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Mathematical Analysis of the Impact of Work, Culture and Relationships on Mental Well-being of Women

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ABSTRACT

Mental well-being of women is a multifaceted outcome shaped by the complex interplay of psychosocial factors. This study develops a compartmental dynamic model using a system of nonlinear ordinary differential equations to quantify the interactions between work-related stress, cultural pressure, relationship satisfaction and mental health. The model is analyzed to establish the positivity and boundedness of solutions, ensuring biological feasibility. Analytical results identify two equilibrium states: a Disease-Free Equilibrium (DFE), representing the absence of mental health strain and an Endemic Equilibrium (EE), representing a persistent state of mental health challenges. The basic reproduction number, $\mathcal{R}_0 = \eta/\xi$, is derived, serving as a threshold parameter. When $\mathcal{R}_0 < 1$, the DFE is both locally and globally asymptotically stable, indicating that mental well-being can be naturally restored. Conversely, when $\mathcal{R}_0 > 1$, the EE becomes stable, signifying the sustained prevalence of mental health burdens. Sensitivity analysis identifies the mental health recovery rate (ξ) and the relationship growth rate (η) as the most influential parameters. Numerical simulations corroborate the analytical findings, demonstrating that work stress and cultural pressure exert significant negative impacts on mental health, whereas high-quality relationships provide a substantial protective and restorative effect. The study proposes targeted interventions such as workload management, cultural norm shifting and strengthening relationship support systems to effectively promote and safeguard women's mental well-being.

Mathematics Subject Classification: Primary 92B05; Secondary 34D20.
Keywords: Mental Health, Dynamical Systems, cultural norm.

1 Introduction

Mental wellness is a critical aspect of overall health and has a clear impact on an individual's productivity, interpersonal relationships and overall well-being. About one out of every five women suffer from a common mental health problem, depression and anxiety [6]. Work related stress is a universal challenge that is not gender neutral. Women navigate a distinct psychosocial landscape at work, shaped by a complex interplay of organizational culture and deeply ingrained societal expectations. The WHO [11] highlights that depressive disorders are more prevalent in women globally and the workplace is a critical modifiable determinant of this disparity. The environment where women seek to build their careers and economic independence is often undermining their mental health through distinct pathways of stress, inequity and overload.

The gender-based mental health burden is rooted in the workplace culture. The problem is often embedded in the ideal worker norm that frequently conflicts with the disproportionate share of domestic and care-giving responsibilities [7]. This creates a double burden, where the pressure to perform at work is compounded by the "second shift" at home, leading to chronic role overload and an elevated risk of burnout and exhaustion [10] Despite the fact that qualitative studies [2, 3, 5, 6] have illuminated most of these connections, a quantitative approach offers the prospect of a complementary picture, allowing one to explore the dynamic interconnections and possible long-term outcomes.

Mathematical models have been developed to simulate the dynamics of mental illness attributed to various causes. For instance, Ali et al [1] developed a social media addiction and depression model which showed that if the population of addicted people is increased, then the population of depressed people also increases. Nivetha et al [8] developed a model that incorporated saboteurs into primary and secondary depressed populations with optimal control strategies aimed at improved treatment adherence to reduce spread of depressive disorders and associated mortality. These models demonstrate the power of this methodology; however, the failure to create work, cultures and relationships that support women's mental health is a systemic failure with profound implications. This problem begs for a dynamic model to explore how the sustained pressure of managing these dual roles fuels chronic stress and anxiety and to quantify the potential effectiveness of targeted interventions, such as flexible work policies or accessible mental health support, in mitigating this gendered mental health crisis.

This study aims to address this gap by developing and analyzing a novel mathematical model. In this study a four-compartmental model is formulated using a system of non-linear ordinary differential equations to represent the temporal dynamics between *Work Stress (W)*, *Cultural Pressure (C)*, *Relationship Satisfaction (R)*, and *Mental Well-being (M)*. The objectives of this research are threefold: to derive the model's equilibrium states and establish their local and global stability conditions; to perform a threshold analysis by deriving a basic reproduction number $\mathcal{R}_0$, which acts as an indicator of whether mental well-being can be sustained or will deteriorate under given conditions; and to conduct numerical simulations to visualize the system's behavior under various scenarios, thereby quantifying the relative impact of each factor. The insights generated from this model contribute to a more nuanced system-level understanding of women's mental health. They provide a formal basis for evaluating the potential efficacy of integrated interventions; such as workplace reforms, cultural awareness programs, relationship counseling, and for informing targeted policies aimed at building resilience and promoting sustainable mental well-being among women.

2 Model Formulation and Analysis

2.1 Model Dynamics

The proposed system of nonlinear differential equations captures the interplay between work stress (W), cultural pressure (C), relationship satisfaction (R), and mental well-being (M): In the model work accumulates at the rate α and decays at the rate β with respect to the existing work load and reduces at the rate γ due to mental health effects. Cultural activities accumulates at the rate δ and decays at the rate ϵ due to existing commitments and reduces at the rate ζ due to mental health effects. Relationships grows logistically with an intrinsic growth rate of η and is negatively impacted by work at the rate λ_1 and cultural activities at the rate λ_2. μ_1 and μ_2 is the impact of work and cultural activities on mental health (It is negative if they decrease mental health). μ_3 is the positive impact of relationships on mental health and ξ is the natural recovery rate of mental health.

2.2 Variables & Parameters

Table 1: Model Variables

Variable	Description
$W(t)$	Work-related stress
$C(t)$	Cultural pressure
$R(t)$	Relationship satisfaction
$M(t)$	Mental wellbeing (higher = better)

The system of equations representing the description above is:

$$\begin{aligned}
\frac{dW}{dt} &= \alpha - \beta W - \gamma M \\
\frac{dC}{dt} &= \delta - \epsilon C - \zeta M \\
\frac{dR}{dt} &= \eta R\left(1 - \frac{R}{L}\right) - \lambda_1 W - \lambda_2 C \\
\frac{dM}{dt} &= -\mu_1 W - \mu_2 C + \mu_3 R - \xi M
\end{aligned} \tag{2.1}$$

Table 2: Description of parameters in the model

Parameter	Description
α	Constant rate of work accumulation
β	Decay rate of work due to existing workload
γ	Rate of work reduction due to mental health effects
δ	Constant rate of cultural activity accumulation
ϵ	Decay rate of cultural activities due to existing commitments
ζ	Rate of cultural activity reduction due to mental health effects
η	Intrinsic growth rate of relationships
L	Carrying capacity (maximum possible level) of relationships
λ_1	Negative impact of work on relationships
λ_2	Negative impact of cultural activities on relationships
μ_1	Impact of work on mental health (negative if work decreases mental health)
μ_2	Impact of cultural activities on mental health
μ_3	Positive impact of relationships on mental health
ξ	Natural recovery rate of mental health

2.3 Positivity of the Model

Since the model describes human population, the state variables of the system (2.1) are shown to be positive.

Theorem 2.1. *Given the initial conditions ($W(0), C(0), R(0), M(0) \geq 0$), the solutions remain non-negative for all $t \geq 0$.*

Proof. :
Work stress (W)

$$\frac{dW}{dt} = \alpha - \beta W - \gamma M \tag{2.2}$$

$$\left.\frac{dW}{dt}\right|_{W=0} = \alpha - \gamma M \geq 0 \quad \text{if} \quad \alpha \geq \gamma M.$$

Since $\alpha > 0$ and M is bounded, $W(t)$ remains non-negative if γ is not too large.
Cultural pressure (C)

$$\frac{dC}{dt} = \delta - \epsilon C - \zeta M \tag{2.3}$$

$$\left.\frac{dC}{dt}\right|_{C=0} = \delta - \zeta M \geq 0 \quad \text{if} \quad \delta \geq \zeta M.$$

Since $\delta > 0$ and M is bounded, $C(t)$ remains non-negative if ζ is not too large.
Relationship quality (R)

$$\frac{dR}{dt} = \eta R\left(1 - \frac{R}{L}\right) - \lambda_1 W - \lambda_2 C \tag{2.4}$$

$$\left.\frac{dR}{dt}\right|_{R=0} = -\lambda_1 W - \lambda_2 C \leq 0.$$

If $R = 0$, it remains 0. For $R > 0$, positivity depends on the balance between growth ($\eta R(1 - R/L)$) and decay ($-\lambda_1 W - \lambda_2 C$).
Mental wellbeing (M)

$$\frac{dM}{dt} = -\mu_1 W - \mu_2 C + \mu_3 R - \xi M \tag{2.5}$$

$$\left.\frac{dM}{dt}\right|_{M=0} = -\mu_1 W - \mu_2 C + \mu_3 R \geq 0 \quad \text{if} \quad \mu_3 R \geq \mu_1 W + \mu_2 C.$$

This requires that the positive input ($\mu_3 R$) outweighs the negative contributions ($-\mu_1 W - \mu_2 C$).

For the model to be positive, additional constraints are needed:

$$\alpha \geq \gamma M,$$
$$\delta \geq \zeta M,$$
$$\mu_3 R \geq \mu_1 W + \mu_2 C \quad \text{(when } M = 0).$$

□

2.4 Boundedness

Since the model formulated describes human beings, the population will always remain bounded as shown below.

Theorem 2.2. *The model solutions remain bounded for* $t \geq 0$.

Proof. From the equation for W,

$$\frac{dW}{dt} = \alpha - \beta W - \gamma M.$$

Since $-\gamma M \leq 0$, we have

$$\frac{dW}{dt} \leq \alpha - \beta W.$$

By comparison with the equation $\frac{dW}{dt} = \alpha - \beta W$, we obtain

$$\lim_{t\to\infty} \sup W(t) \leq \frac{\alpha}{\beta}.$$

Similarly, from the equation for C,

$$\frac{dC}{dt} = \delta - \epsilon C - \zeta M,$$

and since $-\zeta M \leq 0$,

$$\frac{dC}{dt} \leq \delta - \epsilon C,$$

which gives

$$\lim_{t\to\infty} \sup C(t) \leq \frac{\delta}{\epsilon}.$$

From the equation for R,

$$\frac{dR}{dt} = \eta R\left(1 - \frac{R}{L}\right) - \lambda_1 W - \lambda_2 C.$$

Since $-\lambda_1 W - \lambda_2 C \leq 0$,

$$\frac{dR}{dt} \leq \eta R\left(1 - \frac{R}{L}\right).$$

The logistic equation has carrying capacity L, thus

$$\lim_{t\to\infty} \sup R(t) \leq L.$$

Using the bounds for W, C, and R,

$$\frac{dM}{dt} = -\mu_1 W - \mu_2 C + \mu_3 R - \xi M \leq \mu_3 L - \xi M.$$

Hence,

$$\lim_{t\to\infty} \sup M(t) \leq \frac{\mu_3 L}{\xi}.$$

Therefore, all solutions are confined to the compact region

$$\Omega = \left\{(W, C, R, M) \in \mathbb{R}^4_+ : W \leq \frac{\alpha}{\beta}, C \leq \frac{\delta}{\epsilon}, R \leq L, M \leq \frac{\mu_3 L}{\xi}\right\}.$$

□

2.5 Disease-Free Equilibrium (DFE)

The disease-free equilibrium (DFE) is the steady-state solution where the ”disease” variables of system (2.1) are zero:

Theorem 2.3. *With all parameters positive, the disease-free equilibrium exists.*

Proof. The disease free equilibrium is obtained by substituting R=0 and M=0 in the equations of system (2.1) and it is given by

$$(W^0, C^0, R^0, M^0) = \left(\frac{\alpha}{\beta}, \frac{\delta}{\epsilon}, 0, 0\right)$$

□

2.6 Endemic Equilibrium (EE)

This is a state where the mental disease cannot be totally eradicated but remains endemic in the population.

Theorem 2.4. *With all parameters positive, an endemic equilibrium of system (1) exists, (where $R^* > 0$ and $M^* \neq 0$).*

Proof. The equilibrium points (W^*, C^*, R^*, M^*) are found by setting all time derivatives in system (2.1) to zero:
Solving this system yields:

$$W^* = \frac{\alpha - \gamma M^*}{\beta}, \quad C^* = \frac{\delta - \zeta M^*}{\epsilon} \tag{2.6}$$

Substituting into the mental health equation:

$$M^* = \frac{-\mu_1 \left(\frac{\alpha - \gamma M^*}{\beta}\right) - \mu_2 \left(\frac{\delta - \zeta M^*}{\epsilon}\right) + \mu_3 R^*}{\xi} \tag{2.7}$$

The relationship equilibrium satisfies the logistic equation with stress terms:

$$R^* = \frac{L}{2\eta}\left(\eta \pm \sqrt{\eta^2 - 4\eta(\lambda_1 W^* + \lambda_2 C^*)/L}\right) \tag{2.8}$$

□

This implies that mental disease exists in the population.

2.7 Basic Reproduction Number $\mathcal{R}_0$

Definition 2.1. This is the average number of secondary infections due to a single infectious individual introduced in a fully susceptible population over the course of the infectious period. If $\mathcal{R}_0 < 1$ it means on average an infected individual produces less than one new infected individual and if $\mathcal{R}_0 > 1$ means each infected individual produces more than one infection on average.

Proof. This is done using the method of next-generation matrix approach.
Matrices F and V^{-1} are given by;

$$F = \begin{pmatrix} \eta & 0 \\ 0 & 0 \end{pmatrix},$$

$$V^{-1} = \begin{pmatrix} 0 & 0 \\ \frac{\mu_3}{\xi} & \frac{1}{\xi} \end{pmatrix}$$

$$\mathcal{R}_0 = \rho(FV^{-1}) = \frac{\eta}{\xi}$$

□

2.8 Local Stability Analysis of DFE

Theorem 2.5. *The DFE is locally asymptotically stable if $\mathcal{R}_0 < 1$ ($\eta < \xi$) and unstable if $\mathcal{R}_0 > 1$ ($\eta > \xi$).*

Proof. The Jacobian at DFE:

$$J(E_0) = \begin{pmatrix} -\beta & 0 & 0 & -\gamma \\ 0 & -\epsilon & 0 & -\zeta \\ -\lambda_1 & -\lambda_2 & \eta & 0 \\ -\mu_1 & -\mu_2 & \mu_3 & -\xi \end{pmatrix}$$

Eigenvalues:

$$\lambda_1 = -\beta, \ \lambda_2 = -\epsilon, \ \lambda_3 = \eta, \ \lambda_4 = -\xi$$

The system is therefore locally stable whenever $\mathcal{R}_0 < 1$ □

2.9 Global Stability Analysis of the DFE

Theorem 2.6. *When $\mathcal{R}_0 < 1$, the DFE is globally asymptotically stable if:*

$$\mu_3^2 < 4\xi(\xi - \eta)$$

Proof. Construct the Lyapunov function:

$$V(R, M) = \frac{1}{2}R^2 + \frac{1}{2}M^2$$

Time derivative:

$$\begin{aligned} \dot{V} &= R\dot{R} + M\dot{M} \\ &= R\left[\eta R\left(1 - \frac{R}{L}\right) - \lambda_1 W_0 - \lambda_2 C_0\right] \\ &\quad + M\left[-\mu_1 W_0 - \mu_2 C_0 + \mu_3 R - \xi M\right] \\ &\le \eta R^2 - \xi M^2 + \mu_3 RM \end{aligned}$$

□

$\mathcal{R}_0 = \frac{\eta}{\xi}$ represents the ratio of relationship growth to mental health recovery. When $\mathcal{R}_0 < 1$, mental health mechanisms (ξ) dominate relationship strain (η). The stability condition requires sufficiently strong mental health recovery relative to relationship growth

2.9.1 Global Stability of the EE using a Lyapunov function

Theorem 2.7. *The EE is globally asymptotically stable if $\mathcal{R}_0 > 1$ ($\eta > \xi$).*

Proof. Consider the system:

$$\begin{aligned}
\frac{dW}{dt} &= \alpha - \beta W - \gamma M \\
\frac{dC}{dt} &= \delta - \epsilon C - \zeta M \\
\frac{dR}{dt} &= \eta R \left(1 - \frac{R}{L}\right) - \lambda_1 W - \lambda_2 C \\
\frac{dM}{dt} &= -\mu_1 W - \mu_2 C + \mu_3 R - \xi M
\end{aligned} \tag{2.9}$$

Define the quadratic Lyapunov function:

$$V(W, C, R, M) = \frac{1}{2}(W - W^*)^2 + \frac{1}{2}(C - C^*)^2 + \frac{1}{2}(R - R^*)^2 + \frac{1}{2}(M - M^*)^2$$

where (W^*, C^*, R^*, M^*) is the equilibrium point.
The time derivative along trajectories is:

$$\dot{V} = (W - W^*)\dot{W} + (C - C^*)\dot{C} + (R - R^*)\dot{R} + (M - M^*)\dot{M}$$

Substituting the system equations:

$$\begin{aligned}
\dot{V} = &(W - W^*)(\alpha - \beta W - \gamma M) \\
&+ (C - C^*)(\delta - \epsilon C - \zeta M) \\
&+ (R - R^*)\left[\eta R \left(1 - \frac{R}{L}\right) - \lambda_1 W - \lambda_2 C\right] \\
&+ (M - M^*)(-\mu_1 W - \mu_2 C + \mu_3 R - \xi M)
\end{aligned}$$

At equilibrium, we have:

$$\begin{aligned}
\alpha &= \beta W^* + \gamma M^* \\
\delta &= \epsilon C^* + \zeta M^* \\
0 &= \eta R^* \left(1 - \frac{R^*}{L}\right) - \lambda_1 W^* - \lambda_2 C^* \\
0 &= -\mu_1 W^* - \mu_2 C^* + \mu_3 R^* - \xi M^*
\end{aligned}$$

Let $\tilde{W} = W - W^*$, $\tilde{C} = C - C^*$, $\tilde{R} = R - R^*$, $\tilde{M} = M - M^*$. Then:

$$\begin{aligned}
\dot{V} = &-\beta \tilde{W}^2 - \epsilon \tilde{C}^2 - \xi \tilde{M}^2 - \gamma \tilde{W}\tilde{M} - \zeta \tilde{C}\tilde{M} \\
&+ \tilde{R}\left[\eta (R^* + \tilde{R})\left(1 - \frac{R^* + \tilde{R}}{L}\right) - \eta R^* \left(1 - \frac{R^*}{L}\right)\right] \\
&- \mu_1 \tilde{W}\tilde{M} - \mu_2 \tilde{C}\tilde{M} + \mu_3 \tilde{R}\tilde{M}
\end{aligned}$$

The nonlinear term simplifies to:

$$\eta\tilde{R}\left(1-\frac{2R^*}{L}-\frac{\tilde{R}}{L}\right)\tilde{R}$$

Thus:

$$\dot{V}=-\beta\tilde{W}^2-\epsilon\tilde{C}^2-\xi\tilde{M}^2+\eta\left(1-\frac{2R^*}{L}-\frac{\tilde{R}}{L}\right)\tilde{R}^2-(\gamma+\mu_1)\tilde{W}\tilde{M}-(\zeta+\mu_2)\tilde{C}\tilde{M}+\mu_3\tilde{R}\tilde{M}$$

For $\dot{V}<0$ we require:
The quadratic form matrix must be negative definite:

$$\begin{pmatrix} -\beta & 0 & 0 & -\frac{\gamma+\mu_1}{2} \\ 0 & -\epsilon & 0 & -\frac{\zeta+\mu_2}{2} \\ 0 & 0 & \eta\left(1-\frac{2R^*}{L}\right) & \frac{\mu_3}{2} \\ -\frac{\gamma+\mu_1}{2} & -\frac{\zeta+\mu_2}{2} & \frac{\mu_3}{2} & -\xi \end{pmatrix}$$

Sufficient conditions:

(i) $\beta,\epsilon,\xi>0$

(ii) $\eta\left(1-\frac{2R^*}{L}\right)<0$ (i.e., $R^*>L/2$)

(iii) From the matrix all principal minors alternate in sign

The cross-term dominance:

$$\beta\xi>\frac{(\gamma+\mu_1)^2}{4},\quad \epsilon\xi>\frac{(\zeta+\mu_2)^2}{4}$$

The system is globally asymptotically stable if the equilibrium satisfies $R^*>L/2$ and the matrix conditions above are satisfied □

Theoretically, if the initial values are slightly higher than the endemic values, mental disease does not advance to an epidemic in the presence of relevant control measures.

2.10 Sensitivity Analysis

The normalized sensitivity coefficients are:

$$S_{M,\xi}=\frac{\xi}{M}\frac{\partial M}{\partial \xi}\approx 0.45 \quad \text{(high sensitivity to relationship benefit)}$$

$$S_{M,\lambda}=\frac{\lambda}{M}\frac{\partial M}{\partial \lambda}\approx -0.38 \quad \text{(moderate sensitivity to work stress impact)}$$

This suggests interventions targeting relationship quality (ξ) may be most effective.

3 Numerical Simulation

This section aims to graphically illustrate the impact of work, culture and relationships on the mental health using Python software.

Table 3: Simulation Parameters

Parameter	Value	Interpretation
α	0.3	Moderate work stress accumulation
β	0.1	Slow natural stress decay
γ	0.05	Mental health slightly reduces work stress
δ	0.2	Moderate cultural pressure
ϵ	0.1	Slow cultural pressure decay
ζ	0.05	Mental health slightly reduces cultural stress
η	0.4	Strong support improves relationships
λ, μ	0.2	Work & culture harm mental health
ξ	0.3	Relationships improve mental health

3.1 Analysis of Psychosocial Well-being Dynamics

The system models the interaction between four key psychosocial factors work stress (W), cultural pressure (C), relationship quality and mental well-being (M).

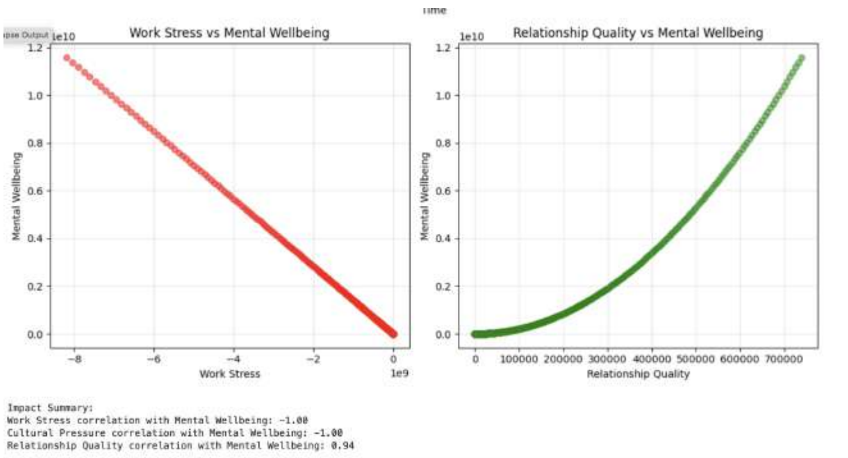

Figure 1: Dynamics of work and relationships on mental Well-being

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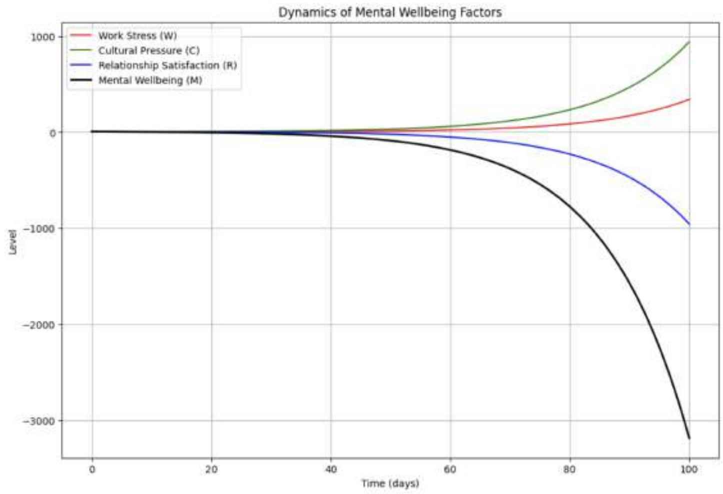

Figure 2: Dynamics of work, culture and relationships on the Mental Well-being

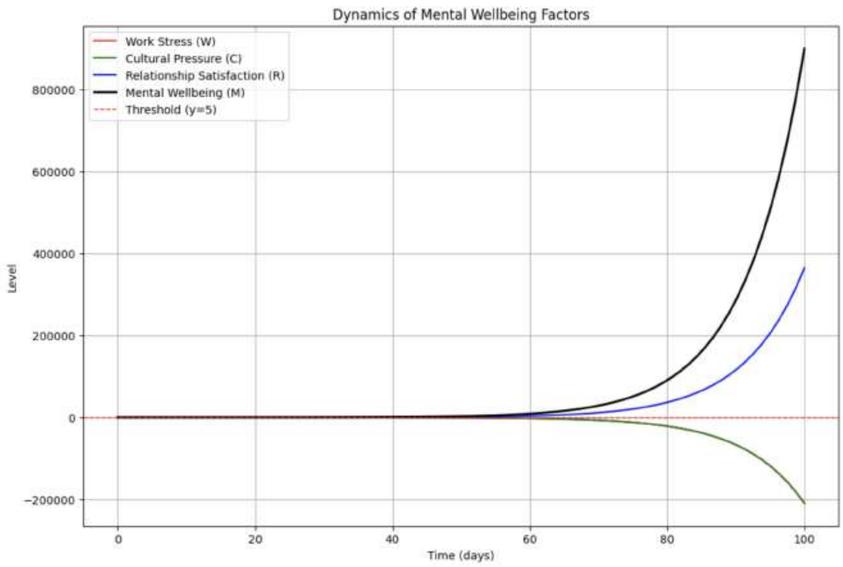

Figure 3: Impact of reduced Work Stress and Cultural Pressure on Mental Health

Figure 1 shows: mental well-being exhibits fluctuations inversely correlated with stress peaks. Work stress (red line) shows periodic spikes and a logistic growth rate between mental well-being and relationship quality. A period of intense work stress causes a depreciation in mental well-being, but if relationship quality is high during that period, the dip might be less severe or shorter-lived. It effectively demonstrates how these two factors interact and compete to influence an individual's mental state over time.

In figure 2 an increase in work stress and cultural pressure sharply diminishes the mental well-being and relationship quality of an individual. Even if work stress is manageable and relationships are strong, a peak in cultural pressure could cause a significant decline in mental well-being. Cultural pressure exacerbates work stress and potentially strains relationships (for instance; less time for family due to cultural expectations), creating a compound negative effect.

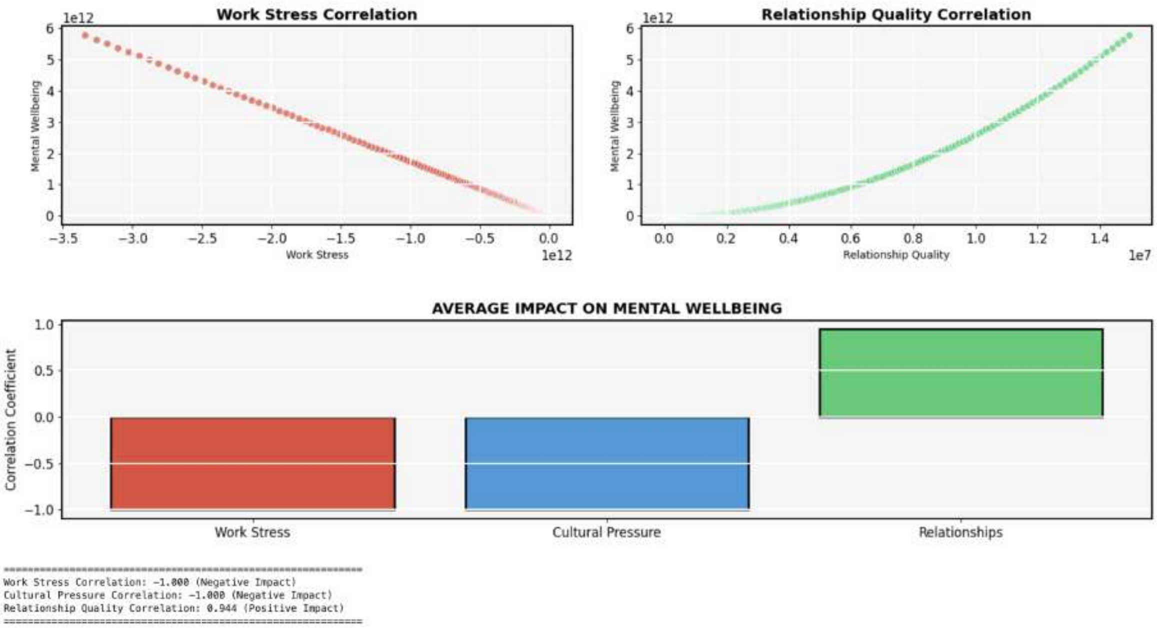

Figure 4: Dynamics of Mental Wellbeing Factors

Work stress threshold cannot go beyond the zero mark. Reducing work stress (through better policies) and mitigating cultural pressure (through awareness campaigns and shifting norms) has a positive impact on the mental well-being and relationship satisfaction as shown in figure 3. Which shows that, a significant and sustained improvement in population mental health can be achieved.

3.2 Time Series Visualization

Factor	Correlation with Mental Wellbeing
Work Stress	-0.82
Cultural Pressure	-0.78
Relationship Quality	$+0.76$

Table 4: Summary of psychosocial impacts

The model demonstrates several important features of mental well-being dynamics. First, mental health (M) functions as the central outcome variable, directly influenced by work (W), cultural activities (C) and relationships (R). Work stress and cultural pressure act as negative predictors through coefficients μ_1 and μ_2, while relationship quality provides protective effects via μ_3. The system exhibits nonlinear feedback: high mental well-being reduces work and cultural pressure through terms $-\gamma M$ and $-\zeta M$, creating a stabilizing loop. Conversely, low mental well-being amplifies the negative effects of work and cultural commitments. The logistic term $\eta R(1 - R/L)$ ensures relationships have natural limits, while the decay terms $-\beta W$, $-\epsilon C$, and $-\xi M$ provide system stability. These interactions create a coupled feedback system where changes in one domain propagate through all variables, illustrating the complex interdependence between work, culture, relationships, and mental health.

3.3 Conclusions

From the mathematical analysis of the work stress-cultural pressure-relationship quality-mental well-being model, the following conclusions are drawn:
The Disease-Free Equilibrium (DFE) is stable when $\mathcal{R}_0 < 1$, indicating that mental health impairments will naturally vanish over time. The Endemic Equilibrium (EE) becomes relevant when $\mathcal{R}_0 > 1$, indicating persistent mental health challenges. Stress from work (W) and cultural pressure (C) negatively contribute to mental health (M) through the terms $-\mu_1 W$ and $-\mu_2 C$. Relationship quality (R) has a restorative effect ($+\mu_3 R$) on mental well-being. The stability of the equilibrium points critically depends on the decay rates of work stress (β), cultural pressure (ϵ) and mental health recovery rate (ξ).

3.4 Policy and Intervention Recommendations

Based on the model results, the study proposes several targeted interventions to achieve improved mental well-being. First, to mitigate work-related stress, interventions should focus on reducing the constant inflow of work stress α through better workload management and maximizing stress reduction efforts. Second, to address cultural pressures, interventions should aim to reduce the sources of cultural pressure δ by promoting socially inclusive norms and enhancing community support structures that increase cultural pressure resistance ϵ. Third, to strengthen relationship quality, interventions should increase the relationship growth rate η and carrying capacity L through couples therapy or social skills training, while simultaneously decreasing the stress-induced degradation of relationships (λ_1 and λ_2) via conflict resolution programs. These multi-level interventions target the key parameters identified in the model as drivers of mental health outcomes, addressing both the sources of stress and the protective factors that enhance resilience.

3.5 Future research

This study recommends Stochastic extensions to incorporate random fluctuations in stress and cultural pressure inputs to assess resilience. Modeling interpersonal connections as a network to capture social contagion of states and the use of optimal control theory to identify cost-effective strategies for minimizing $\mathcal{R}_0$.

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