was evaluated, with the results presented in Table 2. Following Nunnally and Bernstein's (2008) guideline, all variables demonstrated strong reliability, with coefficients exceeding the threshold of .7. Specifically, ASE (.914) and ACB (.902) exhibited excellent reliability, while EA (.828) showed good reliability. This indicates a high level of internal consistency across all scales, ensuring the reliability of the entire questionnaire.

Table 2

Reliability Test

Scale	Respondents	N of Items	Cronbach's Alpha
Academic Self-efficacy (ASE)	359	40	.914
Examination Anxiety (EA)	353	27	.828
Academic Cheating Behaviour (ACB)	366	13	.902

3.3 Data Analysis

The data collected was analysed using both descriptive and inferential statistics. Descriptive statistics included frequency counts, percentages, means, and standard deviations. For inferential analysis, independent sample t-tests and one-way ANOVA were utilized. A normality test was conducted to ensure that the assumptions for performing the t-tests and ANOVA were met, with a significance threshold set at .05.

3.4 Test for Normality

The normality diagnostic for the study was assessed by examining the skewness of the data distribution. According to Field (2009), normality values for skewness should not exceed ±1.96 for small samples ($N < 200$) and ±2.58 for larger samples ($N \geq 200$). Table 3 presents the results of the normality test. For Academic Self-Efficacy (ASE), the mean score was 3.93 with a median of 3.93. The skewness value of -0.17 indicates a minor negative skewness, suggesting a slight elongation on the left side of the distribution. Similarly, Examination Anxiety (EA) had a mean of 3.19 and a median of 3.15, with a skewness value of -0.11, denoting an almost symmetrical distribution. For Academic Cheating Behaviours (ACB), the mean was 2.48, the median was 2.38, and the skewness value of 0.67 suggests a moderate positive skewness, indicating a slight elongation on the right side of the distribution. Since all skewness values fall within the acceptable range, the data was considered normally distributed, supporting the use of parametric statistics for further analysis.

Table 3

Test for Normality

Variable		Statistic	Std. Error
ASE	Mean	3.9289	.02615
	Median	3.925	
	Skewness	-.166	.127
EA	Mean	3.1899	.02933
	Median	3.1481	
	Skewness	-.108	.127
ACB	Mean	2.4772	.04605
	Median	2.3846	
	Skewness	.674	.127

IV. FINDINGS & DISCUSSIONS

4.1 Demographics of Students

The results on the background characteristics of the respondents are shown in Table 4.

Table 4

Background Characteristics of the Respondents

Demography	Subscale	Frequency	Percentages (%)
Gender	Male	188	50.8
	Female	182	49.2
Age	Below 15 years	10	2.7
	16 - 20 years	352	95.1
	21 and above	8	2.2

From Table 4, male respondents (188; 50.8%) slightly outnumbered female respondents (182; 49.2%), indicating a balanced gender representation in the study. In terms of age, the vast majority of the participants (95.1%) were between 16 and 20 years, with only a small proportion (2.7%) being below 15 years and 2.2% aged 21 and above. This suggests that the study was predominantly composed of respondents aged 16-20, reflecting the typical age range of final-year senior high school students.

4.2 Determine the level of Academic Performance Dynamics

Research objective 1 aimed to determine the level of academic performance dynamics (ASE, EA, and ACB). Tables 5, 6 and 7 present the results.

4.2.1 Level of Academic Self-efficacy

Table 5 displays the findings regarding the level of ASE among final-year SHS students.

Table 5

Academic Self-efficacy (ASE)

Statements	Mean	SD
Irrespective of the subject, I am competent in learning.	4.10	.92
I sense that I am quick to pick the points from what I read	4.09	.87
I can do my projects well.	4.11	.82
I can arrange the help of my teachers in learning.	3.94	.97
I can arrange help of my peers for my learning whenever I need it.	4.04	.93
I can usually find out quite a few solutions when I confront with problems in my study.	4.02	.95
During examinations, I can recollect what I have learnt.	4.14	.81
If taught, I can prepare my class notes neatly.	4.17	.79
I am assured that I have a few friends who would be helpful in my study.	4.25	.78
I can accomplish my aims in learning.	4.31	.86
I can develop the reading skills required to learn school subjects.	4.26	.84
I can utilize the available library facility for my study.	3.97	1.03
If I miss some classes for some reason, I can compensate the loss fairly well.	3.92	1.03
I am confident that I can perform well in competitive examinations.	4.05	1.06
I can be calm at the time of the exam as I am conscious of my ability to learn.	4.08	.94
I can usually handle the disturbing situations in the study.	3.74	1.11
If a sudden test is conducted for us without prior notice, I can answer it well.	3.75	1.03
If I try, I can become one of the good grade holders.	4.18	1.01
I can score well in the short answer type questions.	4.10	1.00
However, twisted the question is I can answer them.	3.84	1.02
I cannot read and understand my textbooks well.	3.81	1.15
I feel that I have no ability to keep things unforgotten.	3.68	1.10
I can't manage time efficiently for learning.	3.58	1.09
I fail to find out the necessary sources for my study.	3.62	1.12
I fail to set higher goals in my studies.	3.88	1.00
I can't express ideas well while attending examinations.	3.96	1.01
It is difficult for me to read and understand the textbooks in the English language.	4.03	1.04
Often, I fail to comprehend the actual meaning of what I study.	3.91	1.06
I fail to find time for learning amid sundry chores.	3.66	1.10
I can't arrange the resources for my study from my relatives, neighbours,	3.68	1.12
I may not clarify doubts from my teachers while in class, even if I reach higher classes.	3.77	1.06
I can't answer the essay-type questions well.	3.91	1.01

I experience that I am weak in understanding the classes of my teachers.	3.96	1.42
When I study a new concept, I can't recall the related knowledge from the earlier classes.	3.87	1.06
I observe that I fail to prepare my seminars and assignments in time.	3.79	1.02
I consider that I failed to develop a healthy relationship with my teachers.	3.75	1.08
I can't deal efficiently with the unexpected problems in my study.	3.73	1.08
I can't complete the homework myself without any help from guidebooks, previous notes etc.	3.71	1.09
I can't answer the questions which teachers ask me.	3.89	1.03
I can't accomplish challenging tasks and problems in my study.	3.90	1.85
Overall Mean/ SD	**3.93**	**.50**

Key: *A mean score above 3 denotes that the respondents agreed with the assertion made while a mean score below 3 indicated that the respondents disagreed with the assertion made. This was determined by finding the average of the scale used in the questionnaire [(1+2+3+4+5)/5 = 3].*

From Table 5, the findings indicate that the mean values for all items measuring Academic Self-efficacy (ASE) were above 3.0, suggesting that the respondents generally agreed with the statements about their self-efficacy. The highest mean score was recorded for "I can accomplish my aims in learning" (M = 4.31, SD = .86), indicating strong confidence in achieving academic goals. Similarly, items like "I can develop the reading skill required to learn school subjects" (M = 4.26, SD = .84) and "I am assured that I have a few friends who would be helpful in my study" (M = 4.25, SD = .78) also had high mean scores, reflecting positive self-efficacy perceptions among students in terms of both academic ability and social support.

Conversely, the item "I can't manage time efficiently for learning" had the lowest mean (M = 3.58, SD = 1.09), indicating some challenges with time management. The overall mean score of 3.93 (SD = .50) suggests that the students exhibited a high level of academic self-efficacy across the various dimensions measured. The relatively small standard deviation values imply that the responses were moderately homogeneous, with most students sharing similar levels of confidence in their academic abilities.

4.3 Level of Examination Anxiety

Table 6 presents the results on the level of EA among final-year SHS students.

Table 6

Examination Anxiety (EA)

Statements	Mean	SD
I lose sleep over worrying about examinations.	3.31	1.28
While taking an important examination, I find myself wondering whether the other students are doing better than I am.	3.43	1.24
I have less difficulty than the average college student does in getting test instructions straight.	3.44	1.13
I tend to freeze up on things like intelligence tests and final exams.	3.07	1.27
I am less nervous about tests than the average college student.	3.33	1.22
During tests, I find myself thinking of the consequences of failing.	3.18	1.67
At the beginning of a test, I am so nervous that I often can't think straight.	2.86	1.29
The prospect of taking a test in one of my courses would not cause me to worry.	3.25	1.26
I am more calm in test situations than the average college student.	3.50	1.18
I have less difficulty than the average college student in learning assigned chapters in textbooks.	3.56	2.31
My mind goes blank when I am pressured for an answer on a test.	3.10	1.27
During tests, the thought frequently occurs to me that I may not be too bright.	3.04	1.24
I do well in speed tests in which there are time limits.	3.22	1.10
During a course examination, I get so nervous that I forget facts I know.	3.09	1.20
After taking a test, I feel I could have done better than I did.	3.63	1.20
I worry more about doing well on tests than I should.	3.59	1.17
Before taking a test, I feel confident and relaxed.	3.53	1.18
While taking a test, I feel confident and relaxed.	3.47	1.13
During tests, I have the feeling that I am not doing well.	2.95	1.23
When I take a difficult test, I feel defeated before I even start.	2.92	1.24
Finding unexpected questions on a test causes me to feel challenged rather than panicky.	3.23	1.27

I am a poor test taker in the sense that my performance on a test does not show how much I know about a topic.	2.75	1.25
I am not good at taking tests.	2.65	1.28
When I first get my copy of a test, it takes me a while to calm down to the point where I can begin to think straight.	3.08	1.28
I feel under a lot of pressure to get good grades on tests.	3.12	1.45
I do not perform well on tests.	2.71	1.19
When I take a test, my nervousness causes me to make careless errors.	3.02	1.29
Overall Mean/ SD	**3.19**	**.56**

Key: *A mean score above 3 denotes that the respondents agreed with the assertion made while a mean score below 3 indicated that the respondents disagreed with the assertion made. This was determined by finding the average of the scale used in the questionnaire [(1+2+3+4+5)/5 = 3].*

From Table 6, the findings reveal that the mean values for most items measuring Examination Anxiety (EA) were above 3.0, suggesting that the respondents generally agreed with the statements related to experiencing examination anxiety. The highest mean was recorded for the item "After taking a test, I feel I could have done better than I did" (M = 3.63, SD = 1.20), indicating that a significant number of students tend to experience regret or self-criticism after exams. Similarly, items like "I worry more about doing well on tests than I should" (M = 3.59, SD = 1.17) and "Before taking a test, I feel confident and relaxed" (M = 3.53, SD = 1.18) also scored high, reflecting a mix of pre-exam confidence and post-exam anxiety.

On the other hand, the item "I am not good at taking tests" had the lowest mean (M = 2.65, SD = 1.28), indicating that fewer students perceived themselves as generally poor test-takers. Likewise, the item "I do not perform well on tests" (M = 2.71, SD = 1.19) also had a mean score below 3, suggesting that most students disagreed with this negative self-assessment. Overall, the mean score for examination anxiety (M = 3.19, SD = .56) implies that students experience a moderate level of anxiety during exams. The standard deviation values indicate some variability in the responses, particularly for items related to self-assessment and test performance, but the data generally show a moderate level of examination-related stress among respondents.

4.4 Level of Academic Cheating Behaviours

Table 7 presents results on the level of ACB of final-year SHS students.

Table 7

Academic Cheating Behaviours

Statements	**Mean**	**SD**
Getting notes from people who have previously taken the course.	3.07	1.29
Getting exams from people who have previously taken the course.	3.06	1.31
Getting papers from people who have previously taken the course.	3.02	1.23
Turning a paper, that someone else who took the class in a previous year wrote.	2.65	1.36
Purchasing a paper someone else wrote and turning it in.	2.33	1.40
Making up a story to tell a lecturer to get more time to complete the assignment.	2.48	1.34
Bringing cheat sheets to an exam.	2.07	1.36
Copying answers from another student while taking an exam.	2.27	1.24
Studying from an exam someone else stole.	2.31	1.33
Working on individual assignments with classmates.	2.83	1.30
Saying a paper/test was turned in when it wasn't.	2.21	1.25
Bringing completed blue books to a test.	2.02	1.31
Have a friend sign your name on an attendance sheet when you did not attend the class.	1.93	1.26
Overall Mean/ SD	**2.48**	**.88587**

Key: *A mean score above 3 denotes that the respondents agreed with the assertion made while a mean score below 3 indicated that the respondents disagreed with the assertion made. This was determined by finding the average of the scale used in the questionnaire [(1+2+3+4+5)/5 = 3].*

From Table 7, the results indicate that the majority of the respondents disagreed with the various Academic Cheating Behaviours (ACB) listed, as evidenced by the overall mean score of 2.48 (SD = .89), which is below the threshold of 3.0. This suggests that, on average, students reported relatively low engagement in cheating behaviours.

The highest mean scores were observed for items such as "Getting notes from people who have previously taken the course" (M = 3.07, SD = 1.29) and "Getting exams from people who have previously taken the course" (M = 3.06, SD = 1.31). These suggest that students were more likely to agree with engaging in these behaviours, which might be seen as less severe forms of academic misconduct.

In contrast, items like "Having a friend sign your name on an attendance sheet when you didn't attend the class" (M = 1.93, SD = 1.26) and "Bringing completed blue books to a test" (M = 2.02, SD = 1.31) had the lowest mean scores, indicating strong disagreement with these cheating behaviours, which may be perceived as more serious violations of academic integrity. Overall, the findings suggest that while some students may engage in minor forms of academic dishonesty, such as sharing notes or exams, more overt and serious cheating behaviours like fabricating attendance or using cheat sheets during exams are less common among respondents. The variability in standard deviations highlights that students' engagement in these behaviours varies across the sample, though the general trend points to low levels of cheating.

4.4.1 Mean Difference Testing

Testing Homogeneity of Variances and Mean Equality

Tables 8 and 9 present the results of statistical tests used to assess the assumptions of homogeneity of variances and the equality of means across groups.

Table 8

Levene's Test for Equality of Variances

Dependent Variable	Independent Variable	F	Sig.
ASE	Gender	.633	.427
EA	Gender	.107	.744
ACB	Gender	6.77	.010

Table 9

Robust Test for Equality of Means (Welch)

Dependent Variable	Independent Variable	Statistic	df1	df2	Sig.
ASE	Age	37.3	2	11.952	.000
EA	Age	.892	2	10.941	.438
ACB	Age	2.239	2	11.462	.151

Before conducting an independent sample t-test or one-way ANOVA, it is crucial to verify the assumption of homogeneity of variance, which requires that the variances of the dependent variables are equal across the levels of the independent variables. From Table 8, Levene's test results show that for Academic Self-Efficacy (ASE) and Examination Anxiety (EA), the assumption of homogeneity of variance is met, with non-significant values for gender (p = .427 and p = .744, respectively). However, for Academic Cheating Behaviour (ACB), the test yielded a significant result (p = .010), indicating unequal variances across genders.

In Table 9, the Welch test was conducted for age as an independent variable. The results show a significant difference in ASE across age groups (p = .000), while no significant differences were found for EA (p = .438) and ACB (p = .151). Thus, the assumption of homogeneity of variance was violated for ACB in the gender comparison but met for the other variables, allowing further analyses using t-tests and ANOVA where applicable.

4.5 Difference in Academic Performance Dynamics among Final-Year SHS Students Based on Gender

Research objective 2 examined the difference in Academic Performance Dynamics among final-year SHS students based on gender. The dependent variables include ASE, EA, and ACB, while the independent variable is gender, consisting of male and female students. An independent sample t-test, at a significance level of .05, was employed to test whether there are significant differences between male and female final-year SHS students in terms of ASE, EA, and ACB. This statistical tool was chosen because the independent variable (i.e., gender) consists of two categories, and the dependent variables (i.e., ASE, EA, and ACB) were normally distributed. The results are presented in Table 10.

Table 10

Difference in Academic Performance Dynamics among Final-Year SHS Students Based on Gender

Variable	Gender	N	Mean	SD	T	df	Sig.
ASE	Male	188	3.85	.49	-2.966	368	.003
	Female	182	4.01	.51			
EA	Male	188	3.26	.56	2.424	368	.016
	Female	182	3.12	.56			
ACB	Male	188	2.69	.94	4.854	358.591	.000
	Female	182	2.26	.77			

The results in Table 10 indicate statistically significant differences in the academic performance dynamics of final-year SHS students based on gender for all three variables: Academic Self-efficacy (ASE), Examination Anxiety (EA), and Academic Cheating Behaviours (ACB). For ASE, female students (M = 4.01, SD = .51) reported significantly higher self-efficacy than male students [M = 3.85, SD = .49; t (368) = -2.966, p = .003]. This suggests that female students have a higher sense of confidence in their academic abilities compared to their male counterparts.

For EA, male students (M = 3.26, SD = .56) experienced significantly higher levels of anxiety compared to female students [M = 3.12, SD = .56; t (368) = 2.424, p = .016]. This indicates that male students are more prone to examination-related anxiety. Regarding ACB, male students (M = 2.69, SD = .94) reported significantly higher engagement in academic cheating behaviours than female students [M = 2.26, SD = .77; t (368) = 4.854, p = .000]. This implies that male students are more likely to engage in academic dishonesty compared to female students.

4.6 Difference in Academic Performance Dynamics among Final-Year SHS Students Based on Age

The aim of objective 3 was to examine the difference in Academic Performance Dynamics among final-year SHS students based on age. One-way ANOVA, at a significance level of .05, was used to test whether there is a significant difference among the age groups of final-year SHS students in terms of ASE, EA, and ACB. This statistical tool was employed because the independent variable (i.e., age) consists of three groups (Below 15 years; 16-20 years; and 21 and above), while the dependent variables (i.e., ASE, EA, and ACB) are normally distributed. The results are presented in Table 11.

Table 11

Difference in Academic Performance Dynamics among Final-Year SHS Students Based on Age

Variable	Age Group	N	Mean	SD	ANOVA					
						Sum of Squares	df	Mean Square	F	Sig.
ASE	Below 15	10	3.27	.48	Between Groups	7.890	2	3.945	16.934	.000
	16-20	352	3.96	.49	Within Groups	85.493	367	.233		
	Above 21	8	3.30	.23	Total	93.383	369			
EA	Below 15	10	3.15	.57	Between Groups	.820	2	.410	1.291	.276
	16 -20	352	3.18	.56	Within Groups	116.596	367	.318		
	Above 21	8	3.50	.66	Total	289.576	369			
ACB	Below 15	10	2.42	.50	Between Groups	4.898	2	2.449	3.157	.044
	16 - 20	352	3.25	.89	Within Groups	284.678	367	.776		
	Above 21	8	2.48	1.03	Total	289.576	369			

The results in Table 11 showed statistically significant differences in ASE among the age groups [F (2, 367) = 16.934, p = .000], indicating that final-year SHS students' ASE levels vary significantly based on age. For EA, however, no statistically significant differences were found among the age groups [F (2, 367) = 1.291, p = .276], indicating that the level of anxiety experienced during examinations is relatively uniform across age groups. Regarding ACB, there was a statistically significant difference among the age groups [F (2, 367) = 3.157, p = .044], suggesting that ACB tendencies differ somewhat across age demographics.

Post-hoc comparisons were conducted to determine where the specific differences lie for ASE and ACB. Since ASE did not meet the homogeneity assumption, Dunnett's T3 test was employed. In contrast, Tukey HSD was run for ACB, as it met the homogeneity assumption. Results are shown in Table 12.

Table 12

Post-hoc Comparisons

Variable	Age (I)	Age (J)	Mean Diff (I - J)	Sig	95% CI
ASE	Below 15 years	16 - 20	-.69	.004	(-1.13, -.25)
		Above 21	-.03	.997	(-.49, .43)
	16 - 20 years	Below 15	.69	.004	(.25, 1.13)
		Above 21	.66	.000	(.42, .91)
	Above 21 years	Below 15	.03	.997	(-.43, .49)
		16 - 20	-.66	.000	(-.91, -.42)
ACB	Below 15 years	16 - 20	-.04	.990	(-.70, .63)
		Above 21	-.83	.119	(-1.81, .16)
	16 - 20 years	Below 15	.04	.990	(-.63, .70)
		Above 21	-.79	.034	(-1.53, -.05)
	Above 21 years	Below 15	.83	.119	(-.16, 1.81)
		16 - 20	.79	.034	(.05, 1.53)

The results in Table 12 show the post-hoc comparisons for ASE and ACB among different age groups. For ASE, significant differences were found between the Below 15 years and 16-20 years groups, with a mean difference of -0.69 ($p = .004$), indicating that students aged 16-20 had higher ASE than those below 15. Additionally, the 16-20 years group also exhibited significantly higher ASE compared to the Above 21 years group (mean difference = .66, $p = .000$). In contrast, there were no significant differences between the Below 15 years and Above 21 years groups (mean difference = -0.03, $p = .997$).

For ACB, no significant differences were found between the Below 15 years and 16-20 years groups (mean difference = -.04, $p = .990$). However, a significant difference was observed between the 16-20 years and Above 21 years groups (mean difference = -.79, $p = .034$), indicating that students aged 16-20 exhibited lower ACB than those above 21. The comparison between Below 15 years and Above 21 years did not yield significant results (mean difference = -.83, $p = .119$). Overall, these findings suggest that age significantly affects ASE, particularly showing an advantage for the 16-20 year-old group, while ACB differences are less pronounced, with only one significant comparison noted between the younger and older groups.

4.7 Discussion

With an emphasis on ASE, EA, and ACB, the study sought to understand the dynamics of academic performance among SHS students in their final year. The findings revealed that students generally exhibit high levels of ASE, indicating a strong belief in their academic capabilities. This aligns with Nasa (2014), who emphasized that a high ASE enhances students' academic performance, as students expressed particular confidence in accomplishing learning objectives and developing necessary skills. However, challenges with time management were noted, suggesting that while students feel capable, they may struggle to manage their study time effectively. Students to a modest extent, according to the results, experience EA and there is a notable degree of response variability. As a negative emotional reaction that hinders evaluative performance, high levels of emotional intelligence (EA) can hurt performance (Matthews et al., 2006). Students' propensity to critically analyse their performance following tests is consistent with research by Khalid (2024), which found a negative correlation between test anxiety and academic achievement. This study emphasizes the emotional upheaval that students experience throughout examinations. In terms of ACB, the study's findings indicated that students largely disapprove of engaging in cheating behaviours.

While some responses indicate a willingness to participate in less severe forms of academic dishonesty, such as sharing notes, more infractions that are serious were generally rejected. This finding supports Zhao et al. (2021), who defined ACB as actions taken to gain unfair academic advantages. Moreover, Lacanlale et al. (2022) demonstrated a positive association between academic performance and lower levels of cheating, emphasizing that fostering a culture of integrity is essential for academic success. The findings of this study also revealed significant gender differences in the academic performance dynamics of final-year SHS students, particularly in ASE, EA, and ACB. Female students reported higher levels of ASE compared to their male counterparts, suggesting greater confidence in their academic abilities. This aligns with Nasa's (2014) assertion that high ASE enhances academic performance, supporting Alzabidi et al. (2024), who established a positive relationship between self-efficacy and academic outcomes. In contrast, male

students showed greater levels of exam anxiety than female students, which is in line with the results of Ray and Negi (2024) that boys are more prone to stress connected to exams. Khalid (2024) also draws attention to the detrimental effects of high EA on scholastic achievement. These findings contradict previous research since Javed and Abiodullah (2021) found that women are more likely than men to feel test anxiety. This highlights the complexity of the problem and the requirement for specialised support approaches.

According to ACB, male students were more likely than female students to participate in academic dishonesty were. This finding is consistent with the findings of Zhao et al. (2021) and Isakov and Tripathy (2017), who reported that men are more likely than women to engage in cheating activities, are. This result emphasises how important it is to provide interventions that support academic integrity, particularly for male students. That is in contrast to Sarkar's research from 2022, which found no gender variations in ACB that were statistically significant. The study also revealed significant age-related differences in ASE among final-year SHS students, with those aged 16-20 demonstrating higher self-efficacy compared to their younger and older peers. This finding aligns with Nasa's (2014) definition of ASE as a crucial factor influencing academic performance, reinforcing the notion that greater self-efficacy correlates with improved outcomes. Studies by Alzabidi et al. (2024) further support this, highlighting the strong positive relationship between self-efficacy and academic success, particularly among students in this age group.

On the other hand, EA did not exhibit statistically significant variations among age groups, suggesting that students generally experienced test-related stress. This is in line with other studies by Khalid (2024), Javed, and Abiodullah (2021), who did not discover any appreciable differences in anxiety levels among age groups. There were notable variations in ACB, with students between the ages of 16 and 20 showing less cheating than those above the age of 21. The present study bolsters the conclusions drawn by Zhao et al. (2021) and Ossai et al. (2023) about the significance of cultivating academic integrity in senior high school students. When taken as a whole, these findings demonstrate the intricate interaction between demographic variables and the dynamics of academic achievement among SHS students in Ghana.

V. CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

In conclusion, this study highlights the critical role of ASE, EA, and ACB in shaping the academic performance of final-year SHS students. The findings indicated that students possess high levels of ASE, which correlates positively with their academic capabilities, yet they face challenges in time management. Although students experience modest levels of EA, the variability in responses suggests that emotional intelligence plays a significant role in academic performance. The study also found that while students largely reject serious forms of academic dishonesty, gender differences reveal that female students exhibit higher self-efficacy and lower exam anxiety compared to their male counterparts.

5.2 Recommendations

In light of these findings, it is recommended that educational stakeholders, such as the Ministry of Education and school officials, put programmes in place targeted at improving ASE and controlling EA in students, with a special emphasis on male students who exhibit greater levels of exam anxiety. Students can learn efficient time management techniques and emotional control techniques through workshops and counselling sessions. In addition, it will be crucial to promote an academic integrity culture through integrity seminars and awareness campaigns, especially for male students who are more likely to commit academic dishonesty. By encouraging moral academic behaviour across Ghana's educational system, this all-encompassing strategy can enable students to achieve better academic results.

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Assessing Antecedents of Prospective Accountant Ethical Behaviour across Demographics: The Case of Accounting Students in Ghana

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ABSTRACT

This study examined demographic differences of prospective accountants' antecedent of ethical behaviour (AEB) (morality and religiosity). Kohlberg's Theory of Moral Development guided the study. The study employed a cross-sectional survey design to ascertain how demographics affect future accountants' AEB. Only final-year accounting education students at the University of Cape Coast (UCC) and the University of Education, Winneba (UEW) constituted the study's population. A census method was used to include all final-year students in this field involving 120 final-year accounting education students from UCC and 53 from UEW. A structured questionnaire i.e., closed-ended questionnaire was used to gather comprehensive data on the variables under investigation. Descriptive statistics (frequency counts, percentages, means, and standard deviations), and inferential statistics (One-way ANOVA and independent sample t-tests) were used for data analysis using SPSS version 26. The study revealed that accounting students demonstrated a high level of AEB overall. Furthermore, the results showed that there were no statistically significant differences in AEB among accounting students based on age, gender, and institutional affiliation. The study concluded that there were high levels of ethical commitment among accounting students, with no significant differences observed based on gender, age, or institutional affiliation. The study recommended that accounting programmes should place a stronger emphasis on integrating practical and scenario-based learning opportunities into their ethical education curricula as this will help accounting students engage with real-world ethical dilemmas and enhance their ability to apply theoretical knowledge in professional contexts.

Keywords: Accounting Students, Antecedent of Ethical Behaviour, Morality, Religiosity

I. INTRODUCTION

The field of accounting is inherently linked to ethical considerations, with practitioners frequently encountering complex ethical dilemmas in their professional roles. Ethical behaviour in accounting is not merely a matter of regulatory compliance but is also a fundamental necessity for preserving the credibility of financial information (Arens et al., 2023). This resulted in the adoption of the first code of ethics in 1917 by the American Institute of Accountants, now known as the American Institute of Certified Public Accountants (AICPA). The ethical evolution of the accounting profession has been shaped by numerous factors, culminating in the establishment of comprehensive codes of conduct (Kusolsuk & Mouritsen, 2018).

Morality is regarded as one of these variables that has shaped the ethical evolution of the accounting profession. Morality defines what constitutes "right" and "wrong" behaviour, such as acting fairly and avoiding injustice (Haidt & Kesebir, 2010). According to Rest (1983), morality is not a fixed trait but a dynamic process that shapes an individual's moral behaviour, choices, and actions. This process involves four key components: moral

sensitivity, which refers to the ability to recognise and interpret ethical issues; moral judgement, the capacity to discern right from wrong and make sound decisions; moral motivation, which prioritises moral principles and assigns them significance; and moral character, encompassing traits like courage, persistence, and strength of character. Alleyne et al. (2006) explored moral intensity perceptions among final-year accounting students in Barbados, finding high moral intensity levels across scenarios with no significant gender differences. Similarly, Oboh (2019) examined personal and moral intensity variables affecting ethical recognition, judgment, and intention among accounting professionals. The study found high ethical sensitivity, with male respondents displaying greater ethical sensitivity and judgment. Age also influenced outcomes; professionals aged 30–39 exhibited higher ethical sensitivity, while those aged 40 and above were more inclined to make ethical judgments and decisions. Both studies highlight the role of demographic factors in shaping ethical behaviour in accounting.

Religiosity is also recognised as a factor that has influenced the ethical development of the accounting profession. According to Iannello et al. (2019), religiosity refers to the formal, institutional, and outward expression of an individual's relationship with God, often characterised by the unique customs, beliefs, and worldview of a specific religious group. Nazaruddin et al. (2018) examined how religiosity and gender moderate the relationship between love of money and students' ethical perceptions, finding that accounting students, who were highly religious, viewed questionable accounting actions as unethical. Resty (2018) studied accounting students in Minangkabau and concluded that religiosity positively influenced ethical judgment, with students applying strong religious values to differentiate between ethical and unethical behaviour in decision-making. Sofyani et al. (2016) explored ethical sensitivity and religious commitment among prospective accountants at the University of Muhammadiyah Yogyakarta and found that gender affected both religiosity commitment and ethical sensitivity, with female students exhibiting stronger religiosity commitment. However, Helmy (2018) found that gender did not significantly impact ethical judgment, contradicting the view that women are more ethical than men.

Ideally, it is expected that prospective accountants' ethical behaviour would be shaped by their morality and religiosity, with demographic factors such as age and institutional affiliation further influencing these relationships. Prospective accountants with strong moral foundations would likely demonstrate a clear sense of right and wrong, consistently applying ethical principles in their decision-making, regardless of external pressures. Those with a solid religious grounding may also be more inclined to uphold ethical standards, drawing on their faith to guide their actions in challenging situations. Age may influence antecedents of ethical behaviour (AEB), as older students could exhibit greater moral sensitivity and religiosity. Gender, however, would not show significant differences in AEB, suggesting that both male and female prospective accountants would exhibit similar levels of morality and religiosity. Institutional affiliation could also impact AEB, with students from institutions emphasising ethics and religiosity likely to demonstrate heightened ethical awareness.

1.1 Statement of the Problem

Ghana's position on Transparency International's Corruption Perceptions Index highlights the pervasive nature of corruption in both the public and private sectors. With a score of 40 on a scale from 0 (highly corrupt) to 100 (very clean), Ghana has been considered one of the most corrupt countries since 2017. Although there was a slight improvement to a score of 43 in 2020, this progress has been minimal, indicating that corruption remains a significant issue (Transparency International, 2020). In the 2023 Index, Ghana ranked 70th out of 180 countries, where the highest-ranked country is regarded as having the most transparent public sector (Transparency International, 2023). Despite efforts to enhance ethical practices, the pervasive influence of corruption, as demonstrated by Ghana's ranking on Transparency International's Corruption Perceptions Index, raises critical questions about how moral foundations and religiosity can influence accounting students' ethical intentions. While morality and religiosity are expected to guide prospective accountants in making ethical decisions, the demographic factors of age, gender, and institutional affiliation may further shape these ethical perceptions and behaviours. Research suggests that a strong moral foundation and religiosity can deter unethical actions. However, the growing social acceptance of corruption may weaken students' resolve, particularly as they transition into professional environments. This underscores the need to assess how morality, religiosity, and demographic factors impact the academic and professional ethical behaviour of prospective accountants.

Studies on AEB across demographics have produced conflicting results. For example, Sofyani et al. (2016) found a significant difference in religiosity commitment between female and male students, with females demonstrating stronger religiosity commitment. In contrast, Helmy (2018) examined the influence of ethical orientation, gender, and religiosity on accounting students' ethical judgments and found no support for the idea that women are more ethical than men in their decisions. Alleyne et al. (2006) studied moral intensity perceptions among final-year accounting students in Barbados and found no significant gender differences in perceived moral intensity. Similarly, no age-related differences in moral intensity were observed. However, Sari et al. (2021) investigated the

impact of ethics education on ethical perceptions and found significant differences based on the type of university attended, with students from religious-based universities exhibiting stronger ethical perceptions.

Apart from contradictory findings, the study will address a geographical and population gap as few studies in Ghana have examined demographic differences in morality and religiosity of accounting students. According to literature search over the past decade in Ghana, only one study by Frimpong and Omane-Adjekum (2024) has examined gender differences in accounting students' morality. This clearly indicates that studies in AEB among prospective accountants based on demographic factors such as age, gender and institutional affiliation in Ghana are still in an embryonic stage.

1.2 Research Objectives

The study was guided by the following objectives:

i. To determine the level of prospective accountants' antecedent of ethical behaviour (AEB) (morality and religiosity).
ii. To examine the difference in AEB among prospective accountants based on age.
iii. To examine the difference in AEB among prospective accountants based on gender.
iv. To examine the difference in AEB among prospective accountants based on institutional affiliation.

II. LITERATURE REVIEW

2.1 Theoretical Review

2.1.1 Kohlberg's Theory of Moral Development

Developed by Lawrence Kohlberg in the 1950s, Kohlberg's Theory of Moral Development describes how people develop their moral reasoning over time. Kohlberg, building on Jean Piaget's early research on children's moral development, postulated that moral reasoning develops in three major phases, each of which consists of two stages that correspond to progressively more complex ethical understandings. According to the theory, there are three phases of moral development: pre-conventional, conventional, and post-conventional. People make moral judgements at the pre-conventional level based on the results of their activities, such as avoiding punishment or pursuing rewards. At a conventional level, people prioritise upholding connections and social order by following the norms and trying to win others' approval. People at the post-conventional level are distinguished by their tendency to challenge social conventions in order to promote the abstract ethical concepts of justice, equality, and human rights (Kohlberg, 1973; Kohlberg, 1976; Kohlberg & Hersh, 1977; Kohlberg, 1994; Piaget, 2013).

The Theory of Moral Development by Kohlberg provides important insights into how the ethical conduct of aspiring accountants may change depending on their moral reasoning proficiency. In the present study, morality and religion are important antecedents that influence ethical decision-making. Kohlberg's phases offer a framework for comprehending the ways in which accounting students at various developmental stages approach ethical quandaries. As students advance through Kohlberg's phases, they develop increasingly complex moral reasoning skills, which may result in more morally sound accounting scenarios. In this development, demographic factors including age, gender, and institutional affiliation may be important. For example, more life experience may cause older students to display more advanced phases of moral development, while gender and institutional affiliation may have an impact on the ethical principles and values that direct students' actions. This hypothesis contributes to the understanding of how the interplay of personal morality, religious beliefs, and demographic characteristics shapes ethical behaviour among aspiring accountants.

2.2 Empirical Review

2.2.1 Level of Prospective Accountants' Antecedent of Ethical Behaviour

In Ghana, Frimpong and Omane-Adjekum (2024) investigated morality among accounting students across gender. The results revealed that accounting students have a high level of morality. Nazaruddin et al. (2018) analysed the moderating effect of religiosity and gender on the relationship between love of money and students' ethical perceptions. They reported that accounting students were highly religious and perceived questionable accounting actions as unethical. Similarly, Singh et al. (2020) explored the level of ethical sensitivity and its relationship with religiosity among final-year accounting students in Malaysia. Their findings revealed that accounting students were ethically sensitive and demonstrated a strong connection between their religiosity and ethical decision-making. Resty (2018) conducted a study on accounting students in Minangkabau and concluded that religiosity positively influenced ethical judgment. The study indicated that accounting students exhibited a high level of religiosity, as individuals with strong religious values consistently applied those principles in distinguishing ethical from unethical behaviour in their decision-making processes.

Alleyne et al. (2006) examined the perceptions of moral intensity among final-year undergraduate accounting students in Barbados and found that students exhibited high levels of moral intensity across all scenarios. These scenarios highlighted factors such as magnitude of consequences, probability of effect, temporal immediacy, concentration of effect, proximity, and social consensus, which shaped students perceived moral intensity regarding ethical dilemmas. The researchers observed that students heightened moral intensity in non-audit-related scenarios might have been influenced by self-preservation concerns, particularly given the "health-threatening" nature of the scenario presented. Furthermore, these non-audit-related scenarios demanded less technical knowledge and interpretation in ethical decision-making processes. Similarly, Oboh (2019) investigated the impact of personal and moral intensity variables on ethical recognition, ethical judgment, and ethical intention among accounting professionals. The study revealed that accounting professionals demonstrated high ethical sensitivity and were more inclined to make ethical judgments and decisions.

2.2.2 Prospective Accountants' Antecedent of Ethical Behaviour and Age

Conroy and Emerson (2004) conducted a survey among students from two Southern United States universities, one public and the other a private religious institution, and found that age significantly influenced ethical perception. Their findings revealed that in 11 of the vignettes used, older age was a significant predictor of ethical perception at the 10% level, showing the role of age in ethical decision-making. In contrast, Alleyne et al. (2006), in their study on moral intensity among final-year undergraduate accounting students in Barbados, found no significant differences in perceptions of moral intensity across different age groups. Meanwhile, Oboh (2019) examined the influence of personal and moral intensity variables on ethical decision-making among accounting professionals and reported that age significantly affected ethical recognition, judgment, and intention. Specifically, professionals aged 30–39 demonstrated higher ethical sensitivity, whereas those aged 40 and above were more inclined toward making ethical judgments and decisions.

2.2.3 Prospective Accountants' Antecedent of Ethical Behaviour and Gender

In Ghana, Frimpong and Omane-Adjekum (2024) examined morality among accounting students and found no statistically significant differences in morality levels between male and female students. Similarly, Sofyani et al. (2016) investigated ethical sensitivity and religious commitment among prospective accountants at the University of Muhammadiyah Yogyakarta, revealing that gender influenced both factors. Female students exhibited stronger religiosity commitment, supporting the view that women are generally more ethical, less aggressive, and more risk-averse than men. However, Helmy (2018), in a study on the effects of ethical orientation, gender, and religiosity on accounting students' ethical judgment, found no evidence to suggest that women are more ethical than men, indicating that gender did not significantly affect ethical behaviour. Alleyne et al. (2006) explored perceptions of moral intensity among final-year accounting students in Barbados and reported no significant gender differences in perceived moral intensity. In contrast, Oboh (2019) discovered that gender significantly influenced ethical recognition, judgment, and intention among accounting professionals, with male respondents demonstrating higher ethical sensitivity and a greater tendency to make ethical decisions.

2.2.4 Prospective Accountants' Antecedent of Ethical Behaviour and Institutional Affiliation

Sari et al. (2021) investigated the effectiveness of ethics education in instilling core ethical values and examined differences in ethical perceptions between students in religious and public universities. The findings revealed significant differences, indicating that the type of university students attend influences the development of their ethical behaviour, with religious-based universities fostering stronger ethical perceptions. Similarly, Mujtaba et al. (2011) investigated the ethical maturity of individuals in the public and private sectors in Iranian cities such as Tehran, Karaj, and Kerman, focusing on the role of institutional environment in affecting ethical perceptions. Their findings indicated differences in ethical maturity levels, with public sector individuals scoring higher in personal business ethics compared to those in the private sector, students, and unemployed individuals. This suggests that private institutions may create an environment more conducive to cultivating ethical behaviours.

2.3 Conceptual Framework

The conceptual framework in Figure 1 illustrates the hypothesised relationships between demographic factors (age, gender, and institutional affiliation) and the antecedents of ethical behaviour, specifically morality and religiosity. The framework suggests that age (H1) is expected to have a direct influence on ethical behaviour, implying that ethical perspectives and values may evolve as individuals mature, potentially affecting their sense of morality and religiosity. Additionally, gender (H2) is hypothesised to impact ethical behaviour, with the notion that societal and cultural gender roles may shape ethical beliefs differently. Lastly, institutional affiliation (H3) is proposed to influence ethical behaviour, under the assumption that different institutional environments may foster unique ethical values and

attitudes. This model aims to explore how these demographic variables contribute to understanding the antecedents of ethical behaviour, emphasising morality and religiosity as key components.

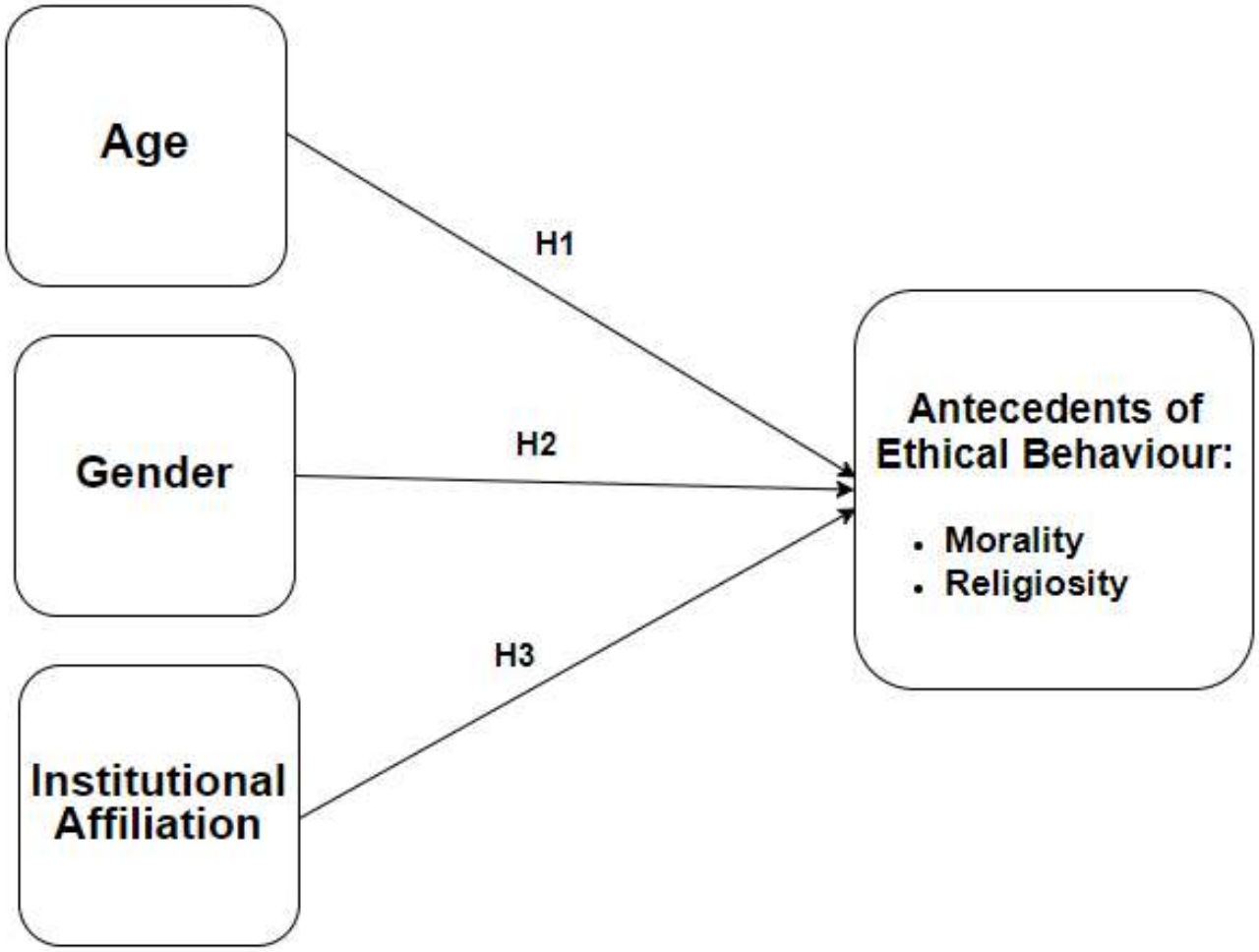

Figure 1
Conceptual Framework Depicting Age, Gender, and Institutional Affiliation Difference in AEB

III. METHODOLOGY

3.1 Research Design and Sampling Procedure

The study employed a cross-sectional survey technique to ascertain how demographics affect future accountants' AEB. Only final-year accounting education students at the University of Cape Coast (UCC) and the University of Education, Winneba (UEW) provided the data. A census method was used to include all final-year students in this field. The focus on final-year students was justified by their extensive knowledge of accounting principles and ethical frameworks, which enables them to provide valuable perspectives on the factors influencing anti-corruption efforts in the accounting sector. Because there were 120 final-year accounting education students from UCC and 53 from UEW overall, this significant group was well covered.

3.2 Instrumentation

The study's instruments were a structured, five-part, closed-ended questionnaire that gathered comprehensive data on the variables under investigation. Section A focused on collecting demographic information, including the participants' age, gender, and institution, in order to provide a baseline image of the sample. Section B included a 20-item scale adapted from Katz and Schmida (1992) to assess the level of religious participation and underlying beliefs among the accounting students. Ten measures assessed religious beliefs, while ten items assessed observance of religious activities. The 12-item Moral Efficacy scale, which was adapted from Albaum and Peterson (2006), May et al. (2014), and Onumah (2019), was used in Section C to measure students' confidence in their capacity to behave morally. Section D evaluated moral meaningfulness using an 8-item scale adapted from May et al. (2014) and Ahinful et al. (2017), focusing on the importance that students place on moral values in their daily lives. Finally, section E

employed a 12-item moral courage scale, which was adapted from Howard (2012), May et al. (2014), and Sonnentag and Barnett (2016), to assess accounting students' readiness to act morally under pressure. The data collected was assessed for reliability of which the results are presented in Table 1. The reliability for the entire questionnaire was .892, which suggested that, largely, the indicators had internal consistency reliability (Nunnally & Bernstein, 2008) which was sufficient for the study.

Table 1
Reliability Test

Scale		Respondents	No. of Items	Cronbach's Alpha
Religiosity	Religious Activities (Practices)	173	10	.684
	Religious Principles (Beliefs)	173	10	.686
Morality	Moral Efficacy	173	12	.610
	Moral Meaningfulness	173	8	.824
	Moral Courage	173	12	.848
	Overall	173	52	.892

3.3 Data Analysis

Both descriptive and inferential statistics were used in the analysis of the data obtained. Descriptive statistics included frequency counts, percentages, averages, and standard deviations. One-way ANOVA and independent sample t-tests were used for the inferential analysis. To make sure the assumptions for using the t-tests and ANOVA were met, a normality test, Levene's test, robust test for equality of means was performed, with a significance threshold of .05.

IV. FINDINGS & DISCUSSION

4.1 Findings

4.1.1 Background Information of the Respondents

The background information of the respondents was analysed using frequency and percentages. The results on the background information of the respondents are shown in Table 2.

Table 2
Background Information of the Respondents

Demography	Subscale	Frequency	Percentages (%)
Gender	Male	110	63.60
	Female	63	36.40
Age	Below 25 years	124	71.70
	25 – 30 years	44	25.40
	Above 30 years	5	2.90
Institution	University of Cape Coast (UCC)	120	69.40
	University of Education, Winneba (UEW)	53	30.60

From Table 2, the study involved 173 accounting students, showcasing a diverse and dynamic demographic profile. Male respondents dominated the sample, representing 63.6%, while females accounted for 36.4%, reflecting the gender composition of the accounting programmes surveyed. Age-wise, majority of the students (71.7%) were under 25 years, suggesting a youthful cohort actively pursuing reading the Accounting Programme. Those aged between 25 and 30 years comprised 25.4%, while a small but notable 2.9% were above 30 years, showing a blend of traditional and non-traditional students in the study. In terms of institutional representation, the UCC emerged as the primary source of participants, contributing 69.4% of the respondents, indicating its prominence in accounting education within the region. The UEW followed with 30.6%, further enriching the diversity of the data.

4.1.2 Test for Normality

Sample size plays a vital role in determining the normality of a dataset. For smaller sample sizes, achieving normality cannot always be guaranteed. For instance, Altman (1991, as cited in Rochon et al., 2012) demonstrated that even a sample of 50 drawn from a normal distribution could appear non-normal. Similarly, Blanca et al. (2013) analysed 693 datasets with sample sizes ranging from 10 to 30, focusing on skewness and kurtosis. Their findings revealed that only 5.5% of these datasets approximated a normal distribution, with skewness and kurtosis values falling between ±.25. These findings show the importance of assessing normality before analysis, even with small

sample sizes. To ensure normality, skewness tests are recommended due to their reliability across both small and large sample sizes (Kim, 2013). In this study, the normality of the data was assessed by examining skewness statistics. According to Kim (2013), for medium-sized samples ($50 < n < 300$), skewness values within the range of ±1.96 to ±3.29 are indicative of normal distribution. With a sample size of 173, the skewness statistics for all variables fell within this acceptable range, and the mean and median values were approximately equal, as presented in Table 3. This confirmation of normal distribution validated the use of parametric statistical tools for data analysis.

Table 3

Test for Normality

Variable			Statistic	Std. Error
Religiosity	Religious Activities (Practices)	Mean	3.128	.040
		Median	3.200	
		Skewness	.376	.185
	Religious Principles (Beliefs)	Mean	3.420	.053
		Median	3.400	
		Skewness	2.636	.185
	Overall	Mean	3.275	.042
		Median	3.300	
		Skewness	1.547	.185
Morality	Moral Efficacy	Mean	3.135	.031
		Median	3.167	
		Skewness	.180	.185
	Moral Meaningfulness	Mean	3.294	.030
		Median	3.250	
		Skewness	-1.364	.185
	Moral Courage	Mean	3.110	.033
		Median	3.083	
		Skewness	-.851	.185
	Overall	Mean	3.166	.026
		Median	3.188	
		Skewness	-1.778	.185

Table 4

Levene's Test for Equality of Variances

Dependent Variable	Independent Variable	F	Sig.
Religious Activities (Practices)	Gender	.168	.682
	Institution	.000	.990
Religious Principles (Beliefs)	Gender	.423	.516
	Institution	.225	.636
Religiosity	Gender	.004	.953
	Institution	.012	.913
Moral Efficacy	Gender	.471	.493
	Institution	.072	.788
Moral Meaningfulness	Gender	.359	.550
	Institution	.000	.991
Moral Courage	Gender	1.564	.213
	Institution	1.834	.177
Morality	Gender	.289	.591
	Institution	.929	.337

Table 5

Robust Test for Equality of Means (Welch)

Dependent Variable	Independent Variable	Statistic	df1	df2	Sig.
Religious Activities (Practices)	Age	.418	2	12.899	.667
Religious Principles (Beliefs)	Age	.282	2	10.684	.760
Religiosity	Age	.166	2	10.886	.849
Moral Efficacy	Age	.496	2	11.304	.622
Moral Meaningfulness	Age	2.454	2	11.205	.131
Moral Courage	Age	.404	2	11.896	.676
Morality	Age	1.194	2	11.550	.338

Parametric mean comparison tests, such as t-tests and ANOVA, require the assumption of equal variance, which ensures that the variances of the groups being tested are consistent. Verifying the assumption of homogeneity of variance is key before conducting these tests. This verification ensures that the variances of the dependent variable (AEB) are uniform across the sub-groups of the independent variables (gender, institution, and age). As shown in Tables 4 and 5, the homogeneity of variance assumption is satisfied, confirming the appropriateness of independent sample t-tests and ANOVA for further statistical analysis.

4.1.3 Level of Prospective Accountants' AEB

Research Objective 1 determined the level of prospective accountants' AEB (morality and religiosity). Mean and standard deviation was used to measure the level of AEB among accounting students. A mean value from 1 to 2 indicates low level, 2.1 to 3 indicates moderate level, and 3.1 to 4 indicates high level. The results are presented in Table 6.

Table 6

Accounting Students' AEB

Variable	Mean	Standard Deviation
Religious Activities (Practices)	3.13	.53
Religious Principles (Beliefs)	3.42	.70
Religiosity (Overall)	3.28	.55
Moral Efficacy	3.14	.40
Moral Meaningfulness	3.29	.39
Moral Courage	3.11	.43
Morality (Overall)	3.17	.34

Key: Low: 1 – 2; Moderate: 2.1 – 3; High: 3.1 - 4

From Table 6, the findings revealed that accounting students demonstrated a high level of AEB overall (religiosity [M = 3.28, SD = .55]; morality [M = 3.17, SD = .34]). In terms of religiosity, accounting students exhibited a high commitment to both religious practices, such as observing sacred days, fasting, and giving tithes, and religious beliefs, including faith in divine authority, moral accountability, and adherence to religious laws. This indicates that religiosity plays a role in shaping their ethical perspectives and its influence is profoundly ingrained in their professional decision-making. Similarly, the morality of the accounting students, assessed through dimensions such as moral efficacy, moral meaningfulness, and moral courage, also reflected a high level. The findings show that accounting students possess a foundational understanding of ethical principles and recognise the importance of morality in their actions by expressing confidence in their ethical knowledge, practical commitment to moral behaviour and their willingness to confront ethical challenges.

4.1.4 Difference in AEB among Prospective Accountants Based on Age

Research Objective 2 examined the difference in AEB among prospective accountants based on age. A One-Way ANOVA, conducted at a significance level of .05, was used to determine whether significant differences exist among the age groups of accounting students regarding their AEB, encompassing morality and religiosity. This statistical tool was appropriate as the independent variable, age, comprised three distinct groups (Below 25 years, 25–30 years, and Above 30 years), while the dependent variable, AEB, was continuous and normally distributed. The results of this analysis are presented in Table 7.

Table 7

Difference in AEB among Accounting Students Based on Age

Variable	Age Group	N	Mean	SD	ANOVA					
						Sum of Squares	df	Mean Square	F	Sig.
RA	Below 25	124	3.13	.55	Between Groups	.060	2	.030	.106	.900
	25-30	44	3.13	.50	Within Groups	48.452	170	.285		
	Above 30	5	3.02	.25	Total	48.512	172			
Variable	**Age Group**	**N**	**Mean**	**SD**	**ANOVA**					
						Sum of Squares	df	Mean Square	F	Sig.
RP	Below 25	124	3.44	.76	Between Groups	.217	2	.109	.220	.803
	25-30	44	3.36	.51	Within Groups	84.010	170	.494		
	Above 30	5	3.36	.81	Total	84.228	172			
Variable	**Age Group**	**N**	**Mean**	**SD**	**ANOVA**					
						Sum of Squares	df	Mean Square	F	Sig.
RR	Below 25	124	3.29	.59	Between Groups	.090	2	.045	.147	.863
	25-30	44	3.25	.44	Within Groups	51.963	170	.306		
	Above 30	5	3.19	.51	Total	52.053	172			
Variable	**Age Group**	**N**	**Mean**	**SD**	**ANOVA**					
						Sum of Squares	df	Mean Square	F	Sig.
ME	Below 25	124	3.12	.44	Between Groups	.112	2	.056	.338	.714
	25-30	44	3.16	.32	Within Groups	28.087	170	.165		
	Above 30	5	3.25	.29	Total	28.198	172			
Variable	**Age Group**	**N**	**Mean**	**SD**	**ANOVA**					
						Sum of Squares	df	Mean Square	F	Sig.
MM	Below 25	124	3.27	.40	Between Groups	.470	2	.235	1.563	.212
	25-30	44	3.32	.37	Within Groups	25.553	170	.150		
	Above 30	5	3.58	.29	Total	26.023	172			
Variable	**Age Group**	**N**	**Mean**	**SD**	**ANOVA**					
						Sum of Squares	df	Mean Square	F	Sig.
MC	Below 25	124	3.10	.43	Between Groups	.111	2	.055	.297	.744
	25-30	44	3.15	.46	Within Groups	31.720	170	.187		
	Above 30	5	3.18	.25	Total	31.831	172			
Variable	**Age Group**	**N**	**Mean**	**SD**	**ANOVA**					
						Sum of Squares	df	Mean Square	F	Sig.
MR	Below 25	124	3.15	.35	Between Groups	.164	2	.082	.703	.496
	25-30	44	3.19	.32	Within Groups	19.826	170	.117		
	Above 30	5	3.31	.22	Total	19.990	172			

RA - *Religious Activities*; RP - *Religious Principles (Beliefs)*; *RR - Religiosity*; ME - *Moral Efficacy*; MM - *Moral Meaningfulness*; MC - *Moral Courage*; MR - *Morality*.

The results, as shown in Table 7, indicated no statistically significant differences in the mean scores across the three age groups for religiosity ($F[2, 170] = .147$, $p = .863$; $p > .05$) and morality ($F[2, 170] = .703$, $p = .496$; $p > .05$). Consequently, no post-hoc comparison tests were conducted. This suggests that ethical values such as religiosity and morality are similar across the various age groups. Regardless of whether students are younger (Below 25 years), in the middle age group (25–30 years), or older (Above 30 years), their levels of religiosity and morality remain relatively similar. Overall, these results challenge the assumption that age-related maturity necessarily leads to higher religiosity or morality.

4.1.5 Difference in AEB among Prospective Accountants Based on Gender

Research Objective 3 examined the difference in AEB among prospective accountants based on gender. The dependent variable in this analysis is accounting students' AEB, while gender serves as the independent variable, with two categories: male and female accounting students. To assess whether there is a significant difference in the AEB between male and female students, an independent sample t-test was conducted at a significance level of .05. This statistical tool was appropriate because the independent variable (gender) has two groups, and the dependent variable (AEB) is normally distributed. The results of the analysis are presented in Table 8.

Table 8

Difference in AEB among Accounting Students Based on Gender

Variable	Gender	N	Mean	SD	t	df	Sig.
RA	Male	110	3.14	.56	.250	171	.803
	Female	63	3.11	.48			
Variable	Gender	N	Mean	SD	t	df	Sig.
RP	Male	110	3.41	.70	-.352	171	.725
	Female	63	3.44	.71			
Variable	Gender	N	Mean	SD	t	df	Sig.
RR	Male	110	3.27	.58	-.105	171	.916
	Female	63	3.28	.49			
Variable	Gender	N	Mean	SD	t	df	Sig.
ME	Male	110	3.15	.38	.823	171	.412
	Female	63	3.10	.45			
Variable	Gender	N	Mean	SD	t	df	Sig.
MM	Male	110	3.32	.41	1.346	171	.180
	Female	63	3.24	.35			
Variable	Gender	N	Mean	SD	t	df	Sig.
MC	Male	110	3.14	.46	1.306	171	.193
	Female	63	3.05	.37			
Variable	Gender	N	Mean	SD	t	df	Sig.
MR	Male	110	3.19	.36	1.375	171	.171
	Female	63	3.12	.30			

RA - *Religious Activities*; RP - *Religious Principles (Beliefs)*; *RR - Religiosity*; ME - *Moral Efficacy*; MM - *Moral Meaningfulness*; MC - *Moral Courage*; MR - *Morality.*

The results in Table 8 indicated that there were no statistically significant differences in AEB between male and female accounting students. Specifically, the mean scores for religiosity ($M = 3.27$, $SD = .58$ for male; $M = 3.28$, $SD = .49$ for female; $t (171) = -.105$, $p = .916$; $p > .05$) and morality ($M = 3.19$, $SD = .36$ for male; $M = 3.12$, $SD = .30$ for female; $t (171) = 1.375$, $p = .171$; $p > .05$) did not differ. This suggests that both male and female accounting students demonstrate similar levels of religiosity and morality, indicating that gender does not influence their antecedents of ethical behaviour. The fact that both male and female students scored similarly on these measures suggests that, in this context, accounting students perceive or engage with ethical principles irrespective of their gender. This result indicates that male and female accounting students share similar ethical foundations in terms of their religiosity and morality.

4.1.6 Difference in AEB among Prospective Accountants Based on Institutional Affiliation

Research Objective 4 examined the difference in AEB among prospective accountants based on institutional affiliation. The dependent variable in this analysis is accounting students' AEB, while the independent variable is institutional affiliation, which consists of two categories: UCC and UEW accounting students. To determine if there is a significant difference in AEB between students from these two institutions, an independent sample t-test was conducted at a significance level of .05. This statistical tool was suitable as the independent variable (institutional affiliation) includes two groups, and the dependent variable (AEB) is normally distributed. The results of the analysis are presented in Table 9.

Table 9

Difference in AEB among Prospective Accountants Based on Institutional Affiliation

Variable	Institution	N	Mean	SD	t	df	Sig.
RA	UCC	120	3.11	.54	-.684	171	.495
	UEW	53	3.17	.52			
Variable	Institution	N	Mean	SD	t	df	Sig.
RP	UCC	120	3.44	.76	.480	171	.632
	UEW	53	3.38	.54			
Variable	Institution	N	Mean	SD	t	df	Sig.
RR	UCC	120	3.27	.58	-.007	171	.994
	UEW	53	3.28	.49			
Variable	Institution	N	Mean	SD	t	df	Sig.
ME	UCC	120	3.15	.41	.919	171	.359
	UEW	53	3.09	.40			
Variable	Institution	N	Mean	SD	t	df	Sig.
MM	UCC	120	3.30	.37	.351	171	.726
	UEW	53	3.28	.43			
Variable	Institution	N	Mean	SD	t	df	Sig.
MC	UCC	120	3.11	.44	-.056	171	.955
	UEW	53	3.11	.42			
Variable	Institution	N	Mean	SD	t	df	Sig.
MR	UCC	120	3.17	.33	.483	171	.630
	UEW	53	3.15	.37			

RA - *Religious Activities*; RP - *Religious Principles (Beliefs)*; *RR - Religiosity*; ME - *Moral Efficacy*; MM - *Moral Meaningfulness*; MC - *Moral Courage*; MR - *Morality.*

The results in Table 9 indicated that there were no statistically significant differences in AEB between UCC and UEW accounting students. Specifically, the mean scores for religiosity (M = 3.27, SD = .58 for UCC; M = 3.28, SD = .49 for UEW; t (171) = -.007, p = .994; p > .05) and morality (M = 3.17, SD = .33 for UCC; M = 3.15, SD = .37 for UEW; t (171) = .483, p = .630; p > .05) did not differ. This suggests that students from both UCC and UEW exhibit similar levels of ethical awareness and moral values regardless of their institution despite the potential differences in the institutional environments. This indicates a shared cultural or educational framework across both institutions that fosters similar ethical perspectives among accounting students.

4.2 Discussion

The study determined the level of prospective accountants' AEB (morality and religiosity). The findings revealed that accounting students demonstrated a high level of AEB overall. Accounting students are deeply committed to religious practices and beliefs, including observing sacred days, fasting, and adherence to divine authority and moral accountability. This strong religiosity aligns with findings by Nazaruddin et al. (2018) and Singh et al. (2020), who discovered the role of religiosity in shaping ethical perceptions and decision-making among accounting students. Similarly, Resty (2018) found that students with high religiosity were more likely to consistently apply religious principles when evaluating ethical dilemmas. These observations show that religiosity does not merely serve as a private belief system but also informs the ethical frameworks accounting students use in professional scenarios. The high religiosity observed among accounting students in the current study may reflect cultural and societal norms within Ghana, where religious values are often interwoven with education and personal conduct. Again, accounting students demonstrated high levels of moral efficacy, moral meaningfulness, and moral courage. These dimensions show their confidence in their ethical knowledge, their commitment to upholding moral principles, and

their readiness to face ethical challenges. This finding corroborates the results of Frimpong and Omane-Adjekum (2024), which also identified high levels of morality among Ghanaian accounting students. The current findings also resonate with Alleyne et al.'s (2006) study, which reported the importance of moral intensity in students' ethical evaluations and decision-making processes. Accounting students' ability to engage with ethical principles suggests a foundational preparation to navigate professional challenges ethically, a critical attribute for individuals entering a profession where ethical dilemmas are commonplace. The results also correspond with the findings of Oboh (2019) who noted that accounting professionals demonstrated high ethical sensitivity and were more inclined to make ethical judgments and decisions as a result of demonstrating high level of morality and religiosity.

Furthermore, the study examined the difference in AEB among prospective accountants based on age, gender, and institutional affiliation. The results showed that there were no statistically significant differences in AEB among accounting students based on age, gender, and institutional affiliation. The results indicated that accounting students' religiosity and morality remain consistent across age groups, with no significant differences observed between younger students (below 25 years), middle-aged students (25–30 years), and older students (above 30 years). This contrasts with studies such as Conroy and Emerson (2004) and Oboh (2019), which suggested that age significantly influences ethical perception and sensitivity, with older individuals displaying heightened ethical awareness. In the context of this study, this reflects a shared educational and cultural framework that fosters a uniform development of ethical values among accounting students, regardless of age. It is possible that the accounting students' ethical training and exposure to similar moral teachings neutralise any age-related variations in ethical perspectives. Additionally, Alleyne et al. (2006) similarly reported no significant differences in perceptions of moral intensity across age groups, supporting the notion that ethical values may not necessarily evolve with age among accounting students. These results challenge the assumption that age-related maturity equates to stronger ethical principles, suggesting instead that ethical development in this context may be more influenced by shared educational and societal values.

Moreover, the study found no significant differences in AEB between male and female accounting students. Both groups exhibited similar levels of religiosity and morality, suggesting that gender does not influence their ethical foundations. This aligns with studies by Frimpong and Omane-Adjekum (2024) and Helmy (2018), which reported no significant gender differences in morality and ethical judgments among accounting students. These findings show the possibility that in this cultural and educational context, gender-neutral ethical training and socialisation have succeeded in promoting equitable ethical standards among students. Conversely, the results differ from those of Sofyani et al. (2016), who observed that female students exhibited stronger religiosity commitment than their male counterparts, attributing this to broader societal perceptions of women as more ethical and risk-averse. This disparity might be explained by cultural differences or the specific characteristics of the sample populations. Furthermore, the current findings align with Alleyne et al. (2006), who found no significant gender differences in moral intensity perceptions among students, reinforcing the conclusion that gender may not be a significant determinant of ethical behaviour in the context of accounting education.

Finally, the lack of significant differences in AEB between accounting students from UCC and UEW suggests uniformity in ethical awareness and moral values across the two institutions. This finding is notable given the potential for institutional differences in curricula, educational environments, or values to influence students' ethical perspectives. The results differ from Sari et al. (2021) who found that students in religiously affiliated universities exhibited stronger ethical perceptions than those in public institutions, indicating the role of institutional culture in shaping ethical behaviour. Similarly, the results of this current contradict with Mujtaba et al. (2011) who indicated differences in ethical maturity levels, with public sector individuals scoring higher in personal business ethics compared to those in the private sector, students, and unemployed individuals suggesting that private institutions may create an environment more conducive to cultivating ethical behaviours. The observed similarity in AEB across institutions may reflect a shared cultural or educational framework in Ghana that transcends institutional boundaries. For instance, national guidelines or standardised curricula in accounting education may ensure consistency in ethical training. Additionally, societal values emphasising morality and religiosity may reinforce these behaviours regardless of the specific institutional environment.

V. CONCLUSION & RECOMMENDATIONS

5.1 Conclusion

The study concluded that the high levels of ethical commitment observed among accounting students suggest that their educational experiences and societal norms have effectively instilled ethical values that prepare them to navigate professional challenges. This shows the capacity of accounting education in Ghana to produce morally and ethically conscious professionals, which is key in a field often fraught with ethical dilemmas. The results imply that ethical principles, such as moral courage and accountability, are being successfully internalised, which enhance ethical decision-making in real-world scenarios. The uniformity in antecedents of ethical behaviour across age groups

suggests that ethical development among accounting students is more influenced by shared educational and cultural frameworks than by age-related maturity. This challenges the assumption that ethical values naturally strengthen with age and instead highlights the importance of targeted ethical education in fostering consistent moral standards across diverse student demographics. The lack of gender differences in religiosity and morality suggests that accounting education and societal values in Ghana have succeeded in promoting equitable ethical standards among male and female students. This underscores the potential for education systems to mitigate gender biases in ethical training and ensure that ethical competence is developed uniformly across genders. Finally, the absence of significant differences in ethical behaviour antecedents between students from UCC and UEW indicates that institutional differences in curricula or environments have minimal impact on ethical development. This suggests that national educational policies and standardised curricula may play a vital role in promoting uniform ethical training, reinforcing the broader cultural and societal emphasis on morality and religiosity.

5.2 Recommendations

The study recommended that accounting programmes should place a stronger emphasis on integrating practical and scenario-based learning opportunities into their ethical education curricula. This will help students engage with real-world ethical dilemmas and enhance their ability to apply theoretical knowledge in professional contexts. Accounting students can develop a deeper understanding of the complexities involved in ethical decision-making, ensuring they are well-prepared for challenges in their careers by providing case studies, role-playing activities, and ethical simulations. Furthermore, institutions should continue to provide equal opportunities for male and female students to develop their ethical and moral values. This will promote inclusivity and ensure that all students are equally prepared to navigate ethical dilemmas in their professional lives. Furthermore, it is recommended that collaboration between institutions, such as the UCC and the UEW, should be encouraged to strengthen the ethical education framework. Institutions can share best practices through joint workshops, conferences, and exchange programmes to standardise ethical training. Such collaboration can help ensure that students from various institutions receive comparable ethical education, fostering a uniform ethical foundation across the accounting profession. Lastly, the study recommended that it is still important to offer tailored support for ethical development across different age groups. Younger students may benefit from mentoring programmes that expose them to ethical principles through the experiences of seasoned professionals, while older students might benefit from advanced ethical training that aligns with their professional experiences and aspirations. This targeted approach can ensure that students of all ages are adequately equipped to uphold ethical standards in their professional lives.

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